Scale Effects of Shallow Foundation Bearing Capacity on Granular Material

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Abstract: Scale effects of shallow foundation bearing capacity on granular materials were investigated to further evaluate the trend of decreasing bearing capacity factor, N_γ, with increasing footing width, B, observed by other researchers. Model-scale square and circular footing tests ranging in width from 0.025 to 0.914 m were performed on two compacted sands at three relative densities. Results of the model-scale footing tests show that the bearing capacity factor, N_γ, is dependent on the absolute width of the footing for both square and circular footings. Although this phenomenon is well known, the current study used a large range of footing sizes tested on well-graded sands to show that the previously reported modifications to the bearing capacity factor, N_γ, using grain-size and reference footing width do not sufficiently account for the scale effect seen in the test results from this study. It also shows that behavior of most model-scale footing tests cannot be directly correlated to the behavior of full-scale tests because of differences in mean stresses experienced beneath footings of varying sizes. The relationship of the initial testing conditions (i.e., void ratio) of the sand beds and mean stress experienced beneath the footing (correlated to footing size) to the critical state line controls footing behavior and, therefore, model-scale tests must be performed at a lower density than a corresponding prototype footing in order to correctly predict behavior. Small footings were shown to have low mean stresses but high N_γ values, which indicates high operative friction angles and may be related to the curvature of the Mohr–Coulomb failure envelope.

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Introduction

The bearing capacity of shallow foundations on granular material has been studied extensively since Terzaghi's original work during the 1930s and 1940s. Although additional considerations to the governing criteria of bearing capacity have been presented, the calculation of the ultimate bearing capacity of a footing has changed very little since Terzaghi (1943) introduced his general equation for ultimate bearing capacity, q_{ult}. The bearing capacity equation is arranged and used such that all three bearing capacity factors are dependent only on the friction angle of the soil. Here, the special case of surface footings on granular soil is considered; thus, only the term (N_γ) related to the unit weight, γ, of the soil is investigated. The corresponding bearing capacity factor, N_γ, has been found to be dependent on the absolute footing size in addition to the friction angle, which is related to relative density, of the soil by some researchers. Most bearing capacity factors have been derived from model-scale footing tests, thus potentially affecting their extrapolation to full-scale footings (i.e., Ingra and Baecher 1982; Dewaiker and Mohapatro 2003). It was shown that actual model-scale footing test results produced higher values of N_γ than theoretical equations and therefore should not be used for the design of full-scale footings without a reduction. In addition to this scale effect, some researchers have indicated that the ratio of the footing width to mean grain size of the soil also influences N_γ determined using small-scale footings (i.e., Kusakabe 1995; Herle and Tejchman 1997).

As N_γ appears to have a scale effect, and is theoretically (e.g., Terzaghi 1943) related only to the unit weight and friction angle of the soil, it is possible that the actual scale effect may be, at least in part, related to the method of determining the friction angle. That is, it is more than possible that the scale effect observed between N_γ and the footing size is directly related to the mean stress felt underneath a footing; i.e., the larger the footing, the higher the mean stress and the lower the friction angle. This can be related to the curvature of the Mohr–Coulomb failure envelope.

The mean stress difference between footings of varying size is also related to the critical state concept, which states that behavior of footings is governed by the initial position of void ratio and mean stress relative to the critical state line. For instance, if two differently sized footings (different mean stresses) are performed on a sand with the same void ratio (same density), the footing closest to the critical state line (the larger footing with the larger mean stress) will behave as it were in a looser soil and will show larger settlement for a certain applied stress than the smaller footing. The smaller footing has a smaller mean stress and therefore is further away from the critical state line and would behave as if it were on a denser soil. Although relative density cannot explain this difference (because both footings were performed on sand

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with the same void ratio), the position of the tests in relation to the critical state line can. This distance from the critical state line to the initial state of a test specimen is called the state parameter, ψ, and was introduced by Been and Jefferies (1985) to combine the effects of stress level, density, compressibility, grain size and shape, and uniformity.

This paper presents and discusses the results of square and circular model-scale footing tests ranging in size from 0.0254 to 0.914 m conducted on two compacted sands and focuses on how the bearing capacity factor, N_γ, is affected by footing size, shape, and sand relative density. A discussion of several hypotheses to explain the scale effect seen in model-scale footing tests on two different sands is also presented.

Background

Bearing Capacity of Shallow Foundations on Granular Material

Terzaghi (1943) was the first to present a theory for the evaluation of the ultimate bearing capacity of rough shallow foundations. Terzaghi expressed the ultimate bearing capacity of a rough, strip footing with the well-known general superposition equation:

$$q_{\text{ult}} = cN_c + qN_q + \tfrac{1}{2}\gamma B N_\gamma \qquad (1)$$

where c=cohesion of soil (kN/m^2); γ=unit weight of soil (kN/m^3); $q=\gamma D_f$(kN/m^2); D_f=depth of footing embedment (m); B=width of foundation (or diameter of circular foundation) (m); and N_c, N_q, N_γ=bearing capacity factors that are nondimensional and were given by Terzaghi (1943) as functions of the soil friction angle, ϕ.

Terzaghi's original bearing capacity equation was for a centrally loaded strip footing of infinite length. This equation may be modified for other footing shapes by adding a shape factor, s_γ. Terzaghi's shape factors for square and circular footings are 0.8 and 0.6, respectively.

For the special case of a footing resting on the surface (D_f =0) of a saturated, clean, granular material (apparent cohesion, c=0), the first two terms of Eq. (1) become zero and Terzaghi's equation reduces to

$$q_{\text{ult}} = \tfrac{1}{2}\gamma' B N_\gamma s_\gamma \qquad (2)$$

where s_γ=shape factor and γ'=buoyant unit weight (kN/m^3). Footing width, B, is involved in the bearing capacity equation because the failure zone beneath the footing is based on the shape of the wedge of soil, which is ultimately based on B. The depth and extent of the failure zone depend on the assumptions of the shape of this wedge of soil and, therefore, the footing width.

Scale Effects of Shallow Foundations

Coarse-grained granular soils have absolute scale, relative to the dimensions of most foundation elements. Fine-grained soils, by virtue of their small size, e.g., micron range, are unaware of the dimensions of a foundation element. Whereas there may be thousands of individual particles of clay under a footing, by comparison, there may only be a few hundred sand grains under the same footing. Although footings on both soils may show scale dependence, the scale for footings on sand is much larger. Currently, the design techniques for determining the bearing capacity of deep and shallow foundations involving granular soils do not account for any scale effects between the soil and the foundation element. This can result in an overly conservative design, which in turn results in excessive costs of foundations.

This scale effect phenomenon was shown as early as 1935 by Berry (1935), who presented results that showed that the bearing capacity of surface model circular footings increased disproportionately with increasing footing size (0.0508, 0.0718, 0.1016, and 0.1437 m) on a dense sand at constant relative density. This meant that the bearing capacity factor, N_γ, decreased with increasing footing size, if N_γ was back-calculated from Eq. (2). The only other possibility to explain this trend is that N_γ is a constant for a constant value of ϕ and an additional term is needed in the bearing capacity equation to account for footing size.

Observations of decreasing N_γ with increasing B were termed the "scale effect" by De Beer (1963). Based on additional observations over the past 40 years, there is considerable evidence that for coarse-grained granular soils (i.e., sands and gravels) the use of Eq. (2) leads to the conclusion that the bearing capacity factor N_γ is dependent on the absolute width of the foundation, B (e.g., Hettler and Gudehus 1988; Ueno et al. 1998; Ueno 2001; Zhu et al. 2001). An extensive compilation of previously reported results of model and prototype scale tests is presented and analyzed in Cerato (2005). Although the scale effect phenomenon seems to be fairly well recognized by many investigators, there is no provision in current design practice to take this behavior into account.

Tatsuoka et al. (1991, 1994) reported that two factors may be responsible for the footing size effect on the bearing capacity factor N_γ. First, the stress level dependency of mechanical properties of sand, which was first suggested by Golder (1941); and second, a change of the ratio B/d_{50}. The second factor may be considered as a "particle size effect." Tatsuoka et al. (1991) used Model 1 g and centrifuge test results as well as Siddiquee's (1991) computer simulations of the 1 gravity (m/s^2) (g) and centrifuge tests to show that the "decrease in N_γ in the 1 g tests known as the scale effect" is a combination of the "pressure-level effect as observed in the centrifuge tests for a constant B_0" and "the particle size effect as the difference between the 1 g and centrifuge tests for an identical $B=nB_0$," where B_0=footing width measured at 1 g and n=acceleration in the centrifuge. Therefore, the centrifuge tests using the prototype sand while under otherwise similar conditions may overestimate the N_γ of the prototype footing (Tatsuoka et al. 1991).

From the results of modeling experiments, there have been reported threshold values of B/d_{50} for bearing capacity tests on shallow foundations above which the particle size effect becomes insignificant (Ovesen 1975; Yamaguchi et al. 1977; Bolton and Lau 1989; Herle and Tejchman 1997). Kusakabe (1995) recommended model testing with $B/d_{50} > 50-100$ to avoid the particle size effect.

It has also been seen from experimental centrifuge tests on rectangular footings with aspect ratios (L/B) of 7, 5, 3, and 1 that there exists a scale effect for the shape factor, s_γ (Kusakabe et al. 1991). The study found that the scale effect was more pronounced when the footing geometry became more three dimensional, i.e., smaller aspect ratio. Kusakabe et al. (1991) showed that the shape factor decreased from 0.6 to 0.4 as the prototype footing width increased up to 3 m (B_0=10 mm, n=30 g) for an aspect ratio of 7.

The actual scale effect seen in small scale footings may be, at least in part, related to the mean stress felt underneath a footing; i.e., the larger the footing, the higher the mean stress and therefore the lower the friction angle. Mean stress increases with increasing footing width and from the curvature of the Mohr–

Table 1. Summary of Sand Characteristics

Soil name	G (Mg/m^3)	$\rho_{\min}$ (Mg/m^3)	$\rho_{\max}$ (Mg/m^3)	$e_{\min}$	$e_{\max}$	d_{10} (mm)	d_{50} (mm)	C_c	C_u	% fines
Brown Mortar	2.69	1.41	1.70	0.58	0.91	0.3	0.6	1.1	2.1	2.0
Winter sand	2.69	1.61	1.96	0.37	0.67	0.2	0.7	1.0	4.5	2.5

Shiraishi (1990) cautioned that any "extrapolatory application" of the experimental equations derived from model-scale footing tests to a prototype foundation may lead to an overestimation of the N_γ^* at times. To avoid this overestimation, Shiraishi (1990) suggested an "engineering practice" equation, shown in the following equation which results in a value of N_γ^* around 30% smaller than his original equation [Eq. (4)]:

$$N_\gamma^* = \frac{0.71 N_\gamma}{B^{0.2}} \tag{5}$$

where N_γ^*=modified bearing capacity factor; N_γ=reference bearing capacity factor; and B=footing width (m).

The reference bearing capacity factors used by Shiraishi (1990) were the factors presented by Terzaghi (1943) and are available in most foundation engineering texts. Shiraishi's (1990) modification suggests that this adjustment is really only important for footings with widths less than about 1 m. It therefore may be especially important to reconsider results of very small scale 1 g and centrifuge footing test results presented in the literature since the 1960s, such as model footings with $B<0.1$ m. Although the two modifications to the bearing capacity factor discussed earlier adjust for footing width and soil grain size, neither takes into consideration whether this correction is dependent on the friction angle of the sand (i.e., relative density).

Current Investigation

In order to evaluate the impact of grain size, footing size, and relative density on the bearing capacity factor N_γ, model-scale square and circular footing tests was performed on two compacted sands having different characteristics, Brown Mortar sand from McLean, Va., and Winter sand from the Connecticut Valley. The sands were characterized using specific gravity, maximum and minimum density, and grain-size tests. The sands were also tested in direct shear to determine friction angles. Results from these tests are summarized in Table 1. The reason for using these two sands was because they had widely different characteristics and footing behavior would be observed and compared. In addition, the sands were readily accessible, and used extensively in various testing programs; a complete set of model-scale footing tests and direct shear tests were sought.

The Winter sand is angular and well-graded, which allows it to be compacted to extremely dense states. Winter sand shows extremely high friction angles when tested in direct shear and also high bearing capacities because of its interlocking tendencies. It is a glacial outwash mined in the Connecticut Valley and is a good foundation material. The Brown Mortar sand is less angular, less well-graded, and cannot be compacted to as dense a state as the Winter sand.

Direct shear testing was performed on both sands in a 0.305 m square direct shear box at the same densities used for the footing tests. A large shear box was used because ASTM D 3080-90, *Standard Test Method for Direct Shear Tests of Soils under Consolidated Drained Conditions*, requires a minimum specimen thickness of six times the maximum particle diameter and a minimum specimen width of ten times the maximum particle diameter in determining what size shear box should be used for testing sands. The minimum specimen width to thickness ratio should be 2:1. The normal stresses used to determine the friction angle ranged from 35 to 250 kPa. The constant volume friction angles were calculated at 10% of the box width. The results of the shear tests are presented in Table 2.

Model-Scale Footing Tests

Model-scale footing tests were performed in three different test pits; the first set was performed in a 0.762×0.762×0.305 m steel box with a concrete base (Fig. 3). Model-scale circular and square footings with widths of 0.0254, 0.0508, and 0.1016 m using two sands, each at three varying densities, were tested in this study. The Brown Mortar sand and the Winter sand were hand compacted, slightly moist, in 0.0508 m lifts with a 0.152 m^2 steel tamper to 1.44, 1.52 and 1.60 Mg/m^3 (D_r=13, 43, and 70%) and 1.68, 1.79, and 1.91 Mg/m^3 (D_r=24, 57, and 87%), respectively. Each test was performed in a fresh test bed.

Tests were performed under saturated conditions with the footings located at the sand surface (D_f=0) to minimize the terms in the Terzaghi bearing capacity equation to Eq. (2); ($q_{\mathrm{ult}} = 0.5\gamma B N_\gamma S_\gamma$). Saturation was achieved by flooding the test pit from the bottom very slowly until the soil bed was saturated. Once the water level reached the surface of the sand bed as measured by a small screened standpipe, saturation was verified. Steel footings were given a rough base by gluing sandpaper to the base and were loaded centrally and vertically at a constant rate of 1×10^{-5} m/s until a settlement of at least $0.1B$ occurred. The ultimate capacity was interpreted as the bearing stress, which produced a relative settlement of 10% B (i.e., s/B=0.1). Although choosing to define q_{ult} at a relative settlement of s/B is completely arbitrary, it (1) is convenient and easy to remember, (2) may actually be close to the average soil strain at failure, (3) forces a fixed value at q_{ult}, and (4) treats the displacement of all footing sizes the same. The bearing capacity factor, N_γ, was back-calculated and plotted versus footing size to observe the scale effect.

Table 2. 0.305 m Square Direct Shear Box Testing Results Using the Constant Volume Shear Strength

Box size			0.305 m
Sand	γ_d (Mg/m^3)	D_r (%)	ϕ (degrees)
Brown Mortar	1.44	13	26.1
	1.52	43	34.0
	1.62	70	38.0
Winter	1.71	24	40.8
	1.83	57	41.5
	1.92	87	46.0

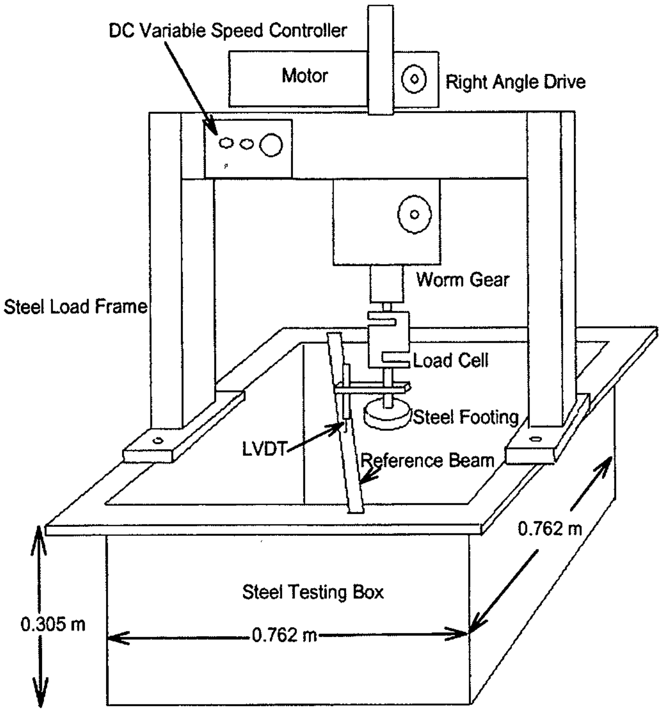


Fig. 3. Test setup for model-scale footing tests

The second set of model-scale surface tests using circular and square footings with widths of 0.152, 0.305, 0.610, and 0.914 m was performed in a 5.4 m by 6.9 m by 6 m deep reinforced concrete test pit on compacted saturated Brown Mortar sand at the Turner Fairbanks Highway Research Center (TFHRC) in McClean, Va. at relative densities of 45, 65, and 70% (McClusky 1996). The final set of model-scale square and circular footings tests with widths of 0.152, 0.305, and 0.381 m was performed at the University of Massachusetts, Amherst (Drury 1999; Mitchell 2000) on compacted Winter sand in a 2.44 m by 2.44 m by 1.22 m reinforced concrete test pit. For each test bed density, three footing tests were set on the surface and loaded centrally and vertically.

Results

Circular and square footings were tested to investigate if any difference might be observed relative to scale effects for different foundation shapes. Typical results of load curves for the footing tests performed in this study are presented in Figs. 4 and 5. These results are for bearing capacity tests on loose and dense Brown Mortar sand and loose and dense Winter sand. The ultimate capacity was taken at the stress at a footing displacement of 10% of the footing width for consistency. Most of the load–displacement curves in this study indicated local or punching type of failure, as opposed to general shear failure. Local shear failure usually occurs with a footing resting on a sand of medium density $(36\% < D_r < 70\%)$, whereas a punching shear failure occurs in very loose sands, typically $D_r < 36\%$ (Vesić 1973). Terzaghi (1943) suggested that if the soil fails by general shear, the bearing capacity is determined by Eq. (1). In order to compute the corresponding bearing capacity factors for local shear failure, Terzaghi (1943) recommended that the friction angle, ϕ, be reduced to $2/3\phi$. Terzaghi made no mention of punching shear.

These curves show that the square footings have a higher bearing capacity than circular footings, which is also the case for tests on both sands and at each density. Terzaghi (1943) suggested shape factors of 0.6 for a circular footing and 0.8 for square footings, which indicates that a square footing will have a bearing capacity approximately 33% larger than a circular footing of the same width. In this study, the square footings had a bearing capacity 30% larger than circular footings of the same width on average.

The ultimate bearing capacities ($q_{\text{ult}} = q$ at $s = 0.1B$) for the first set of model footing tests are presented in Table 3. All tests were performed with a sand thickness of $3B$, which has been shown to be adequate in eliminating any rigid boundary effect below the footing (Cerato and Lutenegger 2006). The model-scale footing tests were also performed with a lateral extent of $3.25B$ or greater. Prandtl's (1921) and Terzaghi's (1943) theoretical solutions for the lateral extent of the influence of the bearing capacity of a strip foundation under general shear on sands ranges from $2.5B$ at a friction angle of 28° to $12B$ at a friction angle of 45°. However, bulging around the footing was seen only in the dense layer foot-

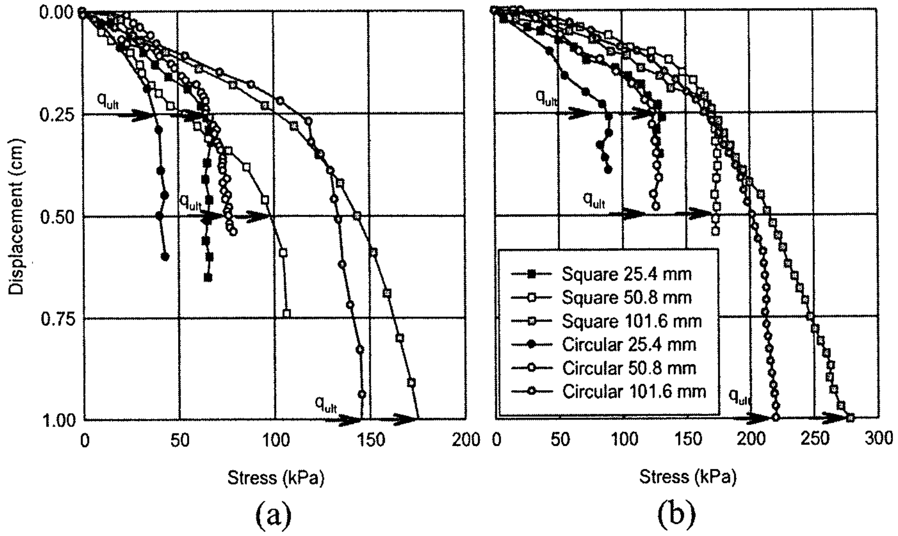
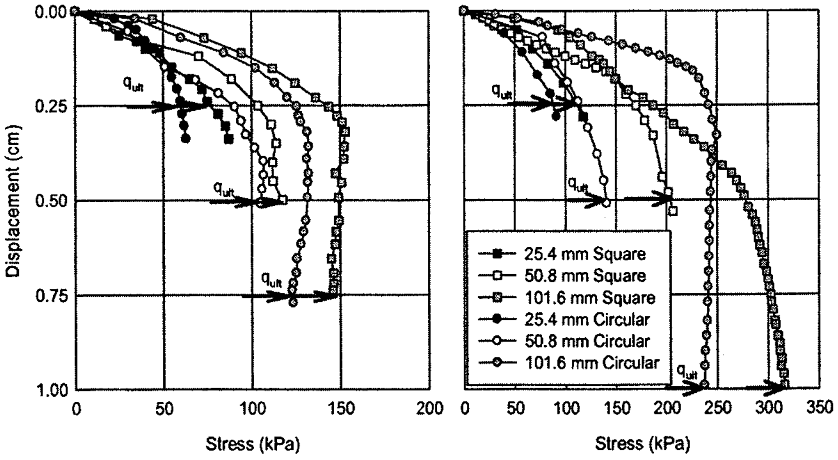


Fig. 4. Brown Mortar sand footing load test curves: (a) loose ($D_r = 13\%$); (b) dense ($D_r = 70\%$)



(a) (b)

Fig. 5. Winter sand footing load test curves: (a) loose (D_r=24%); (b) dense (D_r=87%)

ing tests and the extent of the bulge was never observed to extend to the edges of the testing box; therefore, it can be concluded that the steel box did not affect the bearing capacity of the model-scale footings. N_γ values were back-calculated using Eq. (2) using $s_\gamma=0.8$ and $s_\gamma=0.6$ for square and circular footings, respectively.

From the results obtained on different sands at three relative densities, it can be seen that values of N_γ for both sands decrease with footing size and increase with increasing relative density. The results also show that relative density may have a more pronounced influence on N_γ than grain size and the scale effect is more important for dense sands.

The second set of larger model-scale footing tests was performed to extend the range of footing sizes available to study the apparent scale effect observed in model-scale tests. It was important to extend the range of footing sizes available tested on the same sand at the same densities in order to have a complete set of data where the only variable was footing width. There were very few studies found in the literature that had footing tests ranging from 0.025 to 0.914 m on the same sands at the same densities.

Results of the larger model-scale footing tests performed on Brown Mortar sand are presented in Table 4. A few of the test series were performed specifically for this study; however, many were performed in a number of previous studies (McClusky 1996). The larger model-scale footing test results were combined with the results from the laboratory model-scale footing tests and are shown in Fig. 6. No larger model-scale tests were available for the loose sands.

The model-scale footing tests performed in the steel box show

Table 3. Model-Scale Footing Test Results Performed at UMASS in the Steel Box

	Winter sand					Brown Mortar sand			
Footing shape	B (mm)	D_r (%)	q_{ult} (kPa)	Back-calculated N_γ	Terzaghi N_γ[a]	D_r (%)	q_{ult} (kPa)	Back-calculated N_γ	Terzaghi N_γ[a]
Square	25.4	24	75	710	135.5	13	60	661	10.0
	50.8		120	568			100	551	
	101.6		150	355			170	468	
Circular	25.4		60	757			40	588	
	50.8		105	662			78	573	
	101.6		125	394			145	533	
Square	25.4	57	95	843	293.5	43	90	941	38.0
	50.8		165	732			130	680	
	101.6		220	488			215	562	
Circular	25.4		80	946			60	837	
	50.8		120	710			100	697	
	101.6		180	532			170	593	
Square	25.4	87	110	919	407.4	70	120	1,192	78.6
	50.8		205	856			175	869	
	101.6		325	678			280	695	
Circular	25.4		90	1,002			90	1,192	
	50.8		142	791			125	828	
	101.6		245	682			220	728	

[a]Terzaghi's general shear N_γ based on friction angle calculated in DS.

Table 4. Model-Scale Footing Test Results Performed on Brown Mortar Sand at the FHwA Testing Facility

Test series	Footing shape	B (m)	D_r (%)	q_{ult} (kPa)	Back-calculated N_γ	Terzaghi N_γ[a]
95 SE-2	Square	0.152	45	50	88	38.0
		0.305		81	76	
		0.610		96	42	
		0.914		149	44	
95GA1	Square	0.305	45	65	57	
		0.610		87	38	
		0.914		140	41	
97 SE-1	Square	0.152	54	73	100	
		0.305		120	82	
		0.610		233	80	
100SF2	Square	0.152	65	160	265	78.6
		0.305		400	331	
		0.610		961	398	
		0.914		691	191	
100SF3	Square	0.152	70	280	464	
		0.305		430	356	
		0.610		600	248	
100SF1	Square	0.305	70	431	357	
		0.610		670	278	
		0.914		553	153	
100CFS1	Circular	0.152	64	280	627	
		0.305		551	616	
		0.457		315	235	
		0.457		455	340	
		0.610		475	266	
100SF3	Circular	0.152	70	440	971	
		0.305		320	353	
		0.610		640	353	

[a] Terzaghi's general shear N_γ based on friction angle calculated in DS.

Table 5. Model-Scale Footing Test Results Performed on Winter Sand at UMASS in the Concrete Test Pit

Footing shape	B (m)	D_r (%)	q_{ult} (kPa)	Back-calculated N_γ	Terzaghi N_γ[a]
Square	0.305	57	85	63	293.5
Square	0.152	87	225	313	407
	0.305		207	144	
	0.381		217	101	
Circular	0.152	87	170	315	
	0.305		203	188	

[a] Terzaghi's general shear N_γ based on friction angle calculated in DS.

results that are relatively tightly grouped, whereas the results from the larger model-scale tests are not as repeatable. The model-scale tests were all performed in the smaller box with .0508 m sand lifts, where the density of each lift was closely monitored. The larger model-scale tests were performed in the larger pit with 0.30 m sand lifts. Each of the larger model-scale footing tests were performed in a different pit fill, where the densities could be different by up to 10%. This would make a large difference in the ultimate capacity results and therefore the back-calculated N_γ values for the same size footing tested at the "same" relative density.

Best fit trend lines were drawn through the dense, medium and loose footing tests. The test results show a rapid decrease in N_γ up to $B=0.25$ m after which there is almost no further decrease in N_γ. The N_γ value representing a footing ($B>1$ m) resting on dense Brown Mortar sand is about 190 and a footing ($B>1$ m) resting on medium dense Brown Mortar sand has an N_γ of about 40.

The third set of larger model-scale tests on Winter sand was performed in a previous study and were presented in Drury (1999) and Mitchell (2000). Table 5 presents the results of the larger model-scale footing tests performed on Winter sand. The largest model-scale footing performed in this study was 0.381 m. These results were combined with the results of the model-scale footing tests and are shown in Fig. 7. These data also show that N_γ decreases with increasing footing width and decreases for decreasing relative density, which is consistent with other results found in the literature.

Comparison of Results

The ultimate capacity of footings on Winter sand was almost always greater than the ultimate capacity of the same size footing on a similar density for the Brown Mortar sand because of the differences in gradation, shape, and size. Winter sand was more

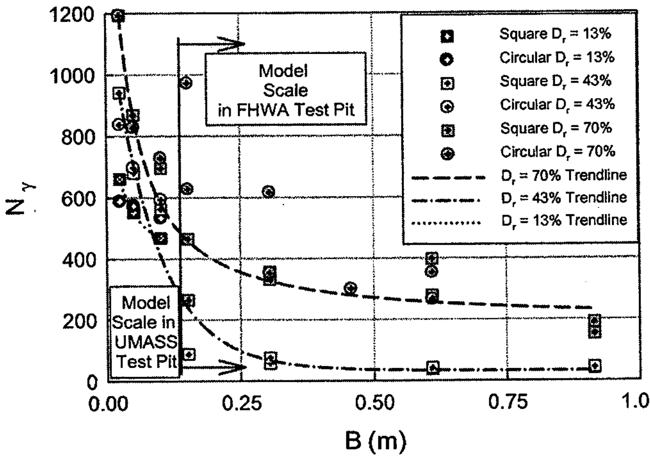


Fig. 6. Combined results of footing tests performed on Brown Mortar sand

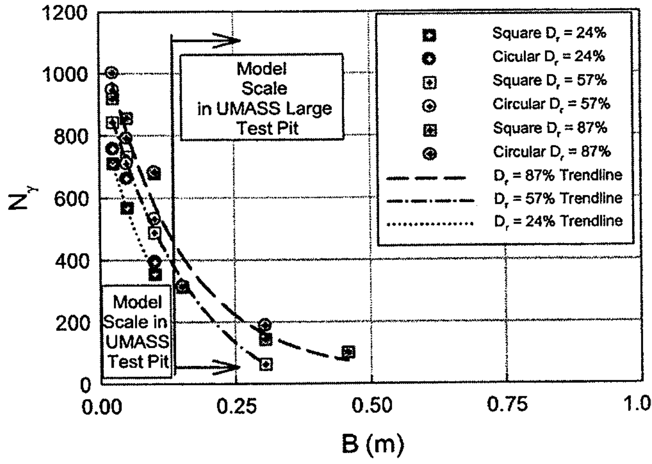


Fig. 7. Combined results of footing tests on Winter sand

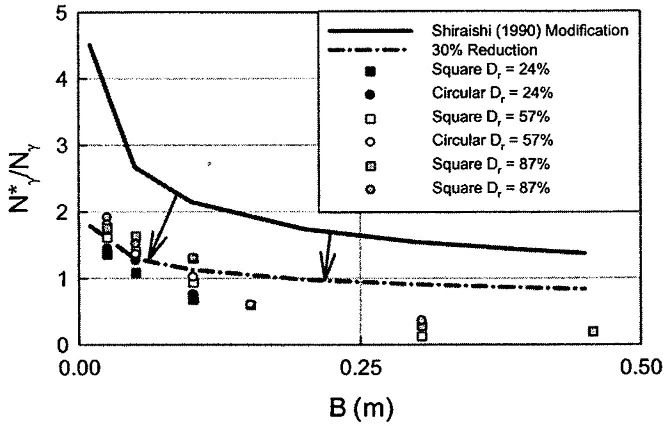


Fig. 8. Shiraishi's (1990) modification of N_γ with Winter sand

angular, more well-graded, and had a larger d_{50} and $d_{\max}$ particle size, and therefore, could achieve a denser state of packing and resist shear to greater loads than could the Brown Mortar sand. Winter sand had a much higher maximum and minimum density than Brown Mortar sand, and could achieve a much higher relative density (87 versus 70% for Brown Mortar), and could not easily achieve a lower relative density than 24%. However, when N_γ was back-calculated using Eq. (2), the N_γ values from the Brown Mortar sand were always higher than the Winter sand because of a lower calculated buoyant unit weight.

As previously noted, some researchers have suggested that the value of N_γ be corrected to account for the "scale effect" seen in model-scale footing tests on sand and introduced "modified" bearing capacity factors. One of those methods, recommended by Shiraishi (1990), suggested that the modified bearing capacity factor could be expressed as a function of the footing width, where N_γ is taken from Terzaghi's values for a rough base. Shiraishi's (1990) theory [Eq. (4)] and subsequent reduction of N^*_γ [Eq. (5)] for larger footings were compared with the results of Winter sand footing tests from this study (Fig. 8). The data were normalized by N_γ at a footing width of 1.4 m, which is Shiraishi's reference footing width. Shiraishi's model [Eq. (4)] overestimates the N^*_γ/N_γ values and his recommended reduction of 30% for larger size footings was applied. Shiraishi's (1990) original modification [Eq. (4)] models the Brown Mortar sand rather well, probably because the grain-size distribution, particle size, and friction angle of the sands at the different relative densities correspond with the sands used to create Eq. (4). Habib's (1974) modification to the bearing capacity factor, N_γ, as a function of grain size has a negligible effect at B/d_{50} ratios less than 200, and no effect at B/d_{50} values greater than 200. Habib's (1974) modification to the bearing capacity factor does not encompass the large scale seen in this study.

It was also observed that the failure mode of the footings tested in this study varied depending on sand type, density, and footing size. In order to explain the different behaviors observed, the footings were related to the critical state concept, which suggests that loose sands, under small mean stresses, will dilate and dense sands under large mean stresses, will contract. The same concept can be related to footings of different widths. For example, if three footings of various sizes were placed on the same sand in the same state, the influence area, and therefore mean stress, of the three footings would be different. The biggest footing would have the largest influence area and the smallest footing would have the smallest influence area.

The critical void ratio versus critical confining stress line was not determined in this study, and the mode of failure and amount of dilation or contracting was not measured for each footing test. Therefore, for a given sand at a given density, the size of the footing where the sand behavior changes from dilative to contractive cannot be determined at this time. However, the overall trend of smaller footings exhibiting different behavior than bigger footings can be well understood. The footings showed a scale effect because the small footings (small mean stress) acted as if they were tested on a dense sand, hence the high N_γ value. However, the larger model-scale footings (larger mean stress) acted as if they were on looser sand, even though the density (void ratio) was the same, hence the lower N_γ value. This scale effect is then explained, in part, by the differing stress states, defined as the distance, ψ, of the footing, from the critical state line.

From this information, it can be gleaned that the behavior of a model-scale test is not directly representative of the behavior of a full-scale footing. As was mentioned before, this occurs because of the differing mean stresses under footings with different widths and also the initial state of the test bed. To directly correlate a model-scale test to a full-scale test, one would have to perform a model-scale test on a looser state of sand than what was required for a full-scale test. As the mean stress of a model-scale test and a full-scale test would be different, the model-scale test must be performed on a looser soil than the full-scale test in order to show the same stress–settlement behavior (equal state parameter, ψ).

Conclusions

Results of model-scale footing tests on two compacted sands indicate that the bearing capacity factor, N_γ, is dependent on the absolute width of the footing for both square and circular footings. N_γ for both sands decreases with footing size and increases with increasing relative density. The results also show that relative density may have a more pronounced influence on N_γ than grain size and that the scale effect is more important for dense sands. One explanation for the scale effect is stress dependency; different sized footings have different mean stresses; the smaller the footing, the smaller the mean stress. This is related to the critical state strength. A small footing (small mean stress) would act as if it were on a denser "state" of soil than a larger footing, even if they were tested on sand with the same void ratio. The stress dependency may also be related to the curvature of the Mohr–Coulomb failure envelope where high friction angles at low stresses and low friction angles at high mean stresses have been observed. This curvature of the Mohr–Coulomb envelope has been well documented and helps to explain why small footings have large N_γ values, and hence, large friction angles corresponding to the dense state of soil in relation (ψ) to the critical state line. Another explanation for scale effects is progressive failure which describes nonconstant shear strain underneath a footing, where the circular failure slip line does not fail all at once, but in pieces, until the whole slip failure zone is mobilized.

Caution should be used when applying model-scale footing test results to the behavior of full-scale foundations. It was shown that actual model-scale footing test results produced higher values of N_γ than theoretical equations and therefore should not be used for the design of full-scale footings without a reduction. When using model-scale footing tests in lieu of more expensive full scale footing tests, the grain size of the sand and the density should be taken into account. For a model-scale test to be representative of a full-scale test, the model-scale test must be performed in sand that is looser than the sand underneath a full-scale test. Often, tests on model footings found in the literature are only representative of full-scale footings in soils that are unrealistically very dense.

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