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# Regulations, regime switches and non-monotonicity when non-compliance is an option: an application to content protection and preference

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## Abstract

Direct regulations have two regimes. In one, all firms behave in the same manner and, in the other, they behave differently. Past work assumed all firms were identical, thereby neglecting the non-monotonicity in comparative statics arising from the regime change.

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## 1. Introduction

Regulations which directly impact the method of production are increasingly common these days. Environmental restrictions where reductions of emissions can be attained only by changing the mix of fuels from dirty (coal) to clean (natural gas) are one example. Policies against discrimination which set aside a fraction of employment or admissions for disadvantaged groups are another. Content preference schemes which need goods to be produced in a certain way to qualify for preferential tariff treatment are a third example.

Such regulations are shown to have two distinct regimes termed the homogeneous and heterogeneous regimes. In the homogenous regime, all firms behave in the same manner, while in the heterogeneous regime, they do not. Making restrictions tighter has opposite effects in the two regimes.

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This result has been overlooked in the literature (see, for example, Grossman, 1981; Mussa, 1984). This oversight comes from assuming that there is only one firm, or that all firms behave in the same manner, neglecting firm level differences in behavior that emerge in equilibrium even with ex ante identical firms. When firms are indifferent between different options, as they will be when profits from the two options are the same, some firms could choose one option while others choose another. As the restriction becomes more severe, the mix of firms meeting the restriction adjusts so that profits remain equal for the two options. In such a regime, comparative static effects are very different from those when all firms behave in an identical manner.

With a content preference scheme which restricts the input mix used by firms if they are to obtain preferential tariff treatment our results are as follows. When the restriction is not very severe, all firms wish to meet it, and more restrictive requirements raise the use of protected inputs and their price and reduce output. As the restriction becomes even more severe firms become as well off meeting the restriction as not. At this point the regime changes and further restrictions reduce the use of protected inputs as well as their price, while raising total output. Similar results are obtained for the other applications mentioned.

Our work is most closely related to that of Grossman (1981) but differs from his as we allow ex ante identical firms to behave differently in equilibrium. By implicitly assuming that all firms behave in the same manner Grossman (1981) misses an essential part of the behavior of firms.

## 2. The model

Under a content preference scheme specified in physical terms, a firm can sell its final output at price  $P$  if it meets the requirement that it use at least a fraction  $k$  of domestic inputs. Its production function is given by  $F(T, m + m^*)$  where  $T$  denotes the level of a fixed factor, and  $m$  and  $m^*$  denote the use of domestic and imported variable inputs. Their prices are denoted by  $P_m$  and  $P_m^*$ , respectively. If the firm fails to meet this restriction then it obtains a lower price  $P/(1+t)$ , where  $t$  is the penalty tariff imposed. We assume that  $P_m^*$ ,  $t$  and  $P$  are exogenous while  $P_m$  is endogenously determined.

$M(P, \min(P_m, P_m^*))$  is the total input demand function for a given firm in the absence of any restriction as only the cheaper input will be used. Suppose there are  $n$  firms in total and  $n$  is fixed. If  $P_m \leq P_m^*$  then all firms will weakly prefer the domestic input and the restriction is automatically met. If  $P_m > P_m^*$  then all firms would prefer to use the imported input if there were no content preference, but may choose to use the domestic one in order to qualify for a higher output price.

If  $P_m > P_m^*$  and the restriction is not met, the firm maximizes

$$\left(\frac{P}{1+t}\right)F(T, m^*) - P_m^*m^*. \quad (1)$$

The maximized value of profits is  $\Pi(P/(1+t), P_m^*)$ . Using the envelope theorem, the demand for inputs by a firm is

$$\Pi_{P_m^*} \left(\frac{P}{1+t}, P_m^*\right) = -M \left(\frac{P}{1+t}, P_m^*\right).$$

If the restriction is met the firm maximizes

$$PF(T, m + m^*) - P_m m - P_m^* m^*$$

subject to  $m/(m + m^*) \geq k$ . Let  $\bar{\Pi}(P, P_m, k)$  be the value function for this problem. If the restriction is met exactly, as it will be when it is binding, we get  $m = [k/(1 - k)]m^*$  as well as  $m + m^* = m^*/(1 - k)$ . Hence,

$$\begin{aligned} P_m m + P_m^* m^* &= P_m \frac{k}{1 - k} m^* + P_m^* m^* \\ &= \frac{(P_m k + (1 - k)P_m^*)}{1 - k} m^* \\ &= (P_m k + (1 - k)P_m^*)(m + m^*). \end{aligned}$$

This gives profits as

$$PF(T, m + m^*) - \phi(k, P_m)(m + m^*), \tag{2}$$

where  $\phi(k, P_m) = kP_m + (1 - k)P_m^*$ . From (1) and (2) it follows that

$$\bar{\Pi}(P, P_m, k) \equiv \Pi(P, \phi(k, P_m)).$$

$\phi(k, P_m)$  can be interpreted as the ‘virtual price’. Exploiting this identity gives

$$\Pi_\phi(P, \phi(k, P_m)) = -M(P, \phi(k, P_m)),$$

which is total input demand by a firm under the restriction.

We can see using the envelope theorem that  $\Pi_P(\cdot, \cdot) = F(T, M(\cdot)) > 0$ , and  $\Pi_\phi(\cdot, \cdot) = -M(\cdot) < 0$ . Also, since the value function for a maximization problem is convex in prices,  $\Pi_{\phi\phi}(\cdot, \cdot) > 0$  and  $\Pi_{PP}(\cdot, \cdot) > 0$  or total demand for inputs is downward sloping and output is increasing in  $P$ . Moreover, as  $\phi(k, P_m)$  is increasing in  $k$  and  $P_m$ ,  $\Pi(\cdot)$  falls as  $k$  or  $P_m$  rises.

When is the restriction met? If  $P_m < P_m^*$  then it is automatically met. If  $P_m > P_m^*$ , but not by a lot, then

$$\Pi(P, \phi(k, P_m)) > \Pi\left(\frac{P}{1 + t}, P_m^*\right),$$

so that firms will be better off meeting the restriction and we are in the homogeneous regime. As  $P_m$  rises there comes a point, at  $P_m = \tilde{P}_m(k)$ , defined by

$$\Pi(P, \phi(k, \tilde{P}_m)) = \Pi\left(\frac{P}{1 + t}, P_m^*\right), \tag{3}$$

where firms are indifferent between meeting the restriction and not doing so. As a result, firm behavior can differ and we are in the heterogeneous regime.

If  $P_m$  rises further, then all firms would prefer not meeting the restriction. Since  $P_m$  is determined by the intersection of demand and supply for domestic inputs,  $P_m$  will never rise above  $\tilde{P}_m(k)$  in equilibrium. If it did, there would be no demand for the domestic input and since supply is positive, this could not be an equilibrium.

Now consider the demand for domestic inputs. The total demand by all firms for inputs,  $nM(P, \cdot)$ , is



### 3. Comparative statics

The comparative statics are driven by the behavior of  $P_m$  in response to changes in  $k$ . The endogenous domestic input price is determined by the intersection of supply and demand as depicted in Fig. 1. Demand shifts with changes in  $k$ . As shown in Lemma 1 the horizontal segment consisting of  $\tilde{P}_m(k)D$  shifts downwards as  $k$  rises.

**Lemma 1.**

$$\frac{d\tilde{P}_m(k)}{dk} < 0.$$

**Proof.** From (3) it follows that  $\phi(k, P_m)$  must be kept constant for (3) to remain true. Since  $\phi(k, P_m)$  is increasing in  $k$  and  $P_m$ , an increase in  $k$  needs a decrease in  $\tilde{P}_m(k)$  for  $\phi(k, P_m)$  to be constant. Hence,  $\tilde{P}_m(k)$  is decreasing in  $k$ .  $\square$

Now we consider the effect on its downward sloping segment,  $DF$ .

**Lemma 2.**

$$\frac{d[nkM(P, \phi(k, P_m))]}{dk} > 0, \text{ if } \epsilon < 1/\theta,$$

where  $\epsilon$  is the elasticity of  $M(P, \phi)$  with respect to  $\phi$  defined as a positive number and

$$\theta = \frac{k(P_m - P_m^*)}{k(P_m - P_m^*) + P_m^*} < 1.$$

**Proof.** That  $nkM(P, \phi(k, P_m))$  is downward sloping follows from  $\phi(k, P_m)$  rising with  $P_m$  which reduces  $M(\cdot)$ . It is easy to see that  $nkM(P, \phi(k, P_m))$  shifts out as  $k$  rises if input demand is not too elastic, since

$$\begin{aligned} \frac{dnkM(P, \phi(k, P_m))}{dk} &= n[M(P, \phi(k, P_m)) + kM_\phi(P, \phi(k, P_m))\phi_k(k, P_m)] \\ &= nM(P, \phi(k, P_m))(1 - \epsilon\theta) \\ &> 0, \end{aligned} \tag{4}$$

if  $\epsilon < 1/\theta$ .  $\square$

An increase in  $k$  to  $k'$  shifts the demand for domestic inputs as depicted in Fig. 1. It shifts the downward sloping component (the homogeneous regime) outwards and the upper horizontal component (heterogeneous regime) downwards. As a result, in the homogeneous regime, an increase in  $k$  raises the domestic input price and usage, but reduces output. In the heterogeneous regime, an increase in  $k$  reduces the domestic input price and usage.

**Lemma 3.** *In the heterogeneous regime, the output of firms who meet the restriction is less than that of those who do not. Moreover, an increase in  $k$  reduces the number of firms who meet the restriction so that total output rises.*

**Proof.** The intuition is that firms who meet the restriction face a higher real input price, i.e.

$\phi(k, P_m)/P > (1 + t)P_m^*/P$ , and so produce less. An increase in  $k$  does not alter  $\phi(k, P_m)$  or total input usage. However, as  $\tilde{P}_m$  falls, the total usage of the domestic input falls though each firm meeting the restriction uses more of it and, as a result, the number of such firms must fall and total output must rise. Details are available on request.  $\square$

**Proposition 1.** *Let content preference be defined in physical terms. For  $0 \leq k < k_0$ , all firms choose to meet the restriction and an increase in  $k$  has no effect on the equilibrium. For  $k_0 \leq k < k_1$ , all firms still choose to meet the restriction, though it is costly to do so. In this region (referred to as the homogeneous regime) an increase in  $k$  raises usage and price of protected input but reduces total output. In contrast, when  $k_1 \leq k \leq 1$ , firms choose to behave differently (referred to as the heterogeneous regime) and an increase in  $k$  reduces usage and the price of protected input but raises total output.*

**Proof.** It only remains to show when each regime occurs, for which we use Fig. 2. At very low levels of  $k$ , for  $k < k_0$ , the intersection of domestic input demand and supply occurs along the lower horizontal part of the demand curve so the equilibrium price of domestic inputs is fixed at  $P_m^*$ . For  $k$  between  $k_0$  and  $k_1$ , the intersection of demand and supply occurs along the downward sloping part of domestic input demand. The restriction is just met by all firms and an increase in  $k$  causes an increase in price and usage of the domestic input and a fall in output. For  $k$  between  $k_1$  and 1, the intersection of demand and supply occurs along the upper horizontal part of demand at  $\tilde{P}_m(k)$  so that we are in the heterogeneous regime and an increase in  $k$  reduces the price and total usage of the domestic input and raises total output of the final good.  $\square$

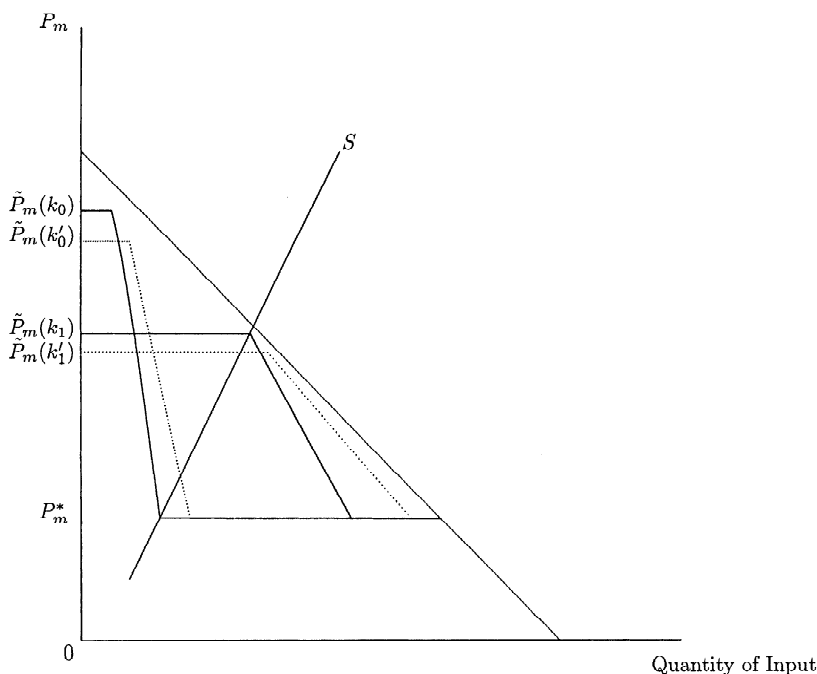


Fig. 2. Regime switch.

Thus, the path of variables such as aggregate output, the price of the domestic input and use of the domestic input are all non-monotonic and have a single common turning point at  $k = k_1$  where the regime switch occurs. It can be shown that our results hold even when content preference is couched in value terms.

Content protection can be analyzed in the same manner. With content protection, access to imported inputs is given at zero tariffs when the physical content restriction is met. Hence, profits from meeting the restriction are  $\Pi(P, kP_m + (1-k)P_m^*)$ . If the restriction is not met, then they are given by  $\Pi(P, P_m^*(1+t_m))$  where  $t_m$  is the penalty tariff. If  $P_m^*(1+t_m) > kP_m + (1-k)P_m^*$ , i.e. if  $P_m < P_m^*[1+(t_m/k)]$ , then it is better to meet the restriction. If  $P_m > P_m^*[1+(t_m/k)]$ , then it is better not to meet the restriction. At  $P_m = P_m^*[1+(t_m/k)]$ , firms are indifferent between meeting the restriction or not and  $P_m^*[1+(t_m/k)]$  corresponds to  $\tilde{P}_m(k)$  in the discussion of content preference. Now the analysis follows as before except for the fact that when  $P_m = P_m^*[1+(t_m/k)]$ , firms who meet the restriction, and those who do not, have the same input cost and hence the same output level. As a result, aggregate output will be constant as  $k$  rises in the heterogeneous regime.

#### 4. Conclusion

Our analysis applies to a number of other problems. Consider, for example, the effects of employment quotas for disadvantaged workers who are identical to all others in their productivity. Employment quotas force employers to hire at least a given share of disadvantaged workers or face penalties. Our analysis suggests that if this share is not too large, wages of disadvantaged workers would rise as the share rose, but once this share reached a certain level, further increases in the share would only reduce the wages of disadvantaged workers.

Alternatively, consider the effect of restricting the use of fuels to cleaner mixes. If firms can either use a relatively non-polluting mix of fuels and face no penalties, or use the cheaper polluting fuel and face penalties, then making this mix move towards cleaner combinations will first raise then reduce the endogenous price of the clean input and will first increase, and then decrease, its usage!

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