

MODEL FOOTING TESTS ON A FINITE LAYER OF GRANULAR SOIL

Amy B. Cerato¹, Alan J. Lutenecker²

¹ Graduate Research Assistant, University of Massachusetts, Amherst, USA

² Professor and Department Head, University of Massachusetts, Amherst, USA

ABSTRACT - The bearing capacity of model footings resting on the surface of a finite layer of granular (cohesionless) soil was investigated. Results of the model scale footing tests show that the modified bearing capacity factor, N'_γ , is dependent on both relative density, D_r , and relative layer thickness, H/B , but appears to be independent of footing shape for square and circular footings.

RÉSUMÉ –

1. Introduction

Bearing capacity theories presented by Prandtl (1921), Terzaghi (1943) and others for the ultimate capacity of a strip footing resting on the surface of a granular (cohesionless) soil assume that the thickness of the bearing stratum is infinite. It has been shown that if the bearing layer is of finite thickness, the bearing capacity increases when an underlying rigid stratum exists within a certain depth. Using Prandtl's (1921) and Terzaghi's (1943) theories of plastic equilibrium, it is possible to determine the maximum theoretical depth of the bearing capacity failure surface and therefore the depth at which the bearing capacity would be free of the influence of the finite layer. This is generally on the order of $1.5B$ to $3B$ for soil friction angles ranging from 25° to 45° .

In order to evaluate the bearing capacity of surface footings on a layer of limited thickness, model square and circular footing tests were performed on a well-graded compacted sand. Test beds were prepared at three different relative densities and footing tests were performed using varying H/B values. This paper presents results of an investigation to evaluate the influence of a finite layer of sand beneath model footings and determine the modified bearing capacity factor, N'_γ , and to determine the influence of relative density and relative layer thickness, H/B , on the bearing capacity. The results of the tests show that N'_γ decreases as H/B increases until a value of H/B of approximately 3. N'_γ is also dependent on Relative Density, D_r , which has not previously been shown. A description of the test methods and material used is given along with the test results.

2. Background

The traditional bearing capacity equation for a footing resting on the surface of a cohesionless granular soil can be stated as:

$$q_{ult} = 0.5\gamma BN_{\gamma}S_{\gamma} \quad [1]$$

where:

- q_{ult} = Ultimate capacity of footing (Mg/m^2)
- B = Footing width (m)
- γ = Soil density (Mg/m^3)
- N_{γ} = Bearing Capacity Factor
- S_{γ} = Shape Factor

For footing load tests taken to failure, B and γ are known and q_{ult} is measured from the tests. The two unknowns are N_{γ} and S_{γ} . Values of the shape factor, S_{γ} , for square or circular foundations are fairly well established and according to all current foundation engineering textbooks, N_{γ} is only dependent on the angle of internal friction, ϕ , of the soil.

2.1 Shape Factors

Terzaghi (1943) suggested shape factors of 0.8 and 0.6 for square and circular foundations respectively, which were implicitly given in his bearing capacity equations. These values assume no interference within the zone of bearing capacity that might be produced by a layer of limited thickness.

Mandel and Salencon (1972) developed a solution for the modified bearing capacity factor, N'_{γ} , using the theory of limit equilibrium as a function of the friction angle, ϕ , and the depth of the rigid base H , divided by the width of the footing, B . They found that for $H/B \geq 1$ the value of N'_{γ} was equal to N_{γ} . Meyerhof (1974) used Mandel and Salencon's (1972) analyses to develop a modified shape factor chart as a function of H/B and ϕ for $H/B \leq 1$. Both of these investigations show that a modification of the general bearing capacity equation is necessary because the rough rigid base interferes with the slip lines and produces a considerably higher ultimate bearing capacity, and therefore, bearing capacity factor, N_{γ} , at layer thicknesses up to $H/B = 1$.

Figure 1 presents Meyerhof's (1974) shape factors for circular foundations for $H/B \leq 1.0$, which are seen to only be dependent on the soil friction angle. Since Meyerhof's theory is based on $N'_{\gamma} = N_{\gamma}$ at $H \geq B$, the shape factor remains constant at 0.6 for any $H/B > 1$ regardless of footing shape (i.e., square or circular).

Meyerhof (1974) modified his solution for circular footing shape factors to accommodate square and rectangular footings with the equation

$$S'_{\gamma(Square)} = \left[1 - \left(1 - S'_{\gamma(Circle)} \right) \bullet \frac{B}{L} \right] \quad [2]$$

Equation 2 states that values of H/B greater than 1 give a shape factor of 0.6 for both square and circular footings, since B/L for a square footing is equal to 1. This varies slightly from Terzaghi's (1943) theory, which states that for an infinite layer, square and circular footings should have different shape factors.

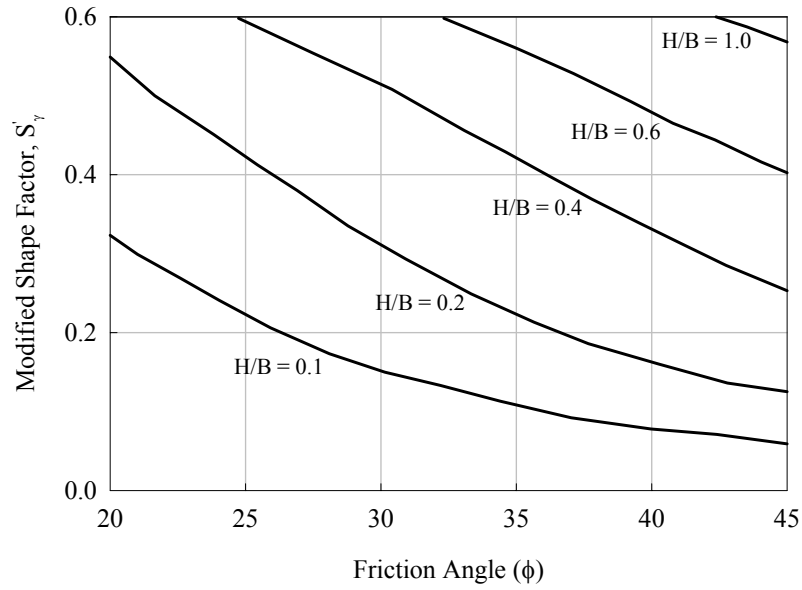


Figure 1. Theoretical Modified Shape Factors for Circular Footings (from Meyerhof 1974).

A variation of Meyerhof's (1974) shape factor was given by Das (1999) as

$$S'_\gamma = 1 - m(B/L) \tag{3}$$

where m is taken from Figure 2 using H/B and the soil friction angle, φ. Based on Meyerhof's work, however, this variation should only be used for circular footings.

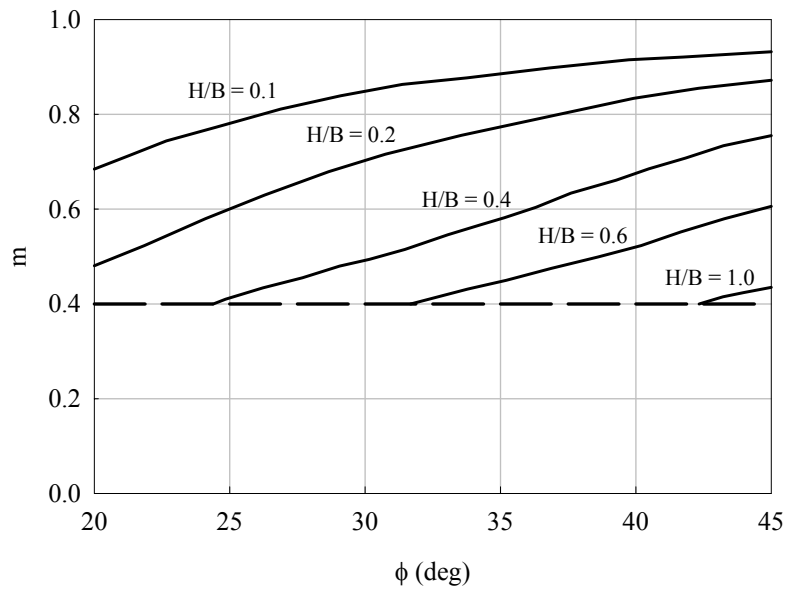


Figure 2: Variation of m Using Meyerhof's (1974) Values (after Das 1999) for Circular Footings.

2.2 N_γ versus H/B – Previous Results

Using Prandtl's (1921) and Terzaghi's (1943) theories of plastic equilibrium, it is possible to determine the maximum theoretical depth of the bearing capacity failure surface of a strip footing and therefore the depth at which the bearing capacity would be free of the influence of the finite layer. Experimental data from the literature show that the zone of influence of a surface footing is approximately between 1.5B and 2B, or 1.5 to 2 times the footing width depending on the sand type and density (e.g. Narahari and Singh 1964; Meyerhof 1974; Tournier and Milovic 1977; Pfeifle and Das 1979; Hanna 1981; Siraj-Eldine and Bottero 1987; Cooke 1988). These tests were all performed on uniformly fine-graded sands. A summary of the testing conditions for each of these investigations is given in Table 1 and the results are shown in Figure 3. The value of $\delta = N'_\gamma/N_{\gamma\infty}$ is used in order to compare all tests from different studies, where $N_{\gamma\infty}$ is equal to the N_γ at the maximum H/B value.

Table 1: Summary Table of Previous Results found in the Literature.

Reference	Footing Shape	Friction Angle, ϕ	Relative Density D_r (%)	Width/Diameter (mm)	H/B Range	D_{50} (mm)
Narahari and Singh (1964)	Square	44	-	76.2	0.33 to 3	0.2
Meyerhof (1974)	Circular	38	-	76.2	0.25 to 2.1	-
Tournier and Milovic (1977)	Square	38	66	200	0.5 to 6.8	0.1 to 0.6
	Square	38	66	350	0.5 to 6.8	0.1 to 0.6
	Square	38	66	500	0.5 to 6.8	0.1 to 0.6
Pfeifle and Das (1979)	Square	43	78	50.8	0.4 to 5.0	0.6
Hanna (1981)	Circular	43	47	50	0 to 2	-
Siraj-Eldine and Bottero (1987)	Circular	35	71	50	0.5 to 2.1	0.7
	Circular	35	71	56.4	0.5 to 2.2	0.7
	Square	35	71	50	0.5 to 2.0	0.7
Cooke (1988)	Circular	37	30	305	0.5 to 3.0	0.2
	Circular	44	44	305	0.5 to 3.1	0.2

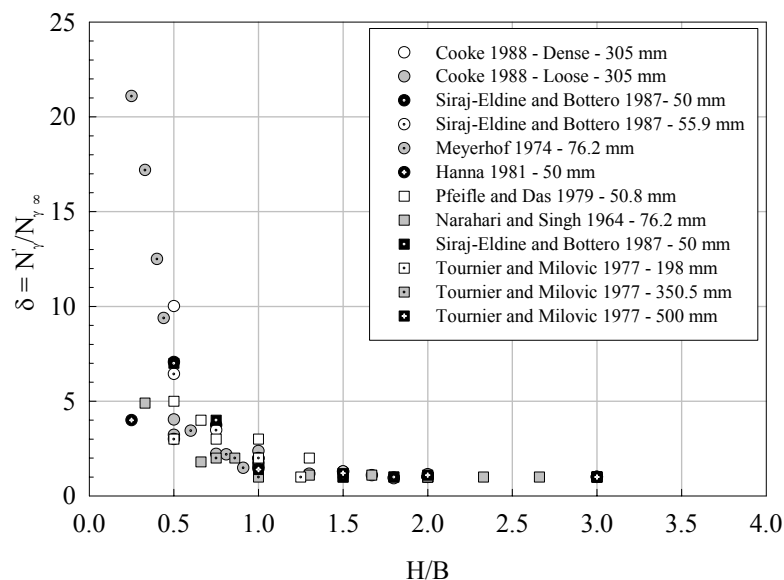


Figure 3. Results of Model and Prototype Scale Footing Tests Compiled from the Literature.

Figure 3 shows that the factor δ decreases as H/B increases until an H/B of approximately 1.5. At H/B values greater than 1.5, δ becomes constant and equal to 1. However, because of the

scatter of results, it is not obvious whether the value of δ is constant for all sand types and densities, or footing sizes and shapes at a given value of H/B .

3. Current Investigation

Model scale square and circular footing tests were performed on a well-graded compacted sand; ($G_s=2.69$, $\rho_{\min}=1.61 \text{ Mg/m}^3$, $\rho_{\max} = 1.96 \text{ Mg/m}^3$, $D_{50} = 0.7 \text{ mm}$, $C_u = 4.5$). Test beds were prepared at three different relative densities corresponding to $D_r = 23.7, 57.2$ and 86.8% , using $H/B = 0.5, 1, 2, 3, 4$.

The model footing tests were performed in a $0.762 \text{ m} \times 0.762 \text{ m} \times 0.305 \text{ m}$ steel box with a concrete base. All tests were performed under saturated conditions with the footing resting on the sand surface ($D_f = 0$). The dimensions of the model footings used were 101.6 mm . The sand was placed in 5.08 cm lifts and spread by hand until level. A $15.24 \text{ cm} \times 15.24 \text{ cm}$ square steel hand tamper was used to compact the sand to the desired density. The footings were given a rough base and were loaded at a constant rate of 0.001 cm/sec until a settlement of $0.1 B$ occurred. The ultimate bearing capacity was defined at a relative settlement s , of $s/B = 0.1$ and the bearing capacity factor, N_γ , was back calculated from the load at failure.

4. Results

Table 2 presents results of the ultimate bearing capacity for the model footing tests. The failure modes for each of the footings varied depending on the sand type and density. As previously mentioned, Terzaghi (1943) recommended shape factors of 0.6 for a circular footing and 0.8 for square footings, which suggests that for footings on an infinitely thick layer, a square footing will have a bearing capacity approximately $0.8/0.6 = 1.33$ times larger than a circular footing of the same width. The results given in Table 2 show that the square footings have a higher bearing capacity than circular footings at each density for all H/B values. On average, $BC_{sq}/BC_{circ} = 1.25$.

Table 2. Model Scale Footing Test Results.

Relative Density	Friction Angle (Direct Shear)	H/B	Square $*q_{ult}$ (kPa)	Circular $*q_{ult}$ (kPa)
$D_r = 23.7 \%$	40.8	4	140	120
		3	150	125
		2	300	250
		1	500	420
		0.5	850	650
$D_r = 57.2 \%$	44.5	4	225	180
		3	220	180
		2	400	325
		1	625	500
		0.5	1000	800
$D_r = 86.8 \%$	46.0	4	325	245
		3	325	245
		2	500	460
		1	800	620
		0.5	1200	950

$*q_{ult}$ defined as q at $s/B = 0.10$

Figure 4 presents the results of tests performed in this study showing square and circular tests at $D_r = 23.7, 57.2$ and 86.8% . From this figure it can be seen that δ is dependent on both H/B and D_r , but is independent of footing shape.

Figure 4 shows that there is clearly a dependence of δ on Relative Density, which had not previously been identified. The variation in δ for different relative densities decreases as H/B increases. That is, D_r becomes less important as H/B tends toward an infinite layer. Values of δ appear to be independent of footing shape for all values of Relative Density and H/B .

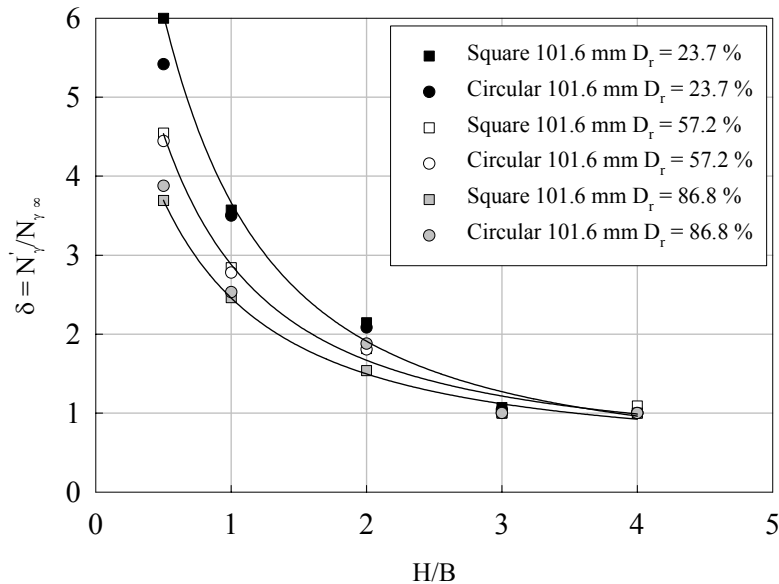


Figure 4: Variation of $\delta = N'_y / N'_{y\infty}$ with D_r , and H/B for Square and Circular Footings.

Mandel and Salencon (1972) derived a modified bearing capacity factor curve for strip footings with varying H/B values with an implied shape factor, S'_y equal to one (1) for a sand with $\phi = 43^\circ$. Figure 5 presents the normalized Mandel and Salencon's (1972) theoretical curve for strip footings along with the same data adjusted using Meyerhof's (1974) shape factors and experimental results from this study including circular and square model footing tests assuming a shape factor of 1. Meyerhof's (1974) solution for S'_y increased the N'_y values at H/B values less than one and remained constant thereafter.

For H/B values between 0.6 and 2.9, the experimental values of N'_y are higher than those predicted by Mandel and Salencon (1972). Das (1999) found that the range was 0.6 to 1.9 using Pfeifle and Das's (1979) experimental data. For H/B values less than 0.6, the experimental values of N'_y are lower than those predicted by theory, which is similar to Das (1999) findings. Das (1999) concluded that this may be due to the crushing of sand grains at high values of ultimate load and the curvilinear nature of the actual failure envelope of soil at high normal stress levels.

Figure 5 suggests that the shape factor for a strip footing is not constant at all values of H/B , which is similar for square and circular footings. The theoretical strip footing curve is constant with H/B values after approximately 1.2, whereas the experimental values for square and circular footings does not become constant until $H/B = 3$. This means that the failure surface of a strip footing would not extend as deep as that of a square or circular footing and that a constant bearing capacity would be achieved after a depth of 1.2 B for a sand with $\phi = 43^\circ$.

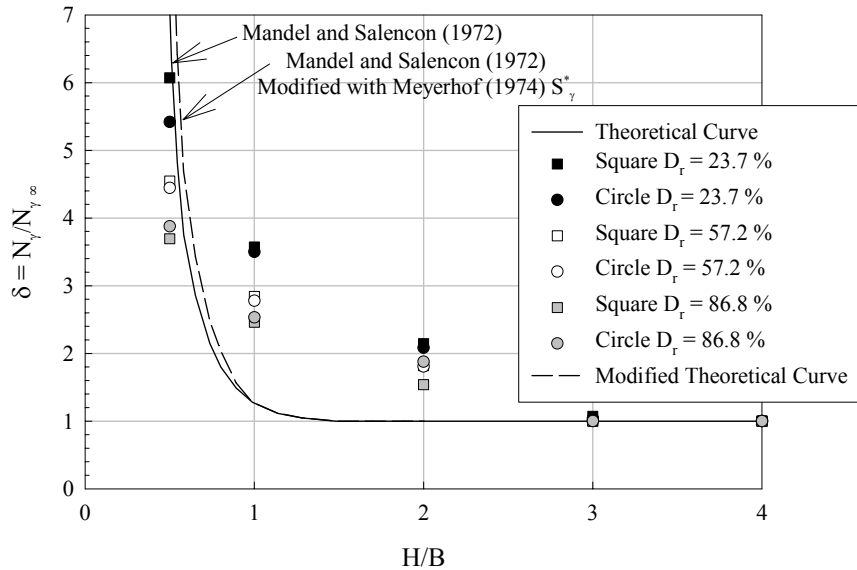


Figure 5: Comparison of Mandel and Salencon (1972) N'_γ Theory for Strip Footings with Experimental Results of N'_γ for Circular and Square Footings using $S'_\gamma = 1$.

5. Conclusions

Results of the model scale footing tests show that in the case of a footing resting on a layer of limited thickness the modified bearing capacity factor, N'_γ , is dependent on both relative density, D_r and H/B but appears to be independent of footing shape at least for square and circular footings. The normalized factor, δ , ($N'_\gamma / N_{\gamma\infty}$) show a dependence on both D_r and H/B . As D_r increases, δ decreases at values of $H/B < 3$ and becomes constant at $H/B > 3$. The results suggest that δ is not constant until $H/B >$ approximately 2.5, which is closer to Prandtl and Terzaghi's theory than previous results have shown. It is hypothesized that the difference in results between previous studies and this study may be related to the mode of failure beneath the footing for the range in relative densities used and the soil type.

From the results of this study, it can be seen that square footings show a higher bearing capacity than the circular footings of the same size, which is generally expected. Mandel and Salencon's (1972) theoretical solution for strip footings show a depth influence much less than either a circular or square footing ($1.2 B < 3 B$).

6. References

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