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THE EFFECTS OF REAL AND NOMINAL UNCERTAINTY ON INFLATION AND OUTPUT GROWTH: SOME GARCH-M EVIDENCE

KEVIN B. GRIER[a] AND MARK J. PERRY[b]*
[a]*Department of Economics, University of Oklahoma, Norman, OK 73019, USA*
[b]*Department of Economics, University of Michigan-Flint, Flint, MI 48502-2186, USA*

SUMMARY
In this paper we use GARCH-M methods to test four hypotheses about the effects of real and nominal uncertainty on average inflation and output growth in the United States from 1948 to 1996. We find no evidence that higher inflation uncertainty or higher output growth uncertainty raises the average inflation rate. We also find no support for the idea that more risky output growth is associated with a higher average real growth rate. Our key result is that in a variety of models and sample periods, inflation uncertainty significantly lowers real output growth. Copyright © 2000 John Wiley & Sons, Ltd.

1. INTRODUCTION

Since Friedman's (1977) Nobel lecture stressing the potential of inflation uncertainty to lower output growth, macroeconomists have identified several potential interactions between *average* inflation or output growth, and *uncertainty* about inflation or output growth. Besides Friedman, there have been at least three other models incorporating these interactions. Cukierman and Meltzer (1986) and Cukierman (1992) develop a game-theoretic model of Fed behaviour that predicts, among other things, that higher inflation uncertainty raises the average inflation rate. Black (1987) in his work on business cycles argues that more specialized economies will exhibit both greater risk and a higher average growth rate. Deveraux (1989) argues that less certain real output growth will lower the optimal degree of private wage indexation, making surprise inflation more effective in raising output. This flattening of the Phillips curve increases the inflationary bias in the economy.

In this paper we use GARCH-M methods to test all four hypotheses in a single model. We simultaneously estimate the conditional means, variances and covariance of inflation and output growth and test for the effects of real and nominal uncertainty on average inflation and output growth. We find no evidence that higher inflation uncertainty or higher output growth uncertainty raises the average inflation rate. We also find no support for the idea that more risky output growth is associated with a higher average real growth rate. Our key result is that *in a variety of models and sample periods, inflation uncertainty significantly lowers real output growth*. We expand on our main result by simulating the effect of an average-sized inflation surprise on subsequent inflation uncertainty and output growth, showing that the effect is sizeable.

The paper proceeds as follows. In Section 2 we consider each of the four uncertainty hypotheses in more detail. Section 3 discusses previous empirical testing in this area and introduces GARCH models and the use of conditional residual variances as parametric measures

* Correspondence to: Mark J. Perry, Department of Economics, University of Michigan-Flint, Flint, MI 48502-2186, USA; e-mail: mjperry@umich.edu

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of uncertainty. Section 4 presents our empirical results. In Section 5 we concentrate further on our strongest result that inflation uncertainty lowers output growth by studying the size and duration of the effect. Section 6 is a summary and conclusion.

2. THEORIES ON UNCERTAINTY AND AVERAGE ECONOMIC PERFORMANCE

In his Nobel address, Friedman explains a possible positive correlation between inflation and unemployment by arguing that high inflation produces more uncertainty about future inflation.[1] This uncertainty then lowers economic efficiency and temporarily reduces output and increases unemployment. He argues that increased inflation uncertainty changes optimal contract length and the degree of indexation and that unemployment may be higher during the transition to this new set of institutional arrangements. Friedman also points to the adverse effect inflation uncertainty has on extracting information from the price system. He argues that more noise in the price system reduces economic efficiency and raises unemployment, at least during some transitional period as firms adapt to the new environment.[2]

Cukierman and Meltzer (1986) and Cukierman (1992) start with the familiar Barro–Gordon model of Fed behaviour.[3] Here the Fed dislikes inflation but also seeks to stimulate the economy with surprise inflation. The lack of a commitment mechanism produces an inflationary bias in equilibrium. Cukierman and Meltzer model both the policy maker's objective function and the money supply process as random variables. Hence the public has an inference problem when observing higher inflation. Has the Fed's weight on increased employment gone up or is the higher inflation due to a random money supply disturbance? Cukierman and Meltzer show that, in their model, increases in inflation uncertainty raise the optimal average inflation rate by increasing the incentive for the policy maker to create inflation surprises. In contrast to Friedman's view that high inflation creates uncertainty, the causation in Cukierman and Meltzer is from inflation uncertainty to higher average inflation.

Deveraux (1989) also starts with the Barro–Gordon model and adds a stochastic element to money growth. He then allows workers to endogenously choose the level of wage indexation that will exist in the economy. Deveraux shows that an exogenous increase in the variability of real shocks lowers the optimal amount of wage indexation. From the perspective of the policy maker, less indexing makes surprise inflation more effective, thus increasing the incentive to create surprises. In equilibrium, the increased incentive to inflate translates into a higher average inflation rate. Even though there will be a gross correlation between average inflation and inflation uncertainty, Deveraux argues that this link is not causal, but derives from the public's response to increased real uncertainty (less indexation) and the Fed's response to declines in private indexation (more surprise inflation). The unique prediction of Deveraux's work is that output growth uncertainty increases average inflation.

[1] Ball (1992) presents a formal model where high inflation raises uncertainty due the increased uncertainty about whether or not policy stabilization will occur. In this paper we concentrate on the second part of the Friedman hypothesis that inflation uncertainty hurts real economic performance. For recent GARCH evidence about the effect of inflation on inflation uncertainty see Baillie, Chung and Tieslau (1996), Grier and Perry (1998) and Grier and Grier (1998).

[2] In support of this argument, Grier and Perry (1996) show that increases in produce price inflation uncertainty cause significant increases in the variance of relative price changes in postwar US data.

[3] Barro and Gordon (1983a,b) assume that the Fed dislikes inflation but values the higher employment that results from surprise inflation. They show that if the public has rational expectations and policy is discretionary that there is an inflationary bias to monetary policy that has no effect on employment.

Black (1987, Chapters 10 and 13) considers the relationship between aggregate risk and return. He argues that the choice of investing in a risky specialized technology will produce an economy with higher average growth and describes a tradeoff between the severity of the business cycle and the average growth rate of output. The testable hypothesis we consider from his work is that greater output growth uncertainty raises the average real growth rate.

These papers are all interrelated in that they each propose some link between uncertainty about inflation or real growth and the average level of inflation or output growth. The four hypotheses are not mutually exclusive. Higher real uncertainty could be associated with both higher average growth (Black) and more inflation (Devereux). Yet the empirical work on these issues is, with few exceptions, limited to testing Friedman's hypothesis that inflation uncertainty lowers output growth.[4]

In this paper, we use GARCH-M modelling to parametrically measure uncertainty and simultaneously test the four possible effects of uncertainty on inflation and output growth in a single statistical model.

3. A GARCH-M MODEL OF INFLATION AND OUTPUT GROWTH

3.1 Using GARCH to Measure Uncertainty

Testing any of the above theories requires the construction of a specific measure of uncertainty. The two methods typically used in the literature are the cross-sectional dispersion of individual forecasts from surveys or a moving standard deviation of the variable under consideration.[5] Neither of these techniques obviously captures the type of uncertainty modelled in the work of Cukierman and Meltzer or Deveraux, where uncertainty is the variance of the stochastic, or unpredictable, component of a variable. As is well known, there can be a very large difference between variability and uncertainty, depending on whether the variability is predictable in the model under consideration. Predictable fluctuations in a variable will show up in moving standard deviation measures although they create no true economic uncertainty.

Survey-based measures summarize the range of disagreement among individual forecasters at a point in time. However, they do not give information about each individual's uncertainty regarding their own forecast. It is possible for each forecaster to be extremely uncertain about future events but for them to submit very similar point estimates. Then, the survey measure would fail to capture the amount of existing uncertainty.[6]

In contrast to the above measures, GARCH techniques specifically estimate a model of the variance of unpredictable innovations in a variable, rather than simply calculating a variability measure from past outcomes (moving standard deviation) or from conflicting individual forecasts. That is, GARCH estimates a time-varying residual variance that corresponds well to the notion of uncertainty in Cukierman and Meltzer and Deveraux. Further, because there is a parametric model, GARCH techniques are useful for at least three other reasons. First, GARCH estimation gives an explicit test of whether the movement in the conditional variance of a variable over time is statistically significant. That is, we can construct a test of the null hypothesis that

[4] Grier and Tullock (1989) in their pooled cross-national time series model show that the standard deviation of GDP growth raises average GDP growth. Caporale and McKiernan (1998) show the same result with an ARCH-M model of annual US output growth from 1871 to 1993.

[5] Holland (1993) and Golob (1993) both contain tables that summarize many of these papers, including the measure of uncertainty employed in each.

[6] See Zarnowitz and Lambros (1987) for a discussion of using survey dispersion to measure uncertainty.

uncertainty is constant over the sample period. At a minimum, one should be able to reject this null hypothesis before doing a time series test of the effect of uncertainty on macroeconomic performance. While survey- or variability-based measures of uncertainty do fluctuate over time, papers using these measures generally do not present tests for whether those fluctuations are statistically significant.

Second, GARCH allows simultaneous estimation of the conditional variance equations and the mean equations for the variables under consideration. Pagan (1984) shows that, when working with generated regressors, simultaneous estimation is more efficient than a two-step process. All the papers using survey- or standard deviation-based measures of uncertainty are forced to use a two-step estimation process.[7]

Third, as we show in our empirical work below, both inflation and output growth exhibit significant conditional heteroscedasticity. This means that OLS estimation of an output growth equation is inefficient whether generated regressors are used or not. Engle (1982) shows that the gain in efficiency from using ARCH instead of OLS when there is significant conditional heteroscedasticity can be very large.[8] All the empirical papers on the real effects of inflation uncertainty surveyed in Golob (1993) use OLS to estimate their output (or unemployment) equation and may contain extremely inefficiently estimated coefficients.

3.2 GARCH(1,1)-M Model of Inflation and Output Growth

In our empirical work, we estimate several bivariate GARCH-M systems for inflation and output growth. The model allows us to simultaneously estimate equations for the means of inflation and output growth that include the conditional variance of both series as regressors, along with the time-varying residual covariance matrix. Several parameterizations of the general multivariate model are possible, including the constant conditional correlation model of Bollerslev (1990). In the constant correlation model, the conditional covariance matrix is time-varying but the conditional correlation across equations is assumed to be constant. The assumption of a constant correlation matrix represents a major reduction in terms of computational complexity and is commonly used in multivariate GARCH models.

Let Π_t represent inflation in period t and Y_t represent output growth in period t, both of which follow autoregressive processes. We first specify single-equation OLS models for inflation and real output growth and test the null hypotheses that each variable has a unit root and a constant conditional variance. Our sample is monthly data from 1948.07 through 1996.12 on producer prices and industrial production. Inflation is the annualized monthly difference of the log of the Producer Price Index [$\Pi_t = \log(PPI_t/PPI_{t-1}) * 1200$]. Real output growth ($Y_t$) is the annualized monthly difference in the log of industrial production [$Y_t = \log(IP_t/IP_{t-1}) * 1200$].

Panel A of Table I reports an OLS inflation equation with six lags of inflation and a twelfth-order moving average term as regressors.[9] The residuals are uncorrelated (Q-statistic $= 9{\cdot}12$ at 12 lags), but the squared residuals show the classic volatility clustering of an ARCH process. At 1

[7] Specifically, the moving standard deviation must be calculated from the inflation series, and the mean and standard deviations of the individual forecasts must be calculated for each eriod from the survey data.

[8] Grier and Perry's (1993) work provides a striking empirical example of the difference in results that can occur when existing conditional heteroscedasticity is modelled.

[9] We chose time series models for inflation and industrial production growth based on inspecting correlegrams for the series, estimating several plausible models, and then choosing the one with the highest adjusted R^2. The optimal number of lags varies slightly for the inflation equations depending on the sample period and whether the PPI or the CPI is used to calculate inflation. The results we obtain about the effect of inflation uncertainty on output growth are not sensitive to

Table I. OLS regressions for inflation and output growth — 1948.07–1996.12

A: Inflation

$$\Pi_t = \underset{(3\cdot25)}{1\cdot13} + \underset{(5\cdot24)}{0\cdot21}\,\Pi_{t-1} + \underset{(4\cdot54)}{0\cdot19}\,\Pi_{t-2} + \underset{(2\cdot17)}{0\cdot09}\,\Pi_{t-3} - \underset{(1\cdot51)}{0\cdot06}\,\Pi_{t-4} + \underset{(2\cdot38)}{0\cdot09}\,\Pi_{t-5} + \underset{(2\cdot62)}{0\cdot10}\,\Pi_{t-6} + \underset{(3\cdot89)}{0\cdot16}\,\varepsilon_{t-12} + \varepsilon_t$$

$R^2 = 0\cdot238$

Log likelihood function $= -1965\cdot34$

Ljung–Box $Q(4) = 0\cdot07$

Ljung–Box $Q(12) = 9\cdot12$

Ljung–Box $Q^2(4) = 199\cdot20$

Ljung–Box $Q^2(12) = 315\cdot58$

B: Output growth

$$Y_t = \underset{(6\cdot58)}{5\cdot35} + \underset{(9\cdot13)}{0\cdot349}\,Y_{t-1} + \underset{(4\cdot90)}{0\cdot139}\,\Pi_{t-1} - \underset{(2\cdot54)}{5\cdot25}\,SPRD_t + \underset{(5\cdot98)}{0\cdot244}\,\nu_{t-12} + \nu_t$$

$R^2 = 0\cdot2296$

Log likelihood function $= -2232\cdot61$

Ljung–Box $Q(4) = 2\cdot55$

Ljung–Box $Q(12) = 12\cdot52$

Ljung–Box $Q^2(4) = 12\cdot14$

Ljung–Box $Q^2(12) = 13\cdot93$

Sample is 582 monthly observations. Π_t is the inflation rate calculated from the Producer Price Index. Y_t is the growth rate of industrial production. $SPRD_t$ is the difference between the 6-month commercial paper rate and the 3-month treasury bill rate. *T*-statistics are in parentheses. $Q(4)$ and $Q(12)$ are the Ljung–Box statistics for fourth- and twelfth-order serial correlation in the residuals. $Q^2(4)$ and $Q^2(12)$ are the Ljung–Box statistics for fourth- and twelfth-order serial correlation in the squared residuals. The critical values at the 0·01 significance level are 11·14 and 23·33 for 4 and 12 degrees of freedom respectively. All data are from DRI Economic Database.

and 4 lags the null hypothesis of a constant error variance is rejected at the 0·01 level. We check for the stationarity of the inflation process via augmented Dickey–Fuller (ADF) tests with a constant and five lags of differenced inflation. The full-sample *t*-statistic is $-5\cdot70$. Over the 1948.07–1972.12 sub-sample the ADF statistic is $-4\cdot51$, and over the 1973.01–1996.12 sub-sample, it is $-4\cdot05$. In each case the null hypothesis of a unit root in the inflation process is rejected at the 0·01 level indicating that inflation is stationary.[10]

The output growth equation in panel B contains one lag of output growth, one lag of inflation, a twelfth-order moving average term and a default risk variable (*SPRD*). Up to four lags of inflation were considered in the output equation, but only the first lag was statistically significant. Given that the level of inflation is often thought to be related to inflation uncertainty, it is important to include the level of inflation in the output equation to distinguish between the effects of inflation on output and the effects of inflation uncertainty on output.[11] We include *SPRD*, the 6-month commercial paper–3-month treasury bill spread because it has been shown to be an excellent predictor of real output growth.[12] The OLS equation again displays uncorrelated

the particular time series models chosen for the variables. In particular, while the MA(12) terms are necessary to ensure clean residuals, dropping them does not affect our main results in any way.

[10] The same unit root tests are conducted on the growth rate of industrial production and that series is also shown to be stationary over the full-sample period and both sub-samples.

[11] We thank an anonymous referee for making this point to us.

[12] Several recent papers (Stock and Watson, 1989; Friedman and Kuttner, 1992; Bernanke and Blinder, 1992) show that interest rate spreads are significant predictors of real activity. One type of interest rate spread, the difference between private and public securities of the same maturity, is often considered a measure of default risk. Stock and Watson (1989) use the 6-month commercial paper rate–6-treasury bill spread. However, 6-month treasury bills were not traded before 1959. Our sample begins in 1948, so we use the 6-month commercial paper–3-month treasury bill spread. Other authors (e.g. Friedman and Kuttner, 1992) also use mismatched maturities to create a pre-1959 spread variable.

residuals but correlated squared residuals. The null hypothesis of a constant residual variance is rejected at the 0·01 level for 4 lags.

Since the correlation in both sets of squared residuals is somewhat persistent, we will use a GARCH(1,1) specification of the error variances of both Π_t and Y_t. Our bivariate GARCH(1,1)-M model for inflation and output growth is:

$$\Pi_t = \beta_0 + \sum_{i=1}^{6} \beta_i \Pi_{t-i} + \beta_7 \sigma^2_{\varepsilon_t} + \beta_8 \sigma^2_{\nu_t} + \beta_9 \varepsilon_{t-12} + \varepsilon_t \tag{1}$$

$$\sigma^2_{\varepsilon_t} = \alpha_0 + \alpha_1 \varepsilon^2_{t-1} + \alpha_2 \sigma^2_{\varepsilon_{t-1}} \tag{2}$$

$$Y_t = \Theta_0 + \Theta_1 Y_{t-1} + \Theta_2 \Pi_{t-1} + \Theta_3 SPRD_t + \Theta_4 \sigma^2_{\varepsilon_t} + \Theta_5 \sigma^2_{\nu_t} + \Theta_6 \nu_{t-12} + \nu_t \tag{3}$$

$$\sigma^2_{\nu_t} = \alpha_3 + \alpha_4 \nu^2_{t-1} + \alpha_5 \sigma^2_{\nu_{t-1}} \tag{4}$$

$$COV_t = \rho_{\varepsilon\nu} \sigma_{\varepsilon_t} \sigma_{\nu_t} \tag{5}$$

Equation (1) describes the mean inflation rate as a function of six lags of inflation, a twelfth-order moving average term and the conditional variances of inflation and output growth. Equation (2) gives the conditional variance of inflation. The GARCH(1,1) specification implies that the conditional error variance of inflation follows an ARMA(1,1) process. We use this estimated variance ($\sigma^2_{\varepsilon_t}$) as our time series measure of inflation uncertainty. Equation (3) describes the conditional mean of real output growth as a function of one lag of output growth, one lag of inflation, a twelfth-order moving average term, a measure of default risk (*SPRD*) and the conditional variances of inflation and output growth. Equation (4) is the GARCH(1,1) equation for the conditional variance of output growth. Finally, equation (5) is the constant conditional correlation model of the covariance between ε_t and ν_t.

We assume that the two error terms, ε_t and ν_t, are jointly conditionally normal with zero means and a conditional variance given by equations (1), (3) and (5). We estimate the system of equations (1)–(5) using the Berndt *et al.* (1974) numerical optimization algorithm to calculate the maximum likelihood estimates of the parameters. Bollerslev (1986) shows that under our assumptions, the BHHH estimate of the asymptotic covariance matrix of the coefficients will be consistent. Given that we have more than 500 observations in our full sample, our estimated asymptotic *t*-statistics should be relatively accurate.

Our statistical model incorporates tests of all four theories discussed above. The coefficient Θ_4 on the conditional variance of inflation ($\sigma^2_{\varepsilon_t}$) in the output equation directly tests Friedman's hypothesis of the effect of inflation uncertainty on real output growth. If inflation uncertainty adversely affects real output growth, Θ_4 will be negative and significant in equation (3). The Cukierman and Meltzer hypothesis that greater inflation uncertainty raises average inflation is tested by looking at β_7, the coefficient on the conditional variance of inflation in the inflation equation. A positive and significant coefficient supports Cukierman and Meltzer. The coefficient (β_8) for the conditional variance of output in the inflation equation is our test of Deveraux's hypothesis that increased real uncertainty raises average inflation. A positive and significant β_8 supports Deveraux. Finally, Black's hypothesis that more risky technology promotes a higher average growth rate is represented by Θ_5, the coefficient on the conditional variance of output growth in the output growth equation. Θ_5 should be positive and significant to confirm Black's hypothesis.[13]

The model described above does not include lagged output growth in the inflation equation. We estimated models with such lags, finding that they were never significant and that our key results were not sensitive to their exlcusion. The model also does not allow the covariance of the errors to affect the conditional means of inflation or output growth. While there are theoretical reasons (described above) for including both conditional variances in each conditional mean equation, there is no such reason for including the covariance, nor would we have any interpretation of the results. For the same reason, the interest rate spread is excluded from the inflation equation. There is an extensive literature on interest rate spreads predicting output growth, but nothing suggests that the spread can predict inflation.

4. EMPIRICAL RESULTS

4.1 Full Sample Results

Table II reports estimates of the GARCH(1,1)-M model shown above. The mean and conditional residual variance equations for inflation are reported in equations (1) and (2) of Table II. The sum of the lagged inflation coefficients is 0·57 here, which is similar to the 0·62 sum in the OLS model. The GARCH(1,1) parameters in the residual variance equation are significant at the 0·01 level, and the relatively large coefficient (0·776) on $\sigma^2_{\varepsilon_{t-1}}$ means that inflation shocks create persistent inflation uncertainty. The GARCH-in-mean variables are both completely insignificant in equation (1), showing that in this sample and specification we find no effect of either inflation uncertainty or output growth uncertainty on average inflation. Our GARCH-M model fails to provide any statistical support for the Cukierman and Meltzer or Deveraux hypotheses.

Equations (3) and (4) in Table II report the estimates of the mean and conditional residual variance of output growth. The lagged output growth coefficients in equation (3) is 0·31, which is similar to the coefficient in the OLS equation (0·349). Our measure of default risk, *SPRD*, is negative and significant at the 1% level in both the GARCH and the OLS models. The lagged level of inflation is positive and significant at the 1% level in both the GARCH and the OLS models. The GARCH(1,1) parameters in the residual variance equation are both significant at the 0·01 level. However, the coefficient on the lagged residual variance is smaller for output growth (0·658) than for inflation (0·776), suggesting that output growth shocks have shorter-lived effects on output uncertainty than inflation shocks have on inflation uncertainty.

The Friedman hypothesis that inflation uncertainty lowers output growth implies a negative and significant coefficient on the residual variance of inflation in the output growth equation. The estimated coefficient on $\sigma^2_{\varepsilon_t}$ in the output equation is negative (−1·03) and significant at the 0·01 level (t-statistic = 3·28). Our results thus provide strong empirical confirmation of Friedman's hypothesis. Inflation uncertainty has a statistically significant negative effect on real output growth in the United States over the 1948.07–1996.12 period. There is no evidence that risky

[13] Grier and Tullock (1989) and Caporale and McKiernan (1998) call testing whether there is a positive relationship between output growth uncertainty and the level of growth a test of Black's hypothesis. Strictly speaking it is a correlation implied by the hypothesis; Black argued that technology choice determined both the risk and average return for an economy.

Table II. Inflation (PPI) and output growth — 1948.07–1996.12

GARCH(1,1)-M system — constant conditional correlations

$$(1)\ \Pi_t = \underset{(1\cdot30)}{1\cdot39} + \underset{(5\cdot76)}{0\cdot26}\,\Pi_{t-1} + \underset{(2\cdot90)}{0\cdot13}\,\Pi_{t-2} + \underset{(1\cdot24)}{0\cdot06}\,\Pi_{t-3} - \underset{(0\cdot63)}{0\cdot03}\,\Pi_{t-4} + \underset{(1\cdot33)}{0\cdot06}\,\Pi_{t-5} + \underset{(2\cdot05)}{0\cdot09}\,\Pi_{t-6} + \underset{(3\cdot82)}{0\cdot13}\,\varepsilon_{t-12} - \underset{(0\cdot12)}{0\cdot026}\,\sigma^2_{\varepsilon_t} - \underset{(0\cdot14)}{0\cdot166}\,\sigma^2_{\nu_t} + \varepsilon_t$$

$$(2)\ \sigma^2_{\varepsilon_t} = \underset{(3\cdot27)}{1\cdot91} + \underset{(5\cdot95)}{0\cdot175}\,\varepsilon^2_{t-1} + \underset{(24\cdot82)}{0\cdot776}\,\sigma^2_{\varepsilon_{t-1}}$$

$$(3)\ Y_t = \underset{(2\cdot96)}{5\cdot98} + \underset{(8\cdot22)}{0\cdot31}\,Y_{t-1} + \underset{(5\cdot04)}{0\cdot20}\,\Pi_{t-1} - \underset{(4\cdot54)}{4\cdot74}\,SPRD_t - \underset{(3\cdot56)}{0\cdot15}\,\nu_{t-12} - \underset{(3\cdot28)}{1\cdot03}\,\sigma^2_{\varepsilon_t} + \underset{(0\cdot98)}{3\cdot45}\,\sigma^2_{\nu_t} + \nu_t$$

$$(4)\ \sigma^2_{\nu_t} = \underset{(4\cdot67)}{16\cdot69} + \underset{(6\cdot01)}{0\cdot191}\,\nu^2_{t-1} + \underset{(13\cdot3)}{0\cdot658}\,\sigma^2_{\nu_{t-1}}$$

$$(5)\ COV_t = \underset{(0\cdot74)}{0\cdot033}\,\sigma_{\varepsilon_t}\sigma_{\nu_t}$$

	Inflation eqn	Output eqn	Cross-eqn
$Q(4)$	0·62	5·49	3·10
$Q(12)$	11·31	15·98	9·05
$Q^2(4)$	2·41	2·49	–
$Q^2(12)$	19·92	5·70	–

Log likelihood function = − 4014·47

Sample is 582 monthly observations. Π_t is the inflation rate calculated from the Producer Price Index. Y_t is the growth rate of industrial production. $SPRD_t$ is the difference between the 6-month commercial paper rate and the 3-month treasury bill rate. T-statistics are in parentheses. $Q(4)$ and $Q(12)$ are the Ljung–Box statistics for fourth- and twelfth-order serial correlation in the residuals. $Q^2(4)$ and $Q^2(12)$ are the Ljung–Box statistics for fourth- and twelfth-order serial correlation in the squared residuals. The critical values at the 0·05 significance level are 9·48 and 21·02 for 4 and 12 degrees of freedom respectively. All data are from DRI Economic Database.

output growth is positively correlated with average output growth since the coefficient on $\sigma^2_{\nu_t}$ in the output equation (3) in Table II is insignificant.

We calculate Ljung–Box Q-statistics at 4 and 12 lags for the levels, squares and cross-equation products of the standardized residuals for the estimated GARCH(1,1)-M system.[14] The results reported in Table II show that the time series models for the conditional means and the GARCH(1,1) model for the residual conditional variance-covariance adquately captures the joint distribution of the disturbances. The conditional correlation coefficient is close to zero, suggesting that the residual covariance between equations is not statistically significant.[15]

[14] For all our GARCH estimations, we report $Q(4)$ and $Q(12)$ statistics, which test for serial correlation at four and twelve lags in the standardized inflation residuals, the standardized output growth residuals and the cross-products of those two series. We also report $Q^2(4)$ and $Q^2(12)$ statistics, which test for fourth- and twelfth-order serial correlation in the squared inflation and output growth residuals. The Q^2-tests are designed to see whether our GARCH model accounts for all the conditional heteroscedasticity in the two series.

[15] We have also estimated our GARCH-M models using an alternative model of the conditional covariance matrix. Baba *et al.* (1989, unpublished manuscript) propose a 'positive definite parameterization', where the residual covariance matrix (H_t) is given by: $H_t = C_0^T C_0 + C_1^T(\varepsilon_{t-1}\nu_{t-1})C_1 + C_2^T H_{t-1} C_2$. Here H_t is the 2×2 conditional covariance matrix, C_0, C_1, and C_2 are all 2×2 coefficient matrices with C_0 symmetric and T indicating matrix transposition. Results from this parameterization are quite similar to those reported in the text using the constant correlations parameterization. Inflation uncertainty is still a negative and significant determinant of output growth, the sum of the coefficients on lagged inflation does not materially change and the residual diagnostics are similar and acceptable.

Table III. Inflation (PPI) and output growth — 1948.07–1972.12

GARCH(1,1)-M system — constant conditional correlations

$$(1)\ \Pi_t = \underset{(0.71)}{1.17} + \underset{(3.87)}{0.22}\,\Pi_{t-1} + \underset{(3.21)}{0.19}\,\Pi_{t-2} + \underset{(1.36)}{0.08}\,\Pi_{t-3} + \underset{(2.37)}{0.05}\,\Pi_{t-4} - \underset{(0.27)}{0.123}\,\sigma^2_{\varepsilon_t} + \underset{(0.10)}{0.138}\,\sigma^2_{\nu_t} + \varepsilon_t$$

$$(2)\ \sigma^2_{\varepsilon_t} = \underset{(1.40)}{0.667} + \underset{(2.05)}{0.061}\,\varepsilon^2_{t-1} + \underset{(24.3)}{0.915}\,\sigma^2_{\varepsilon_{t-1}}$$

$$(3)\ Y_t = \underset{(2.01)}{9.42} + \underset{(4.59)}{0.315}\,Y_{t-1} + \underset{(2.08)}{0.250}\,\Pi_{t-1} - \underset{(2.92)}{7.16}\,SPRD_t - \underset{(4.23)}{0.254}\,\nu_{t-12} - \underset{(2.81)}{2.07}\,\sigma^2_{\varepsilon_t} + \underset{(0.97)}{3.45}\,\sigma^2_{\nu_t} + \nu_t$$

$$(4)\ \sigma^2_{\nu_t} = \underset{(3.87)}{34.86} + \underset{(3.63)}{0.210}\,\nu^2_{t-1} + \underset{(6.27)}{0.552}\,\sigma^2_{\nu_{t-1}}$$

$$(5)\ COV_t = \underset{(0.69)}{0.051}\,\sigma_{\varepsilon_t}\sigma_{\nu_t}$$

	Inflation eqn	Output eqn	Cross-eqn
$Q(4)$	0·10	3·37	3·24
$Q(12)$	9·04	8·88	6·10
$Q^2(4)$	1·78	0·91	–
$Q^2(12)$	6·56	3·14	–

Log likelihood function = − 2069·14

Sample is 582 monthly observations. Π_t is the inflation rate calculated from the Producer Price Index. Y_t is the growth rate of industrial production. $SPRD_t$ is the difference between the 6-month commercial paper rate and the 3-month treasury bill rate. T-statistics are in parentheses. $Q(4)$ and $Q(12)$ are the Ljung–Box statistics for fourth- and twelfth-order serial correlation in the residuals. $Q^2(4)$ and $Q^2(12)$ are the Ljung–Box statistics for fourth- and twelfth-order serial correlation in the squared residuals. The critical values at the 0·05 significance level are 9·48 and 21·02 for 4 and 12 degrees of freedom respectively. All data are from DRI Economic Database.

4.2 Pre-1973 Results

To investigate the robustness of our results, we estimate the GARCH(1,1)-M model discussed above on a pre-oil-shock sub-sample consisting of observations from 1948.07 through 1972.12. The results are shown in Table III for the constant residual correlation model. Note that the conditional variance equations for inflation and output growth over the sub-sample as shown in Table III are similar to the full-sample results reported in Table II. While the degree of inflation uncertainty changes over the sample, the process generating it is relatively stable. Both inflation and output growth exhibit significant and persistent conditional heteroscedasticity even in this early period.

The negative effect of inflation uncertainty on output growth also obtains in the sub-sample. The relevant coefficient in Table III is −2·07 and is negative and significant at the 0·01 level ($t = 2.81$). The positive definite residual covariance matrix parameterization produces the same basic results (coefficient of −1·53 and t-statistic of 2·65).

4.3 Post-1953 Results

Given the possible effects of wartime price controls on inflation before 1953, and Cosimano and Jansen's (1988) finding that inflation does not exhibit significant conditional heteroscedasticity in a post-1953 sample period, we replicate our analysis on a 1954.01–1996.12 sub-sample. These results are reported in Table IV. As shown in equation (2) of Table IV, we continue to find a significant degree of conditional heteroscedasticity in inflation, and as shown in equation (3),

Table IV. Inflation (PPI) and output growth — 1954.01–1996.12

GARCH(1,1)-M system — constant conditional correlations

(1) $\Pi_t = \underset{(0.71)}{0.92} + \underset{(4.90)}{0.25}\,\Pi_{t-1} + \underset{(2.14)}{0.10}\,\Pi_{t-2} + \underset{(1.57)}{0.09}\,\Pi_{t-3} - \underset{(0.67)}{0.03}\,\Pi_{t-4} + \underset{(0.54)}{0.03}\,\Pi_{t-5} + \underset{(2.39)}{0.11}\,\Pi_{t-6} + \underset{(3.57)}{0.13}\,\varepsilon_{t-12} + \underset{(0.56)}{0.138}\,\sigma^2_{\varepsilon_t} + \underset{(0.19)}{0.325}\,\sigma^2_{\nu_t} + \varepsilon_t$

(2) $\sigma^2_{\varepsilon_t} = \underset{(3.35)}{2.13} + \underset{(6.04)}{0.177}\,\varepsilon^2_{t-1} + \underset{(22.90)}{0.760}\,\sigma^2_{\varepsilon_{t-1}}$

(3) $Y_t = \underset{(3.19)}{6.36} + \underset{(5.32)}{0.26}\,Y_{t-1} + \underset{(1.64)}{0.08}\,\Pi_{t-1} - \underset{(4.96)}{5.48}\,SPRD_t - \underset{(2.84)}{0.12}\,\nu_{t-12} - \underset{(2.51)}{0.76}\,\sigma^2_{\varepsilon_t} + \underset{(0.78)}{3.41}\,\sigma^2_{\nu_t} + \nu_t$

(4) $\sigma^2_{\nu_t} = \underset{(4.45)}{13.79} + \underset{(5.50)}{0.189}\,\nu^2_{t-1} + \underset{(13.52)}{0.670}\,\sigma^2_{\nu_{t-1}}$

(5) $COV_t = \underset{(0.54)}{0.026}\,\sigma_{\varepsilon_t}\sigma_{\nu_t}$

	Inflation eqn	Output eqn	Cross-eqn
$Q(4)$	0·18	6·73	4·71
$Q(12)$	14·41	15·32	11·02
$Q^2(4)$	2·96	5·50	–
$Q^2(12)$	21·59	11·44	–

Log likelihood function = − 3474·92

Sample is 516 monthly observations. Π_t is the inflation rate calculated from the Producer Price Index. Y_t is the growth rate of industrial production. $SPRD_t$ is the difference between the 6-month commercial paper rate and the 3-month treasury bill rate. T-statistics are in parentheses. $Q(4)$ and $Q(12)$ are the Ljung–Box statistics for fourth- and twelfth-order serial correlation in the residuals. $Q^2(4)$ and $Q^2(12)$ are the Ljung–Box statistics for fourth- and twelfth-order serial correlation in the squared residuals. The critical values at the 0·05 significance level are 9·48 and 21·02 for 4 and 12 degrees of freedom respectively. All data are from DRI Economic Database.

inflation uncertainty continues to lower output growth (coefficient of −0·76 and t-statistic of 2·51).[16] Our result that inflation uncertainty has adverse real effects is fairly robust to the choice of sample period.

4.4 Results Using CPI Inflation

Given that we are working with industrial production as our output growth measure, it seems natural to construct our inflation and inflation uncertainty measures from the index of producer prices. However, since many other studies use CPI-based inflation measures, in this sub-section we replicate the results in Tables II–IV using this alternative inflation measure.

Table V presents the full-sample results, the pre-oil shock period, and our post-1953 sub-sample system.[17] For the full-sample period, the PPI and CPI results are virtually identical. The effect of inflation uncertainty on output growth is still negative and significant (coefficient of −2·09, t-statistic of 2·94). The result of greater inflation uncertainty causing lower output growth also prevails strongly in the pre-1973 sub-sample using the CPI (coefficient of −3·05, t-statistic of 2·68). The only case where this result weakens slightly is in the post-1953 period (coefficient of −1·93, t-statistic of 2·01).

[16] Although our results differ from those of Cosimano and Jansen, it should be noted that their sample was quarterly instead of monthly, and did not include the 1990s.

[17] To save space, only the relevant coefficients are reported in Table V for the GARCH-M models using the CPI measure of inflation. In addition, Q-test results for fourth- and twlefth-order serial correlation in the residuals and squared residuals are not reported, but the residual diagnostics remain acceptable in all estimations.

Table V. Inflation (CPI) and output growth

Sample period	1948.01–1996.12	1948.01–1972.12	1954.01–1996.12
Estimates of selected coefficients from GARCH-M system of equations			
β_7	0·20	0·13	0·46
(inflation uncertainty in inflation eqn)	(0·92)	(0·38)	(1·43)
β_8	−0·76	−0·53	−1·15
(output uncertainty in inflation eqn)	(1·32)	(0·43)	(1·02)
Θ_4	−2·09	−3·05	−1·93
(inflation uncertainty in output eqn)	(2·94)	(2·69)	(2·01)
Θ_5	3·63	7·60	4·83
(output uncertainty in output eqn)	(1·17)	(0·72)	(0·94)

T-statistics are in parentheses. Inflation is calculated from the Consumer Price Index. All data are from DRI Economic Database.

In all our results, the hypothesis that virtually always receives significant statistical support is Friedman's claim of a negative link between inflation uncertainty and real output growth. In the next section we focus on this link by providing some insight into the size and duration of the effect implied by our GARCH-M model.

5. DISCUSSION

Our work shows that increases in inflation uncertainty are significantly associated with lower rates of real output growth in the United States during the 1948–1996 period. To gauge the quantitative importance of the relationship, consider the effect of an inflation surprise on inflation uncertainty and output growth in the bivariate GARCH(1,1)-M model presented in Section 2 (equations (1)–(5)). An inflation surprise in period t, ε_t, raises the conditional variance of inflation in period $t+1$. The initial increase in inflation uncertainty is given by: $\Delta\sigma^2_{\varepsilon_{t+1}} = \alpha_1(\varepsilon_t^2)$. The effect of the inflation surprise in period t on inflation uncertainty will persist over time due to the autoregressive term in the conditional variance equation (i.e. equation (2)) with coefficient α_2. The larger is α_2, the longer a single inflation shock will affect the conditional variance.

Initially the higher conditional variance lowers output growth (in equation (3)) by Θ_4 times the higher variance. For positive shocks, this initial effect is partly offset by the positive coefficient on lagged inflation, Θ_2, in the growth equation (for negative shocks, the initial negative effect is reinforced). However, output growth is also an autoregressive process and the coefficients on lagged output growth create additional real persistence of the effect of inflation uncertainty.

Figure 1 illustrates the effect of a positive inflation shock. We assume a one-time inflation surprise of +4·00 percentage points, which is the average absolute value of the inflation residuals, and use the coefficients α_1, α_2, Θ_1, Θ_2 and Θ_4 from the model estimated in Table II. The inflation shock occurs in period zero. Inflation uncertainty rises and then declines, while output growth falls and then rises. It takes almost two years for the real effects of the inflation surprise to disappear. Two months after the shock, output growth is depressed by about 3 percentage points (on an annualized basis), and after 6 months output growth is still about 0·8 percentage point lower than in the absence of the shock. The negative effect of inflation uncertainty on industrial production growth is non-trivially large and also statistically significant.

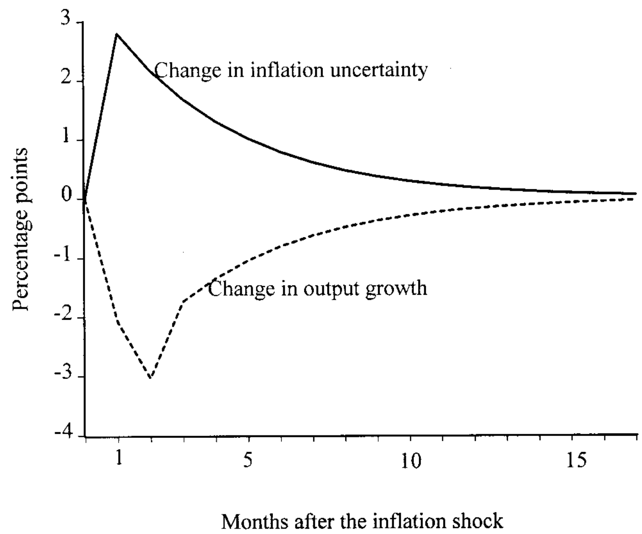

Figure 1. The effect of a 4% positive inflation surprise on inflation uncertainty and output growth

Given the model in equation (2) (Table II), a *permanent* 1 percentage point rise in inflation uncertainty would result in about a 1 percentage point permanent decline in output growth (holding the average level of inflation constant). This effect is rather larger than the one found in Holland (1988) where he estimates the effect of a 4 percentage points increase in uncertainty to depress output growth by 2 percentage points.[18]

6. CONCLUSION

In this paper we use a bivariate GARCH-M model to simultaneously examine the relationship between uncertainty and average outcomes for inflation and output growth. We first show that there is significant, persistent conditional heteroscedasticity in both variables and then present tests of four hypotheses about how uncertainty influences inflation or real growth.

Two of these hypotheses are extensions of the Barro–Gordon model of Fed behaviour. They predict that the causation runs from uncertainty to average inflation. In the Cukierman and Meltzer model the optimal Fed response to greater inflation noise is to raise the average inflation rate. In the Deveraux model, the optimal Fed response to increased output uncertainty is to raise the average inflation rate. Our results fail to find any effect of either type of uncertainty on average inflation. In addition, we find no statistical evidence that the conditional variance of output growth is positively correlated with average output growth as suggested by Black. However, we find that, in every estimation and in every sample period, the conditional variance of inflation significantly lowers average output growth as argued by Friedman.

[18] Brunner (1993) has suggested that models claiming that inflation uncertainty lowers output growth may suffer from reverse causation, where lower output growth raises uncertainty about the future path of inflation. We have conducted preliminary two-step tests of this idea by estimating a GARCH(1,1) model of inflation uncertainty, and then pairwise Granger causality tests for inflation uncertainty and output growth. We find that Granger causality runs uniquely from uncertainty to output growth using anywhere from 1 to 20 lags in the regressions. In future work, we hope to develop a simultaneous test for possible reverse causation.

We believe that the negative effect of inflation uncertainty on output growth uncovered here is large enough and significant enough to merit serious attention. The effect we find is not dependent on the exact sample, model or specification used, and it is not trivially small. It is not, however, directly predicted or explicitly ruled out by any formal macroeconomic theory.[19] It is perhaps ironic that the hypothesis with the least elaborate theory behind it receives the most statistical support among the four considered here. There is an urgent need for further work to provide a stronger theoretical basis or reasoning for the strong negative statistical link between inflation uncertainty and output growth.

DATA APPENDIX — SUMMARY STATISTICS

Variable	Mean	Min	Max	Std deviation
Inflation — PPI	3·16	−22·1	67·55	8·17
Inflation — CPI	3·88	−11·1	21·5	4·05
Industrial production	3·45	−51·7	76·3	12·87
Spread	0·665	0·08	4·17	0·44

All data are from DRI Economic Database. Sample period is monthly, from 1948.07 to 1996.12. Monthly inflation rates are calculated from the Producer Price Index and the Consumer Price Index respectively at annual rates. Industrial Production is the monthly growth rate of the Industrial Production Index at an annual rate. Spread is the difference between the 6-month commercial paper rate and the 3-month treasury bill rate.

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[19] Friedman's (1977) Nobel address is what generated the empirical literature on inflation uncertainty and output growth. However, his speech focuses mostly on the efficiency effects of inflation uncertainty and refers to the employment effects as 'less clear'. Okun (1971) also has an extremely informal discussion of the possible adverse effects of uncertain inflation.

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