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Specimen Size and Scale Effects of Direct Shear Box Tests of Sands

ABSTRACT: The direct shear test has survived over the past 50 years in geotechnical engineering applications because of its simplicity and repeatability. Many laboratories perform direct shear box tests on sands to determine the friction angle ϕ', or shear strength of the sand for engineering design purposes. However, there are different size shear boxes in use today and the effect of the varying specimen size on the resulting friction angle used in foundation deign has never before been investigated thoroughly. Five sands with different properties were tested in three square shear boxes of varying sizes (60 mm, 101.6 mm, and 304.8 mm), each at three relative densities (dense, medium, and loose). Results of the direct shear tests show that the friction angle ϕ' can be dependent on specimen size and that the influence of specimen size is also a function of sand type and relative density. The tests indicate that for well-graded, angular sands, ϕ' decreases as box size increases and that the influence of box size is dependent on relative density. The paper provides a description of the test methods and presents the test results.

KEYWORDS: Direct shear box, friction angle, sand, Mohr-Coulomb, scale effects

Introduction

Scale effects of shallow foundation bearing capacity on granular material have been studied for many years and modifications to the bearing capacity to account for this observed scale effect have been suggested (Habib 1974; Shiraishi 1990). This observed scale effect shows that the bearing capacity factor N_γ, is a function of footing width B, in addition to the friction angle ϕ' and unit weight γ of the soil. However, very little explanation has been offered as to why this scale effect occurs. The fundamentals of the scale effect can be seen when the third term of Terzaghi's (1943) well known general superposition equation for the ultimate bearing capacity of a rough, strip footing ($q_{\text{ult}}=cN_c+qN_q+0.5\gamma BN_\gamma$) is rearranged to back calculate N_γ. In theory, this equation suggests that the value of N_γ should be constant with footing width. As the footing width increases, so does the ultimate capacity, and therefore, N_γ remains constant. However, as can be seen from the model scale and prototype scale centrifuge strip footing test results presented by Kimura et al. (1982) as shown in Fig. 1, the value of N_γ is not constant, but rather decreases as footing width increases.

Throughout the literature, it has been shown that N_γ appears to have a scale effect with footing width. If N_γ is theoretically related only to the unit weight γ and friction angle ϕ' of the soil, it is possible that the actual scale effect may be, at least in part, related to the confining stress felt underneath a footing, i.e., the larger the footing, the higher the mean confining stress at the same depth underneath footings of different sizes if the footing pressure is the same for all footings considered. This makes sense because a large footing has a deeper zone of influence than a small footing.

An example of this was shown from circular rigid plate tests performed on sand ($\phi=31.5°$) with diameters of 24, 45, and 63 cm (after Koegler and Sheidig 1938, from Harr 1977). The dissipation of the contact stresses with depth was measured and isobars were drawn to show the stresses beneath a rigid circular plate. The contact stresses at depth were recorded as a percentage of the total applied force divided by the area of the rigid circular plate. The stress distribution was measured to be 2.26p, 2.25p, and 1.8p if the stress was defined at a relative depth of 10 % of the footing width. This interpolation shows that the stresses decrease with increasing footing width when stresses are calculated at the same relative depth. According to the Koegler and Sheidig (1938) study, the actual bearing or contact pressure measured at the same relative depth is lower for bigger footings, meaning then, that the friction angle would be lower.

Another possible explanation of the scale effects observed in small-scale footing tests is related to the assumed shear zone formation in the active region directly beneath the footing. The propagation of the shear zone in a small footing may not be fully developed because of the geometry of the active region, resulting in a higher friction angle ϕ. Shear bands are well defined narrow regions of intensely sheared material in which significant decreases

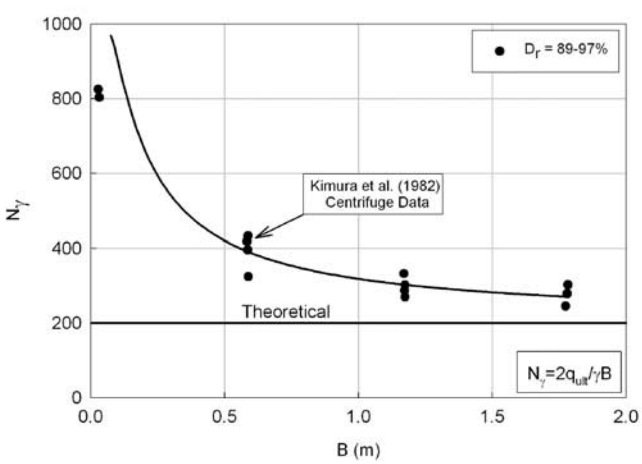

FIG. 1—*Theoretical versus experimental results of N_γ and footing width B.*

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in density occur (Scarpelli and Wood 1982; Vermeer 1990). These shear bands combine to form an overall mechanism called the shear zone. The actual propagation of the shear zone is affected by the relative density, confining pressure and grain size distribution of the soil (Saada et al. 1999).

Granular material deformation and shear zone formation have been studied experimentally in triaxial, biaxial and direct shear tests (i.e., Finno et al. 1997; Saada et al. 1999; Alshibli and Sture 2000; Karakouzian et al. 2001; Lade 2003), however, shear zone formation underneath footings has not been experimentally tested. Visual studies and interpretation of shear zone formation underneath rough, surface model-scale footings in order to determine the effect on scale will be studied in the near future. However, an established laboratory test that could possibly simulate the shear zone propagation behavior of soil underneath various size footings in the field was found to be the direct shear box test. Direct shear tests are relatively simple to perform and easily modified in terms of aspect ratio (H/L) and box width. Direct shear box tests were performed using three different size boxes with different sands to determine if the friction angle was affected by specimen size. Varying the specimen size was intended to mimic the different size footings which have shown a definite scale effect (Cerato 2005). If the friction angle determined from the shear box was affected by the size of the shear box then it was possible that the results could be related to the observed scale effect of the bearing capacity factor N_γ and footing width B.

Apart from gaining a better understanding of footing scale effect on granular material, the specimen size effect of direct shear testing also served a second purpose: to determine whether the current practice of using small shear boxes to find friction angles used in foundation design is appropriate. The size of the direct shear test specimen can affect the resulting friction angle. Two previous studies (Parsons 1936; Palmeira and Milligan 1989) tested a few sands in different size shear boxes but found different results. Parsons (1936) results of Ottawa and crushed quartz showed a slight friction angle decrease with increasing box size, but Palmeira and Milligan's (1989) results using Leighton Buzzard Sand showed no difference in friction angle results with increasing shear box size. It is the purpose of this study to determine the scale effects seen in different size shear boxes using a range of sands at varying densities in order to validate the observed scale effects in small scale footing tests and to show that current strength testing procedures in small direct shear boxes may overestimate the friction angle of a soil by a significant amount depending on the sand type and relative density.

Background

Specimen size or scale effects in direct shear testing were shown as early as 1936. Parsons (1936) presented test results for a crushed quartz and a clean uniform sand which showed that a larger shear box produced lower values of friction angle. Using three different size shear boxes (60×60 mm, 100×120 mm, and 120 mm ×200 mm), with soils prepared in a loose state, tests showed that the friction angles decreased slightly with increasing box size. The rectangular boxes were sheared across the 120 mm surface. Friction angles obtained for the crushed quartz varied from 30.7° to 31.5° (Fig. 2) and the uniform sand varied slightly more from 28.5° to 31.0°. Both sets of data are presented in Table 1. Very small normal stresses were applied to obtain the failure envelopes (0.015 to 0.1 kg/cm^2 or 1.5 to 9.8 kPa).

In a more recent study that appears to give conflicting results,

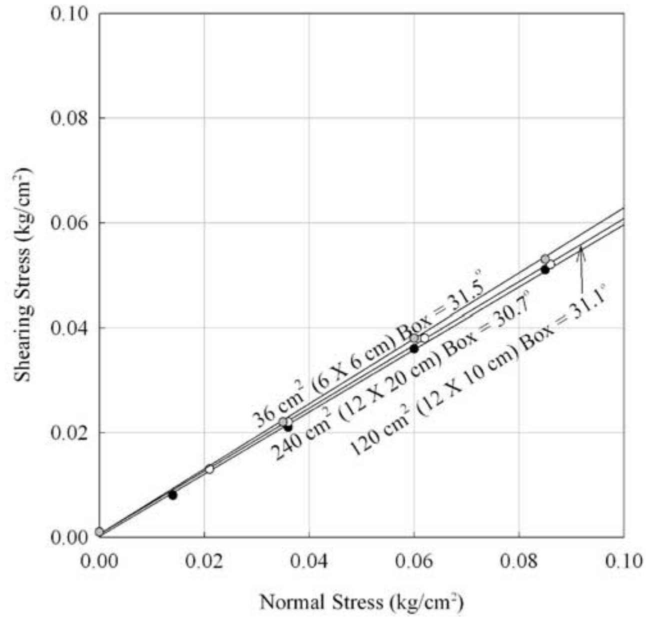

FIG. 2—*Influence of specimen size on friction angle of crushed quartz* (*after Parsons 1936*).

Palmeira and Milligan (1989) performed tests on dense Leighton Buzzard Sand 14/25 (particle size 0.6–1.2 mm; D_{50}=0.8 mm, D_r =87 %) in small, medium, and large shear boxes and found that there were no significant influence of box size on the resulting friction angles (Table 2). The smallest and largest size boxes were square. The middle sized box was rectangular and no information was given as to what direction the box was sheared. Therefore, it is unclear how large the shearing surface actually was for that particular rectangular box. They did find, however, that the thickness of the shear zone at the sample mid-height was significantly affected by the scale of the test as well as the post-peak behavior presented by the sample.

TABLE 1—*Effect of shear box size of crushed quartz and ottawa sand* (*after Parsons 1936*).

Shear Box Dimensions L (mm)×W (mm)	Crushed Quartz Friction Angle Degree	Ottawa Sand Friction Angle Degree
60×60	31.5	31.0
120×100	31.1	29.6
120×200	30.7	28.5

TABLE 2—*Effect of shear box size on shear strength of dense Leighton Buzzard Sand* (*from Palmeira and Milligan 1989*).

Shear Box	Dimensions L (mm)×W (mm)×H (mm)	t=Shear Zone Thickness at Mid-Height (mm)	ϕ' (°)
Small	60×60×32	9±2	50.1
Medium	252×152×152	18±2	50.2
Large	1000×1000×1000	100±10	49.4

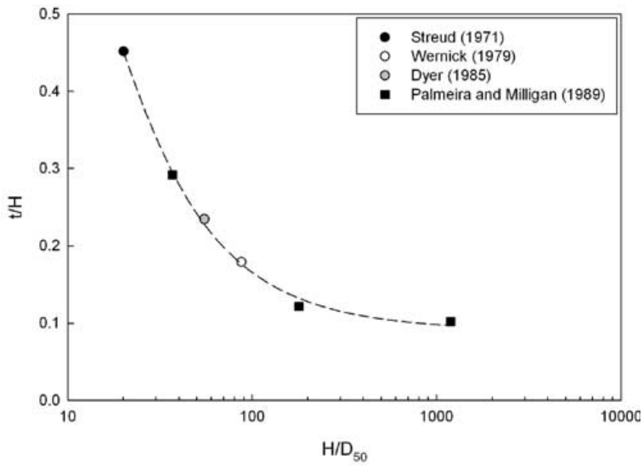

FIG. 3—*Influence of scale on shear zone thickness* (*after Palmeira and Milligan 1989*).

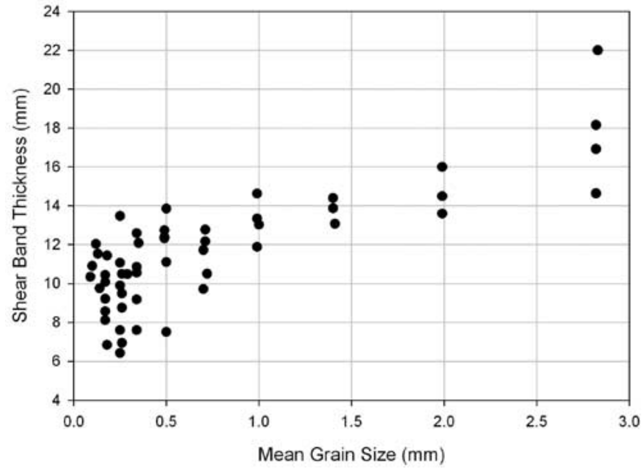

FIG. 4—*Shear zone thickness as a function of mean grain size* (*after DeJaeger 1994*).

Influence of Specimen Size on Shear Zone Thickness

The small layer of sand that is involved in the shearing process (shear zone) has been shown to be significantly affected by the scale of the test shear (Palmeira and Milligan 1989). The shear zone encompasses many shear bands that propagate from the edges of the shear box. The overall mechanism of localization is much larger than individual shear bands, therefore the term "shear zone" is used instead of "shear band" in describing the area where localization occurs. The study obtained the shear zone thickness by visual inspection through the Perspex wall of the shear boxes and by thin wires placed vertically inside the sample crossing the central plane. The effect of the scale of the test on the shear zone thickness is presented in Fig. 3, which is a compilation of test data from their study as well as from the literature (compiled by Palmeira and Milligan 1989). The ratio between the shear zone thickness and the sample height (t/H) is plotted against the sample height to mean particle diameter ratio (H/D_{50}) for several tests with Leighton Buzzard Sand. The data was originally presented this way to show the increase in the thickness of the shear zone in relation to sample height when the size of the sample decreases in relation to the soil particle size. For Leighton Buzzard Sand in these tests, as the sample height increases, the thickness of the shear zone increases. Historically, the shear zone (made up of multiple shear bands) has been defined as a sole function of the mean grain size, d_{50} (Roscoe 1970; Bridgwater 1980; Scarpelli and Wood 1982; Hartley 1982; Mühlhaus and Vardoulakis 1987; DeJong et al. 2003). If this were true, then theoretically, the same sand in different size boxes would have the same size shear zone.

Roscoe (1970) reported that the thickness of shear zones observed in simple shear tests is about $10d_{50}$. This was confirmed theoretically by Bridgwater (1980) using a statistical mechanics approach. Scarpelli and Wood (1982) measured the width of the shear zone of Leighton Buzzard Sand using radiographs. They found that the ratio of shear zone thickness to average grain size was not constant along the length of the shear zone and reached a value of $10d_{50}$ or greater only where dilation had ceased and constant volume shearing was proceeding. Similar ratios of shear zone thickness width to particle size for fully developed shear zones have been found by Hartley (1982) in tests on three different sands (average grain sizes of 0.12, 0.40, and 0.90 mm). When the boundary conditions of the test allow a shear zone the freedom to develop, its tip is typically 5 to 10 times the average grain size. Mühlhaus and Vardoulakis (1987) determined from a plasticity analysis using Cosserat theory that a typical estimate of shear zone thickness was sixteen particle diameters.

Another study, which took into account the mean grain size when determining the shear zone thickness was presented by DeJaeger (1978 and 1991), who theoretically computed the critical void ratio and the minimum thickness disturbed by shear. The results show that a shear zone has a minimum thickness of 5 to 10 mm for fine sand and to 20 mm for a coarse one (Fig. 4). The shear zone thickness increases with angularity and also with grain size. The shear zone thickness is a significant factor which is correlated with the strain necessary to obtain the failure (DeJaeger 1994; Kita and Okamura 2000). The above-cited studies on shear zone thicknesses all used similarly dimensioned shear boxes, which may be a reason for the similar values of shear zone thickness obtained. The only study that used a significantly larger shear box was Palmeira and Milligan (1989) and the results from that study showed a considerable increase in shear zone thickness (100 mm).

Hight and Leroueil (2003) presented data from tests performed on dense Leighton Buzzard Sand in different size shear boxes. Instead of varying the width of the box, they used a 254 mm square box and varied the height of the specimen between 50.8 and 154.9 mm. They presented their results as the ratio of shear stress to normal stress versus the aspect ratio, H/L, and showed that as the aspect ratio increased (the height increased relative to the width), the ratio of the shear stress to the normal stress increased (Fig. 5). The increase of the height of the shear box influences the stress distribution of the specimen shear plane during the test which increases the normal stress at the top of the box and reduces it at the bottom in a more pronounced way than what would occur in standard direct shear boxes. This nonuniform stress distribution is a consequence of the moment of the shear force applied to the upper half of the box which is transferred to the soil specimen by the internal wall of the upper half of the box. This may be one reason for the influences of the box aspect ratio on friction angles obtained in Hight and Leroueil's (2003) study. Hight and Leroueil's (2003) study show that it is important to take the aspect ratio into account when performing direct shear boxes and understand that varying this parameter may result in different values of friction angle.

As much as it is important to keep the height of the box large enough in relationship to mean grain size for a shear zone to form,

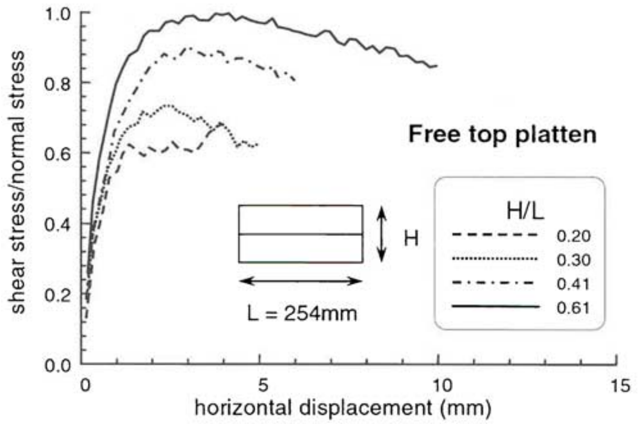

FIG. 5—*Influence of aspect ratio on the determination of friction angle (from Hight and Leroueil 2003).*

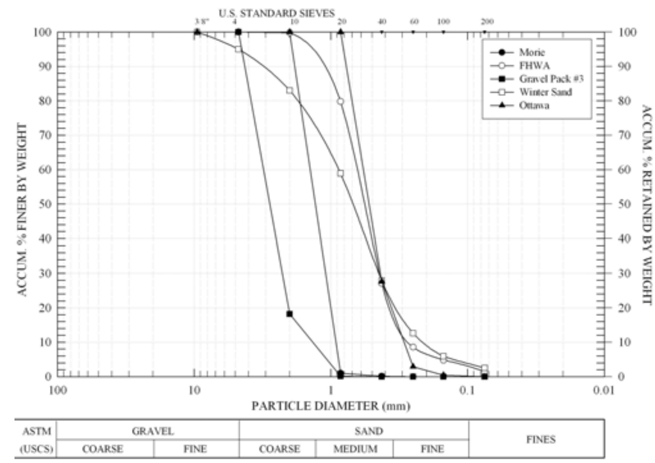

FIG. 6—*Grain-size distribution for the sands used in this study.*

it is also important to shear sands in boxes with adequate length, for full propagation of the shear zone. There also appears to be an end effect in direct shear boxes. Scarpelli and Wood (1982) suggest an order of magnitude of $100d_{50}$, from tests performed in long shear boxes. Stone and Muir Wood (1992) suggest a shear box length of about $176d_{50}$, determined from rotating blade experiments.

ASTM D 3080-90, Standard Test Method for *Direct Shear Tests of Soils Under Consolidated Drained Conditions* requires a minimum specimen thickness of six times the maximum particle diameter and a minimum specimen width of 10 times the maximum particle diameter in determining what size shear box should be used for testing sands. A minimum specimen width to thickness ratio 2:1 is required. Tests conducted by Hight and Leroueil's (2003) tests all meet this criteria, but the tests still result in varying friction angles for dense Leighton Buzzard Sand. These results indicate that perhaps the minimum specimen width to thickness ratio specified by ASTM Standard D 3080-90 should be increased.

Summary of Previous Work

A few researchers investigated the influence of specimen size in shear box testing on the shear zone thickness and resulting friction angle. It was seen that length, width, and height (aspect ratio) of the shear box affected both the shear zone thickness and the resulting friction angle. The early work on specimen size and scale effect in direct shear box tests utilized both square and rectangular boxes in the same test series, which, although showed decreasing friction angle with increasing box size, did not account for what effect the shape of the box may have had on the results. Another question mark in the existing data is the extremely low normal stress range utilized by Parsons (1936). Accuracy and repeatability of results using such low normal stresses in conventional direct shear machines would be unlikely.

Investigation

Since the scale effect of shallow foundation bearing capacity on granular material may be explained by the differing confining stresses, and therefore, different friction angles found beneath different size foundations, a research program testing the friction angles in different size direct shear boxes was implemented to study scale effect of friction angles as a function of specimen size and to relate the results to the scale effect found with N_γ and footing width B. Five sands with different properties were used. Three of the sands were clean sands with significantly different values of d_{50}. The other two sands were natural sands found in the Connecticut Valley and McLean, Virginia.

Sand Properties

The sands used in this investigation were characterized using specific gravity, maximum and minimum density, grain-size, and two-dimensional (2D) image analysis tests. Results from these tests are summarized in Table 3 and Fig. 6. Specific-gravity tests were performed in general accordance with ASTM D 854-92 Standard Test Method for *Specific Gravity of Soils*. Maximum and minimum index density tests were performed in general accordance with ASTM D 4253-93, Standard Test Method for *Maximum Index Density and Unit Weight of Soils Using a Vibratory Table* and ASTM D

TABLE 3—*Sand characteristics.*

Soil Name	G Mg/m^3	ρ_{min} Mg/m^3	ρ_{max} Mg/m^3	e_{min}	e_{max}	d_{10} (mm)	d_{50} (mm)	d_{max} (mm)	C_c	C_u	Percent fines	USCS
Brown Mortar	2.69	1.41	1.70	0.58	0.91	0.3	0.6	1.7	1.1	2.1	2.0	SW
Winter Sand	2.69	1.61	1.96	0.37	0.67	0.2	0.7	5	1.0	4.5	2.5	SW
Ottawa	2.62	1.55	1.80	0.46	0.69	0.3	0.5	0.9	1.0	1.9	0	SP
Morie Sand	2.62	1.47	1.65	0.59	0.78	0.9	1.3	2	1.0	1.8	0	SP
Gravel Pack #3	2.61	1.43	1.56	0.67	0.83	1.5	2.8	5	0.3	2.1	0	SP

FIG. 7—*Magnified pictures of the sand grains:* (*a.*) *Brown Mortar,* (*b.*) *Winter,* (*c.*) *GP#3,* (*d.*) *Morie, and* (*e.*) *Ottawa.*

4254-93, Standard Test Method for *Minimum Index Density and Unit Weight of Soils and Calculation of Relative Density*, respectively. Grain-size analyses were performed in general accordance with ASTM D 422-63 Standard Test Method for *Particle-Size Analysis of Soils*. These results indicate that both the Brown Mortar and Winter Sands are well-graded, while the other three sands are considered clean, poorly graded sands.

2D Digital Image Analysis

The importance of grain shape and size in the mechanical behavior of sands is well known and has been documented in numerous studies (Kolbuszewski 1948; Winterkorn and Fang 1975; Zelasko et al. 1975; Kuo and Freeman 2000; Kato et al. 2003, etc.). Definitions that classify the sand particle shape into categories of sphericity and angularity have been presented and are used by engineers and geologists to determine maximum packing or the degree to "which individual particles are in contact with or interlocked with their neighbors" (Krumbein and Sloss 1963). Knowing the average sand shape helps explain shear strength tendencies whether underneath a shallow footing or in a shear box.

The sands in this study were analyzed using digital image analysis techniques that treated the particles as two-dimensional objects (Fig. 7). A Nikon COOLPIX Digital Camera attached to a Scienscope optical microscope was used to capture the digital image. Thirty grains from each sand type were analyzed to ensure representative values were reported. The digital pictures were loaded onto a computer and opened into Adobe Photoshop, a photo editing software. Once the image size and scale were related by pixel count and converted to the correct format, the pictures were loaded into commercially available software, Scion Image. This program was set to measure individual grains for perimeter length, area, and major and minor axes.

The four grain-shape parameter results from the digital analysis are presented in Table 4. These results are average values of thirty tests on each sand type.

Ottawa and Brown Mortar Sand are classified as rounded, while Morie and Winter Sand are classified as subangular. GP#3 is classified as very angular, indicative of manufactured sand with rough edges. The digital image analysis technique used in this study was helpful in determining average shape parameters for all five sands. These parameters were useful in explaining shear behavior and interlocking tendencies.

TABLE 4—*Mean particle shape analysis for five sands.*

Sand Type	Form Factor	Angularity	Aspect Ratio	Roundness
Brown Mortar	0.75	1.34	1.32	0.79
Winter	0.69	1.45	1.33	0.34
Ottawa	0.85	1.18	1.29	0.63
Morie	0.75	1.34	1.34	0.24
GP#3	0.73	1.37	1.37	0.09

Direct Shear Box Tests

Direct shear box tests were performed on each soil in general accordance with ASTM D 3080-90, Standard Test Method for *Direct Shear Tests of Soils Under Consolidated Drained Conditions* for three different relative densities. According to ASTM D 3080-90, the direct shear box test has several particle-size to box-size requirements when preparing specimens for testing. It is recommended that the minimum specimen width should not be less than ten times the maximum particle-size diameter and the minimum initial specimen thickness should not be less than six times the maximum particle diameter. The minimum specimen width-to-thickness ratio should be 2 to 1. Other works in the literature are much stricter on the particle-size to box-size requirement. For instance, Jewell and Wroth (1987) suggest a ratio of shear box length to average particle size in the range of 50 to 300.

Three different size shear boxes were used. The first square shear box had a width of 60 mm and a depth of 26.4 mm (aspect ratio, $H/L=0.44$). The samples were sheared at a constant rate of 0.25 mm/min, which is consistent with the standard rate for drained tests on sands. Tests were conducted in a water bath with the sample completely submerged in water to assure that the samples had no cohesion. This was identical to the testing protocol of the surface footing tests. The thickness of the 60 mm shear box did not meet the ASTM minimum criteria of six times the maximum particle diameter, or 30 mm (maximum particle size was 5 mm), for the Winter Sand or Gravel Pack #3. The second square shear box had a width of 101.6 mm and a total depth of 40.64 mm ($H/L=0.40$). In the two smaller boxes, five tests were performed for each density with five increasing normal stresses (38, 68, 95, 122, and 150 kPa) using a dead-weight system. The third square shear box had a width of 304.8 mm and a depth of 177.8 mm ($H/L=0.58$). Five shear tests were performed for each density at five varying normal stresses (69, 103, 138, 172, and 207 kPa) for each relative density. These normal stresses in the larger box are slightly different than the normal stresses used in the smaller two boxes because the hydraulic arm of the 304.8 mm shear box could not apply a normal stress less than 69 kPa.

The sample height H and width W to maximum grain size D_{max} ratios for the sands of this study are summarized in Table 5. Tests conducted on GP #3 and Winter Sand using the smallest shear box (60 mm) did not meet the minimum specifications of ASTM D 3080-90. Four of the five sands did not meet Jewell and Wroth's (1987) minimum particle-size to box-size ratio of 50 in the 60 mm shear box and the Winter and GP#3 sands fell below their specifications of W/D_{max} in the 101.6 mm box.

TABLE 5—*Summary of the minimum box size requirements in relationship to maximum particle diameter.*

Soil Name	D_{max} (mm)	60 mm Box H/L=0.40		101.6 mm Box H/L=0.40		304.8 mm Box H/L=0.58	
		H/D_{max}	W/D_{max}	H/D_{max}	W/D_{max}	H/D_{max}	W/D_{max}
Brown Mortar	1.7	16	36	60	60	105	179
Winter	5.0	5[a]	12	20	20	36	61
Ottawa	0.9	31	72	119	119	209	359
Morie	2.0	13	30	51	51	89	152
GP#3	5.0	5[a]	12	20	20	36	61

[a]Indicates test does not meet minimum specifications of ASTM 3080 Standard.

Method of Sand Deposition

In this study, two sands were prepared by moist compaction, two sands were prepared by dry pluviation and the Gravel Pack was prepared by dry compaction. Winter Sand and Brown Mortar Sand were moist compacted because previous testing programs (Drury 1999; Mitchell 2000) used sand beds for prototype scale footing tests using this method and it was continued so that results could be compared. Ottawa and Morie sands were dry pluviated and the Gravel Pack #3 was spooned and tamped to ensure a uniform test specimen (Table 6).

The specimens in the two small shear boxes were pluviated using a stationary pluviator with a constant fall height of 36.8 cm. This height was chosen because (1) a large range of relative densities could be achieved with this fall height (25 to 100 %), (2) constant relative densities were achieved at one particular angle and mesh setting, and (3) the relative density was consistent throughout the thickness of the sample specimen. This fall height was 14 and 9 times greater than the shear box thicknesses respectively. These two pluviators worked on the premise of three identical circular openings located at each point of an equilateral triangle on two separate plates. The plates could be rotated up to 60°, which, depending on the angle chosen, controlled the amount of sand that was pluviated. The plates rotated from 0 (the closed position) to 60° (all three circular openings matched achieving maximum porosity). Below the plates, different sized metal meshes were inserted as diffusion screens to help achieve the desired relative density. Each sand had to be calibrated on each size box; however, the individual particles in Gravel Pack #3 had too large of a d_{50} value to be used with these particular pluviators, therefore, a known amount of GP#3 was placed in each shear box by the spooning and tamping technique according to the desired relative density.

The large shear box was pluviated using a raining device that moved on a stationary track system. The hopper used a variety of sieve and porous plate configurations as well as variable fall heights to achieve the desired specimen relative density. The minimum fall height that could be achieved with this pluviation method was 20.3 cm and the maximum fall height was 45.7 cm. A traveling pluviator was used with the large shear box because the thickness of the soil sample was such (178 mm) that a stationary pluviator would have created an extreme density gradient within the sample. Therefore, 25.4-mm-thick lifts were deposited and the pluviator was raised after each lift to keep the fall height constant. The pluviator opening was 41-cm long by 2.54-cm wide. Numerous porous plates and metal meshes were slid into this pluviator opening to achieve the required density. Gravel Pack #3 proved difficult to pluviate because of the large particle size and therefore the funnel and tamping technique was used with a constant fall height to place the specimen.

TABLE 6—*Summary of the sample preparation techniques in the direct shear box testing program.*

Sand Type	Box Size (mm) 60	101.6	304.8
Brown Mortar	Moist compaction	Moist compaction	Moist compaction
Winter	Moist compaction	Moist compaction	Moist compaction
Ottawa	Stationary pluviator	Stationary pluviator	Traveling pluviator
Morie	Stationary pluviator	Stationary pluviator	Traveling pluviator
GP#3	Dry compaction	Dry compaction	Dry compaction

Results

The constant volume friction angles for each sand at each density in all three boxes are presented in Table 7. In most tests, the failure envelope went through zero because the tests were run under saturated conditions. However, in some of the dense, well-graded specimens, the failure envelope showed some amount of cohesion (c =1.5 to 20 kPa). The friction angles were calculated accordingly. It is recognized that friction angle may show curvature depending on relative density over a large range of normal stresses, as was noted in some triaxial compression tests (Baker 2004). However, within the stress range used in these tests, the Mohr-Coulomb (M-C) failure envelope was approximated as linear. The friction angle was obtained by linear regression. For a presentation of Mohr-Coulomb failure envelopes as well as vertical versus horizontal displacements (used to determine constant volume conditions) during each test see Cerato (2005).

The friction angles were obtained as a function of the shear stresses at the end of each test. The vertical displacement results were checked to make sure a constant volume state (no additional vertical displacement) had been reached, and at this point of no volume change the friction angles were obtained. However, in some instances, the sand continued to dilate and a constant volume was never reached. In these instances, the friction angle was determined at 10 % of the box width for consistency. If tests ended before 10 % of the box width had been sheared then the results were extrapolated. In some of the tests at a high density, the shear strength peaked prior to reaching a deformation of $0.1B$. These peak strength values are presented in parenthesis. Large amounts of dilatation, and therefore, higher peak strengths, occurred mostly in the dense specimens, always in the smallest two boxes. The peak friction angles found in the dense soil specimens show more depen-

TABLE 7—*Direct shear box testing results using the constant volume shear strength.*

	γ_d (Mg/m^3)	D_r (%)	Box Size (mm) 59.9 ϕ (deg)	101.6 ϕ (deg)	304.8 ϕ (deg)
	1.44	13	36.5	28.5	26.1
Brown Mortar	1.52	43	37.3 (39.7)	36.5	34.0
	1.62	75	39.0 (48.3)	38.4	38.0
	1.71	34	47.4	42.0	40.8
Winter	1.83	66	49.7	43.5	41.5
	1.92	91	51.6	48.0	46.0
	1.6	23	30.7	30.5	30.3
Ottawa	1.68	56	35.0 (35.6)	35.0	35.0
	1.76	86	36.0 (39.0)	36.0	36.3
	1.51	25	38.0 (40.0)	37.0 (39.0)	36.0
Morie	1.56	55	38.5 (41.0)	38.5 (40.5)	37.0
	1.61	85	40.5 (45.0)	39.2 (43.8)	37.5
	1.46	25	42.0 (43.5)	36.5	34.0
Gravel Pack #3	1.50	55	44.5	41.0 (42.5)	40.2
	1.54	85	45.5	43.0	42.0

Note: Parentheses indicate peak friction angle.

dence on size then the constant volume friction angles; however, since not all specimens exhibited peak behavior, the constant volume friction angle was used for comparison.

The results show that loose specimens, as well as dense specimens were affected by box size. In some cases, the results show that the constant volume friction angles in loose specimens were actually affected more than the dense specimens, which is opposite of what was expected. However, when analyzing the difference in the peak shear, it can be noted that the friction angles of the dense specimens are affected as much or more than the loose specimens. The one exception is the GP#3 and that may be a function of how the specimens were prepared.

The constant volume friction angle for each sand as a function of box size is presented in Fig. 8. As the box size increased, the friction angle decreased in most cases. It can be argued that the friction angles obtained from the 101.6- and 304.8-mm boxes are similar, with differences probably falling in the typical scatter of this type of test (±2°). The results of the smallest shear box (60 mm), however, show much higher friction angles in four of the five sands tested. This could be a consequence of lower values of H/D_{max} and W/D_{max} for the soils tested in this box (Table 5). It can be seen that Ottawa Sand was the only sand that fulfilled the more stringent particle-size to box-size ratio presented by Jewell and Wroth (1987) and was also the only sand that showed no friction angle dependence on specimen size.

Comparison of Results

The results from this study show that there is a scale effect present in direct shear tests and that the scale effect depends on the sand type and the relative density. These comprehensive results help to explain the findings in the previous two studies that investigated scale effects in direct shear boxes, which were somewhat inconclusive. In the Parsons (1936) study, it was unclear whether specimen size effect would have increased with a wider range of normal stresses. The small differences found in the friction angles of the different size shear boxes were probably a function of three factors: (1) the sands were tested at a loose state (low relative density), (2) the study used clean, fine sand with small particles in relationship to box size, and (3) the study used very low normal stresses. Although Palmeira and Milligan (1989) prepared the Leighton Buzzard 14/25 Sand to a dense state, the W/D_{max} ratios for each of the three direct shear boxes was greater than 50 (D_{max}=1.2 mm). It has been stated in the literature by Jewell and Wroth (1989) and also seen in this study using Ottawa Sand, that there is little or no specimen size influence on friction angle once the W/D_{max} ratio reaches 50 or beyond.

It was mentioned above that similarities in shear zone propagation and friction angle dependence on specimen size may tie the

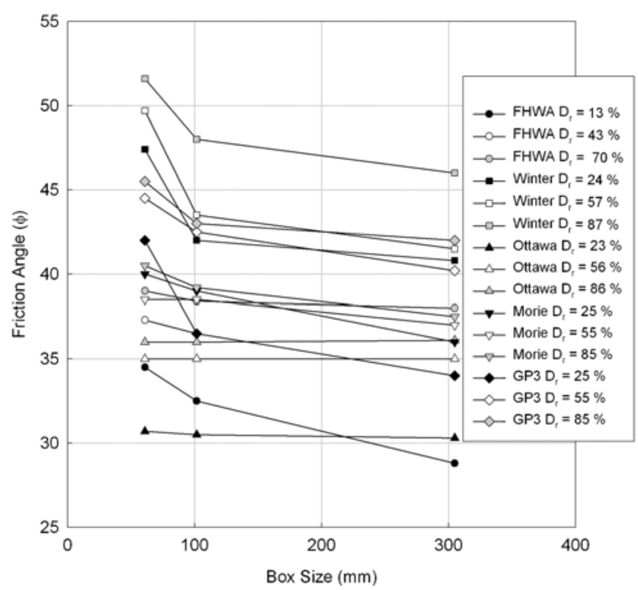

FIG. 8—*Influence of box size on constant volume friction angle.*

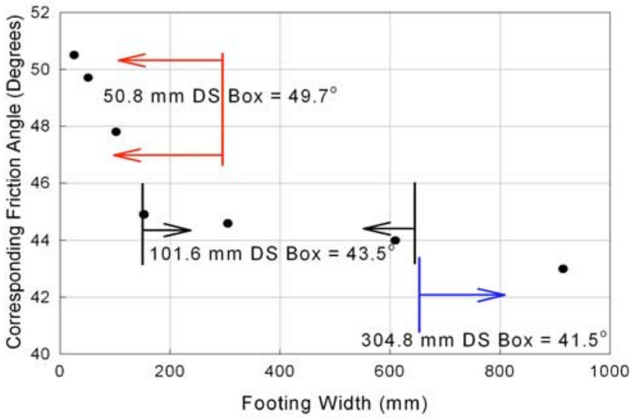

FIG. 9—*Apparent relationship between shear box scale effect and footing scale effect.*

scale effect in the direct shear boxes to the scale effect seen in model and prototype scale footing tests (Cerato 2005). In order to show this relationship, square model and prototype scale footing test results performed on Winter Sand are shown in Fig. 9. The Winter Sand footing load test at $D_r=66\%$ used back calculated N_γ values to find the corresponding friction angles from Terzaghi's (1943) bearing capacity factor chart. These corresponding friction angles are shown decreasing as footing width increases. A similar phenomenon occurred within the direct shear testing program on Winter Sand at the same relative density. The 60-mm shear box produced a friction angle of 49.7°. This corresponded well with the three smallest footings (25.4, 50.8, and 101.6 mm). The middle size direct shear box (101.6 mm) produced a friction angle of 43.5°. This corresponded well with footings between the sizes of 152.4 and 600 mm. The large 304.8-mm direct shear box produced the lowest friction angle of 41.5°, which corresponded well with footings greater than 600 mm.

In a small-scale footing test, it is thought that the shear zone forming in the active region directly beneath the footing is arrested by the small footing width; there is not enough room underneath the footing for the shear zone to fully propagate. The same phenomenon of arrested shear zones has been shown in direct shear box tests (i.e., Scarpelli and Wood 1982; Stone and Muir Wood 1992), and is both a function of box length and height. When the shear box is in motion, multiple shear bands start forming at the edges and continue toward the middle of the specimen. At a fixed displacement, the soil particles closer to the edges of the box will feel a greater strain than the soil particles in the middle of the specimen as a result of progressive failure. The bigger the box, the larger the horizontal shear rupture, the more room the soil particles have to rearrange and the more room the shear zone has to fully develop and to dilate to a critical state. Essentially, the shear zone has room to fully propagate in a larger shear box and is therefore a more realistic representation of the strength found in field conditions. Not only is the friction angle a function of box length, but it is a function of box height (Hight and Leroueil 2003). The smaller the height of the direct shear box, the larger the shear zone compared to the box thickness. If the sand particles are sufficiently large, there will not be enough room in the box to develop individual shear bands within the shear zone and the result of the test will show a high friction angle. At some box height H, the shear zone can fully develop and any specimen thickness should theoretically yield the same friction angle.

It is believed that the scale effect observed between the friction angle and direct shear box specimen size does help to explain the scale effect observed between the bearing capacity factor N_γ and footing size. Just as in the direct shear test study where the scale effect depended on the type and relative density of the sand, the scale effect seen with the model and prototype footing tests also depended on the type and relative density of the sand (Cerato 2005). The shear zone formation underneath the active region of footing tests was never quantified. However, the results of the direct shear box testing program has lead to the re-evaluation of the shear zone both experimentally and by utilizing finite element modeling. It is the hope of these combined studies that the difficult topic of scale effects in granular material may be understood.

Along with helping to understand the scale effect in small scale footings on granular material, the direct shear box testing program in this study yielded other valuable information. Data on five different sands were presented that show that the friction angle may vary up to almost 10° between a 60 and a 305 mm box depending on the type of sand and the density. This is a considerable decrease in friction angle and may significantly influence any design in which friction angle is used, e.g., bearing capacity of shallow foundations. It is customary in the consulting industry to test the friction angle of sand in either a circular or square 60 mm box. Therefore, it is proposed that the friction angle should be determined using a direct shear box with a W/D_{max} ratio of 50 or greater, which is much more strict than the recommendations from ASTM Standard D 3080-90, which uses a W/D_{max} of 10. If the soil particle-size to box-size ratio falls below 50 as presented by Jewell and Wroth (1987) and seen in this study with four of the five sands in the 60-mm box and two of the five sands in the 101.6-mm box, the friction angle should be reduced by 10 % in order to compensate for the over-predicted friction angle due to H/D_{max} and/or W/D_{max} constraints. The Ottawa Sand tested in this study, with a particle-size to box-size ratio of approximately 70 (W/D_{max}), showed no friction angle dependence on size, and therefore would need no reduction in friction angle.

From the results of this study, it is recommended that ASTM D 3080-90, Standard Test Method for *Direct Shear Tests of Soils Under Consolidated Drained Conditions* be revisited and further limit the values of H/D_{max} and W/D_{max}. As was seen in the results reported by Parsons (1936), Palmeira and Milligan (1989) and also the results on fine Ottawa Sand reported in this study, the friction angle was only slightly affected by specimen size. This likely occurred because the sand particles were small and rounded and had adequate H/D_{max} and W/D_{max} ratios. For both well-graded sands reported in this study as well as the clean, angular GP#3 sand, the friction angle was significantly affected by specimen size. The large diameter particles and packing structure of the angular particles did not allow for free movement during shearing and therefore the confining effects of the shear box were felt, resulting in up to a 10° increase from the 304.8-mm box to the 60-mm box. In all three cases, the minimum W/D_{max} presented by Jewell and Wroth (1987), which is considerably stricter than the ASTM Standard D 3080-90 recommendation, was not met in the 60- and 101.6-mm shear boxes.

Conclusions

Five sands with different properties were tested in three square shear boxes of varying sizes, each at three densities; dense, medium, and loose. The friction angle was seen to increase with increasing relative density in each of the three boxes. The constant volume (residual) friction angle decreased or remained constant, with increasing box size depending on the type of sand and the relative density. The results show that loose specimens, as well as dense specimens were affected by box size. The friction angles of the well-graded angular natural sands and the angular poorly graded manufactured gravel pack, GP#3, were most affected by specimen size. The constant volume friction angle of the Ottawa Sand, which was the most rounded and uniform sand, was only slightly affected by specimen size. This was most likely a result of adequate H/D_{max} and W/D_{max} ratios. The average particle shape analysis of the five sands was helpful when discussing the behavior observed in the direct shear tests.

The trend of decreasing friction angle with increasing direct shear box size modeled the decreasing friction angle with increasing footing size found in model and prototype scale foundation tests very well. It is thought that constraining the shear zone propagation in both the small footing tests and the small direct shear box tests are related and may help to explain the scale effects seen in footing tests on granular material.

Friction angles obtained from small diameter/width direct shear samples must be used with caution in footing design. It was shown for a few of the five sand types that the friction angle may be up to 10° higher in a 60-mm box than in a 305-mm box. This variation in friction angle would vary a footing design drastically. The friction angle, therefore, should be determined using the largest size specimen possible, with H/D_{max} and W/D_{max} ratios greater than 6 (ASTM Standard 3080-90) and 50 (Jewell and Wroth 1987), respectively. It is unclear if the ASTM Standard 3080-90 minimum H/D_{max} ratio of 6 is adequate and more testing needs to be done to confirm that ratio. If meeting these minimum size requirements is not possible, then the friction angle should be reduced by 10 % to ensure an accurate strength parameter. The ASTM D 3080-90, Standard Test Method for *Direct Shear Tests of Soils Under Consolidated Drained Conditions* should be revisited to increase the particle-size to box-size ratios of H/D_{max} and W/D_{max} to ensure that specimen size will not affect friction angle results.

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