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Oligopolistic Competition, Technology Innovation, and Multiproduct Firms

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Abstract

Firms' proliferation behavior in a differentiated product market is studied using an oligopolistic competition model with multiproduct firms. The model has the following characteristics: (1) the elasticity of substitution across firm's own products and the elasticity of substitution across different firms are allowed to differ; (2) the product managers of the same firm behave cooperatively rather than independently; (3) the number of firms is determined by a free-entry condition and so is endogenous. If the elasticity of substitution across the firm's own products increases, it is shown that the firm proliferates less and the number of firms in the market increases. If the elasticity of substitution across different firms increases, firms proliferate more and the number of firms in the market decreases.

1. Introduction

This paper investigates firms' innovative behavior in differentiated oligopolistic markets using a *compound* CES utility function. In contrast to other oligopolistic competition models with multiproduct firms, the model in this paper has the following characteristics: (1) the elasticity of substitution across firm's own products and the elasticity of substitution across different firms are allowed to differ; (2) the product managers of the same firm behave cooperatively rather than independently; (3) the number of firms is determined by a free-entry condition and so is endogenous rather than exogenous.

Monopolistic competition models have been used to examine a wide range of issues. The Dixit and Stiglitz (1977) model, in particular, has played a central role in many studies.[1] However, this monopolistic competition model is viewed as a "small departure from the Marshallian apparatus" (Dixit, 2000, p. 3). In the Dixit–Stiglitz model, strategic interactions are minimal: the effect of a firm's price decision on the market price index is ignored and each firm is restricted to produce only one product, excluding firm's choice about the number of products to produce. The monopolistic competition model is suitable for markets with a large group of relatively small firms. However, it hardly describes oligopoly markets where several multinational companies dominate the markets and each giant firm produces a large number of products. Such a market structure would require a full-fledged oligopolistic competition model, which is the purpose of this paper.

The market structure in many industries is oligopolistic competition with multiproduct firms. The US automobile industry is dominated by GM, Ford, and Chrysler, and each of them produces both vertically and horizontally differentiated products. For example, GM produces more than 50 different car models that span the quality

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spectrum. At least 85% of the brands of ready-to-eat breakfast cereals in the US market are produced by the four largest US manufacturers: Kellogg, General Mills, General Foods, and Quaker Oats. A handful of multinational companies occupy a large share of the world computer market with several hundred brands of computers.

The environment considered in this paper is a two-stage game in which all firms simultaneously choose the number of products to produce in the first stage. A *compound* CES utility function is considered in the second stage where δ, the elasticity of substitution across the products produced within the same firm, is different from θ, the elasticity of substitution across the products produced by different firms. Firms make price decisions to maximize total profits across all of their products.

The effect of the price for each product on the firm's price index and the market price index are considered. Each firm faces the same constant marginal cost of production, c, which is independent of the number of products chosen. The marginal cost of innovating a product is assumed to be a constant, f, and is identical for all firms. With each product's profit being positive in equilibrium, the sum over all of the firm's products yields its gross profits. Assuming that the entry cost of the firm is k, then the number of firms is given by the zero-profit condition.

In a symmetric equilibrium, I show that the oligopolist tends to charge less when the market is less differentiated and more when the oligopolist has more market power. An increase in the level of proliferation raises the firm's market power; therefore, it is shown that the level of proliferation has a positive effect on the firm's own prices and a negative effect on other firms' prices.

The number of products produced by each firm increases as the marginal proliferation cost decreases and tends to infinity as the marginal proliferation cost is vanishing. If the elasticity of substitution across a firm's own products increases, it is shown that the firm's proliferation level decreases and the number of firms in the market increases. On the other hand, if the elasticity of substitution across different firms increases, the firm's proliferation level increases and the number of firms in the market decreases.

Baldwin and Ottaviano (1998) have recently broken the Dixit–Stiglitz identity between the number of firms and the number of products by studying multiproduct multinational firms. In their two-country model, they consider one multiproduct firm per country and represent that firm's total output by a function of the number of products. Using the technique of calculus of variations, they show that multinational firms place some of their factories abroad in order to reduce the cannibalization effect which any given product has upon the firm's other products.

Brander and Eaton (1984) look at duopoly product choice from a possible constellation of four products arranged in two groups of two products each, with intragroup cross elasticities of demand being more elastic than intergroup ones. Raubitschek (1987) studies proliferation with multiproduct firms, using a two-stage process based on monopolistic competition equilibrium in quantities. She shows that the number of brands each firm possesses in equilibrium is inversely related to the cost of introducing a new brand. However, two rather restrictive assumptions are made in her model. First, the number of firms is exogenous and secondly, the firm's product managers operate independently. Champsaur and Rochet (1989) prove the existence of a duopoly equilibrium in a model of vertical product differentiation, with each firm producing a nonoverlapping continuum of the product spectrum. Anderson and Palma (1992) prove the existence of a symmetric equilibrium with multiproduct firms using a nested logit model of demand. A two-stage quantity competition model of divisionalization is examined by Baye et al. (1996). They show that the firms choose multiple divisions in the equilibrium, while the divisionalization choice, followed by Cournot competition, leads to a limit of infinite divisions.

The model-setting considered in this paper is close to that of Raubitschek (1987) and Baye et al. (1996). In contrast to their assumptions that the number of firms is fixed and that managers make quantity decisions independently, this paper assumes that a firm makes price decisions to maximize the sum of profits for all of its products and that a free-entry condition determines the number of firms in the market. In other words, the product managers of the same firm behave cooperatively rather than independently.

In strategic analysis, it is usually the case that the results of price competition are opposite to those of the quantity competition. However, the result that firms will proliferate if the marginal proliferation cost is sufficiently low are supported by quantity competition models of Raubitschek and Baye, Crocker, and Ju, and verified by the price competition model in this paper.

An alternative interpretation of the model is discussed in section 4, where firms produce a single product with multiple characteristics. In this case, competitions in vertical innovation result in multiple firms producing differentiated products with the same level of quality in equilibrium.

2. The Model

Consider an industry with m multiproduct firms, whereby each firm i produces n_i differentiated products. Here, the total number of products in the market is $N = \Sigma_{i=1}^{m} n_i$, with a free entry, so that m is determined by the zero-profit condition. N represents the level of horizontal market innovation, while n_i denotes the level of the horizontal innovation of an individual firm. In section 4, a model of vertical innovation will be considered where n_i indicates the level of quality, and m becomes the level of proliferation. The number of firms is assumed to be no less than two, hence $m \geq 2$. A firm's behavior is based on a simultaneous-move, two-stage game of complete information. In stage one, each of the m firms simultaneously chooses its number of products. In stage two, firms then choose prices for their products. The marginal cost of production is identical for all products, which is assumed to be constant, c. All firms possess identical technologies for forming products and, furthermore, the marginal cost of innovating a product, f, is constant and identical for all firms.

The representative consumer has a utility function $U = U(x_0, V)$, where V has a *compound* CES function form. Hence:

$$V = \left[\sum_{i=1}^{m} x_i^{\alpha}\right]^{\frac{1}{\alpha}}, \tag{1}$$

$$x_i = \left[\sum_{j=1}^{n_i} x_{ij}^{\rho}\right]^{\frac{1}{\rho}}, \tag{2}$$

where x_0 is the *numéraire* good, x_{ij} ($i = 1, \ldots, n_i$; $j = 1, \ldots, m$) is the quantity consumed of each product, and x_i represents the composite good of firm i.[2] Assuming $U(.)$ is homothetic in its arguments, a two-stage budgeting is then valid. Let Y denote the fixed amount that is spent by the consumer population on the industry's products, with $P = (p_{11}, \ldots, p_{1n_1}, \ldots, p_{m1}, \ldots, p_N)$ constituting the price vector of the industry. The consumer's problem becomes

$$\max_{x_{ij}} V(x_{11},\ldots,x_N)=\left(\sum_{i=1}^{m}\left[\left(\sum_{j=1}^{n_i}x_{ij}^{\rho}\right)^{\frac{1}{\rho}}\right]^{\alpha}\right)^{\frac{1}{\alpha}} \tag{3}$$

$$\text{s.t.} \sum_{i=1}^{m}\sum_{j=1}^{n_i}p_{ij}x_{ij}\le Y, \tag{4}$$

where V is the subutility derived from the consumption of the industry's products. For concavity, α and ρ must be less than 1. Let $\theta = 1/(1-\alpha)$ and $\delta = 1/(1-\rho)$. Thus, both θ and δ are greater than 1.

The price index that corresponds to firm i, q_i, is given by

$$q_i=\left[\sum_{j=1}^{n_i}p_{ij}^{1-\delta}\right]^{\frac{1}{1-\delta}}$$

and the price index that corresponds to the industry, q, is given by

$$q=\left[\sum_{i=1}^{m}q_i^{1-\theta}\right]^{\frac{1}{1-\theta}}.$$

I prove in the Appendix that the consumer's utility-maximization problem can be decomposed into two steps. In step one, the consumer maximizes x_i subject to the expenditure constraint, Y_i, on the products of firm i. In step two, the consumer maximizes utility as a function of composite goods subject to the budget constraint on composite goods with corresponding price indexes.

The demand function for x_{ij} can be written as $x_{ij} = Y_i/(p_{ij}^{\delta}q_i^{1-\delta})$, while a staged budgeting procedure gives that $Y_iq_i^{\theta}q^{1-\theta} = q_iY$ (equation (A5) in the Appendix). Therefore:

$$x_{ij}=Y_i\big/\left(p_{ij}^{\delta}q_i^{1-\delta}\right)=Y\big/\left(p_{ij}^{\delta}q_i^{\theta-\delta}q^{1-\theta}\right). \tag{5}$$

The profit of the jth product of firm i may be written as

$$\pi_{ij}(P,H)\equiv x_{ij}(p_{ij}-c)-f, \tag{6}$$

where H is an m-dimensional vector with the arbitrary component n_i. As the result, the gross profit of firm i is the sum of the profits of its n_i products:

$$\pi_i(P,H)=\sum_{k=1}^{n_i}\pi_{ik}(P,H)=\sum_{k=1}^{n_i}x_{ik}(p_{ik}-c)-n_if. \tag{7}$$

3. Market Equilibrium of Proliferation

This model is solved by backwards induction, solving first for the equilibrium prices resulting in stage two for a given set of products. Given the optimal stage-two prices, one then can solve for the Nash-equilibrium number of products selected by each firm in stage one.

Stage-Two Price Decisions

In stage two, each firm in the market takes for granted both the number of products and prices of other competing units, and then chooses its own prices to maximize profits. The market is assumed to be oligopolistic so the effects of each p_{ij} on q_i and q are considered. Therefore, firms' price decisions are not negligible for the market.[3]

Transforming the demand function (5) into logarithms we have

$$\ln x_{ij} = \ln Y - \delta \ln p_{ij} - (\theta - \delta)\ln q_i - (1-\theta)\ln q. \tag{8}$$

Note that $\partial \ln q_i / \partial \ln p_{ij} = (p_{ij}/q_i)^{1-\delta}$. The effect of p_{ij} on market price index q is

$$\frac{\partial \ln q}{\partial \ln p_{ij}} = \frac{\partial \ln q}{\partial \ln q_i}\frac{\partial \ln q_i}{\partial \ln p_{ij}} = (q_i/q)^{1-\theta}(p_{ij}/q_i)^{1-\delta}. \tag{9}$$

We then have the following expressions for the elasticities:

$$\eta_{jj} = \frac{\partial \ln x_{ij}}{\partial \ln p_{ij}} = -\delta - (\theta-\delta)(p_{ij}/q_i)^{1-\delta} - (1-\theta)(q_i/q)^{1-\theta}(p_{ij}/q_i)^{1-\delta} \tag{10}$$

$$\eta_{kj} = \frac{\partial \ln x_{ik}}{\partial \ln p_{ij}} = -(\theta-\delta)(p_{ij}/q_i)^{1-\delta} - (1-\theta)(q_i/q)^{1-\theta}(p_{ij}/q_i)^{1-\delta}, \quad \text{for } k \neq j \tag{11}$$

where $(p_{ij}/q_i)^{1-\delta}$ measures the effect of changing p_{ij} on q_i and is denoted as γ_{ij}, and $(q_i/q)^{1-\theta}$ measures the effect of changing q_i on q and is denoted as ϕ_i. The firm's price decision has no effect on the market if $\phi_i = 0$, while the firm's price decision has full effect on the market if $\phi_i = 1$. Therefore, ϕ_i may indicate the firm's market power.

Using (7), the profit-maximization condition for firm i then becomes

$$\begin{aligned}\frac{\partial \pi_i}{\partial p_{ij}} &= \sum_{k=1}^{n_i}(p_{ik}-c)\frac{\partial x_{ik}}{\partial p_{ij}} + x_{ij} = \sum_{k=1}^{n_i}\frac{(p_{ik}-c)x_{ik}}{p_{ij}}\eta_{kj} + x_{ij} \\ &= -\sum_{k=1}^{n_i}\frac{(p_{ik}-c)x_{ik}}{p_{ij}}\left[(\theta-\delta)+(1-\theta)\left(\frac{q_i}{q}\right)^{1-\theta}\right]\left(\frac{p_{ij}}{q_i}\right)^{1-\delta} \\ &\quad -\frac{(p_{ij}-c)x_{ij}}{p_{ij}}\delta + x_{ij} \\ &= 0.\end{aligned} \tag{12}$$

Note that $x_{ij} = Y/(p_{ij}^{\delta} q_i^{\theta-\delta} q^{1-\theta})$ and rewrite the above first-order condition as

$$\begin{aligned}&-\sum_{k=1}^{n_i}(p_{ik}-c)x_{ik}\left[(\theta-\delta)+(1-\theta)\left(\frac{q_i}{q}\right)^{1-\theta}\right]\frac{1}{q_i^{1-\delta}} + \frac{Y}{q_i^{\theta-\delta}q^{1-\theta}} \\ &\quad = \frac{(p_{ij}-c)}{p_{ij}}\frac{Y\delta}{q_i^{\theta-\delta}q^{1-\theta}}.\end{aligned} \tag{13}$$

The left-hand side of equality (13) is the same for all $j = 1, \ldots, n_i$. Therefore, all p_{ij} are equal and denoted as p_i. Hence, $p_{ij} = p_i$ for $j = 1, \ldots, n_i$, which gives $(p_{ij}/q_i)^{1-\delta} = 1/n_i$. It is immediately seen that all x_{ij} are equal from (12) and denoted as x_i. Using the definitions of q_i and q, we know that

$$\phi_i = (q_i/q)^{1-\theta} = \frac{n_i^{\frac{1-\theta}{1-\delta}} p_i^{1-\theta}}{\left(\sum_{i=1}^{m} n_i^{\frac{1-\theta}{1-\delta}} p_i^{1-\theta}\right)}$$

and is less than 1. Now (12) can be simplified as

$$\frac{p_i}{p_i - c} = \theta + (1-\theta)\frac{n_i^{\frac{1-\theta}{1-\delta}} p_i^{1-\theta}}{\sum_{i=1}^{m} n_i^{\frac{1-\theta}{1-\delta}} p_i^{1-\theta}} \quad \text{for } i = 1, \ldots, m, \tag{14}$$

$$= \theta - (\theta - 1)\phi_i = (1-\phi_i)\theta + \phi_i \quad \text{for } i = 1, \ldots, m, \tag{15}$$

which shows that the firm's price is positively related to the firm's market power, ϕ_i, and negatively related to the elasticity of substitution across different firms, θ. The oligopolist tends to charge more when it has more market power and tends to charge less when the market becomes less differentiated. The firm's markup, $(p_i - c)/p_i$, is inversely proportional to $\theta - (\theta - 1)\phi_i$.

The m equations in (14) give the equilibrium prices $p_i = p_i(\delta, \theta, n_1, \ldots, n_m)$ for $i = 1, \ldots, m$, which are the functions of the elasticity of substitution across the firm's own products, elasticity of substitution across different firms, and the levels of proliferation by all firms.

The firm's profit function (7) now can be written as

$$\pi_i = n_i\left[\frac{Y}{p_i^{\delta} q_i^{\theta-\delta} q^{1-\theta}}(p_i - c) - f\right] = \frac{Y n_i^{\frac{1-\theta}{1-\delta}} p_i^{1-\theta}}{\sum_{i=1}^{m} n_i^{\frac{1-\theta}{1-\delta}} p_i^{1-\theta}} \frac{(p_i - c)}{p_i} - n_i f \tag{16}$$

$$= \frac{Y\left(\frac{p_i}{p_i - c} - \theta\right)}{1-\theta}\frac{(p_i - c)}{p_i} - n_i f = Y\left[1 - \frac{\theta c}{(\theta - 1)p_i}\right] - n_i f, \tag{17}$$

where equality (14) has been used to get (17).

When the firm i's level of proliferation, n_i, is changed in the first stage, both firm i's own price, p_i, and other firms' price, p_j for $j \neq i$, will be affected. It is interesting to investigate such effects by a static comparison. Summing all equations in (14), we have

$$\sum_{i=1}^{m} \frac{p_i}{p_i - c} = 1 + (m-1)\theta, \tag{18}$$

which shows that the sum of inverse markups over all firms is independent of firms' proliferation levels. Differentiating the above equation subject to n_i gives

$$\sum_{k=1}^{m} \frac{c}{(p_k - c)^2}\frac{\partial p_k}{\partial n_i} = 0. \tag{19}$$

Dividing the ith equation by the jth equation in (14), we have

$$\left(\frac{p_i}{p_i - c} - \theta\right)\Bigg/\left(\frac{p_j}{p_j - c} - \theta\right) = \left(n_i^{\frac{1-\theta}{1-\delta}} p_i^{1-\theta}\right)\Bigg/\left(n_j^{\frac{1-\theta}{1-\delta}} p_j^{1-\theta}\right) \quad \text{for } j \neq i. \tag{20}$$

Cross-multiplying the terms in both sides of the above equation and then differentiating subject to n_i gives

$$\left[\left(\frac{p_i}{p_i-c}-\theta\right)(1-\theta)n_j^{\frac{1-\theta}{1-\delta}}p_j^{-\theta}+n_i^{\frac{1-\theta}{1-\delta}}p_i^{1-\theta}\frac{c}{(p_j-c)^2}\right]\frac{\partial p_j}{\partial n_i}$$
$$=\left[\left(\frac{p_j}{p_j-c}-\theta\right)(1-\theta)n_i^{\frac{1-\theta}{1-\delta}}p_i^{-\theta}+n_j^{\frac{1-\theta}{1-\delta}}p_j^{1-\theta}\frac{c}{(p_i-c)^2}\right]\frac{\partial p_i}{\partial n_i}$$
$$+\left(\frac{p_j}{p_j-c}-\theta\right)\frac{1-\theta}{1-\delta}n_i^{\frac{\delta-\theta}{1-\delta}}p_i^{1-\theta}. \tag{21}$$

The symmetric assumption gives that $p_i = p$ and $n_i = n$. Simplifying equation (19) gives

$$(m-1)\frac{\partial p_j}{\partial n_i}+\frac{\partial p_i}{\partial n_i}=0. \tag{22}$$

Using these results, equation (21) can be simplified as

$$\left[\left(\frac{p}{p-c}-\theta\right)(1-\theta)+\frac{pc}{(p-c)^2}\right]\left(\frac{m}{m-1}\right)\frac{\partial p_i}{\partial n_i}$$
$$=-\left(\frac{p}{p-c}-\theta\right)\left(\frac{1-\theta}{1-\delta}\right)pn^{-1}. \tag{23}$$

Equation (14) gives $p/(p-c) = \theta + (1-\theta)/m$, which also implies that $c/(p-c) = [(m-1)(\theta-1)]/m$. Using these results, equation (23) can be written as

$$\left[\frac{m(m-1)\theta+(\theta-1)}{m-1}\right]\frac{\partial p_i}{\partial n_i}=\frac{(\theta-1)p}{(\delta-1)n}. \tag{24}$$

$\partial p_i/\partial n_i$ is clearly positive from (24). Therefore, $\partial p_j/\partial n_i$ is negative from (22). We summarize this result in the following proposition.

PROPOSITION 1. *In a symmetric equilibrium, an increase in the level of a firm's proliferation raises the firm's own prices and reduces other firms' prices.*

Equation (15) reveals that the oligopolist tends to charge more when it has more market power. An increase in the level of proliferation n_i raises the market power ϕ_i, which then elevates p_i. The increase in p_i raises firm i's markup. Then equality (18) implies that other firms' markups must decrease; therefore, other firms' prices are reduced.

Stage-One Proliferation Decisions

In stage one, firm i takes as given the proliferation choices of the other $(m-1)$ firms, and chooses n_i to maximize its profits. On the one hand, more products indicate higher proliferation costs, thus reducing the incentive for innovation. On the other hand, creating a new product will increase the firm's market share, thereby tending to increase both profits and incentives to form new products. Thus, the firm may have an incentive to innovate products, provided that the cost of doing so is sufficiently low.

Assuming the total number of products is "large," the integer constraint is then ignored.[4] Using (17), the derivative of firm i's profits subject to n_i is

$$\frac{\partial \pi_i}{\partial n_i} = \frac{Yc\theta}{(\theta-1)p_i^2}\frac{\partial p_i}{\partial n_i} - f = 0. \tag{25}$$

To simplify matters, we will consider a symmetric equilibrium in which all firms choose $n_i = n$ products and charge price $p_i = p$.[5] Equation (14) then gives

$$p = \frac{c[(m-1)\theta+1]}{[(m-1)(\theta-1)]} \tag{26}$$

and correspondingly

$$x = \frac{Y}{mnp} = \frac{Y(m-1)(\theta-1)}{cnm[(m-1)\theta+1]}. \tag{27}$$

Substituting (24) and (26) into (25), the first-order condition for the profit-maximizing amount of proliferation may be written as

$$n = \frac{Y(m-1)^2(\theta-1)\theta}{f(\delta-1)[(m-1)\theta+1][\theta m(m-1)+\theta-1]}. \tag{28}$$

Free-Entry Equilibrium

In a free-entry equilibrium, each firm must maximize profits, but no operating firm may acquire positive profits. Assuming the entry cost of the firm is k, $\pi_i = k$ then holds, yielding[6]

$$Y\left[1 - \frac{\theta c}{(\theta-1)p}\right] - fn \tag{29}$$

$$= \frac{Y}{(m-1)\theta+1} - fn = k. \tag{30}$$

The symmetric Nash-equilibrium proliferation of each firm, n^*, and the equilibrium number of firms, m^*, are implicitly defined by a solution of equations (28) and (30).

Substituting (28) into (30), we have

$$\frac{1}{(m-1)\theta+1}\left[1 - \frac{(m-1)^2(\theta-1)\theta}{(\delta-1)[\theta m(m-1)+\theta-1]}\right] = \frac{k}{Y}. \tag{31}$$

Equation (31) reveals that $m^* = m(\delta, \theta, Y, k)$ is independent from the marginal proliferation cost f, while equation (30) can be written as

$$\frac{1}{f}\left[\frac{Y}{(m-1)\theta+1} - k\right] = n. \tag{32}$$

Therefore, n^* increases as f decreases and tends to infinity as f is vanishing. This result is summarized in the following proposition.

PROPOSITION 2. *The number of products produced by each firm increases as the marginal proliferation cost decreases. Furthermore, the number of products produced by each firm tends to infinity as the marginal proliferation cost is vanishing.*

The above proposition clearly shows that firms will produce multiple products in the equilibrium when the marginal proliferation cost is sufficiently low. The closed-form solutions for n^* and m^* are quite messy. Accordingly, I use comparative-static analysis to examine the effects on equilibrium associated with changes in the parameters in the model.

I first analyze the effects on an equilibrium number of firms. Denote the left-hand side of equation (31) as $L(m, \delta, \theta)$, which can be written as

$$L(m,\delta,\theta)=\frac{1}{(m-1)\theta+1}\left[1-\frac{(\theta-1)\theta}{(\delta-1)\left[\theta\left(1+\frac{1}{m-1}\right)+\frac{\theta-1}{(m-1)^2}\right]}\right] \tag{33}$$

$$=\frac{1}{(m-1)\theta+1}\left[1-\frac{(m-1)^2}{(\delta-1)\left[\frac{m(m-1)}{\theta-1}+\frac{1}{\theta}\right]}\right], \tag{34}$$

where equality (33) follows from dividing the numerator and denominator by $(m-1)^2$ in the bracket of (31), and equality (34) follows from dividing the numerator and denominator by $(\theta-1)\theta$ in the bracket of (31). Equality (33) shows that $L(m, \delta, \theta)$ is decreasing in m and increasing in δ, while (34) shows that $L(m, \delta, \theta)$ is decreasing in θ. Total differentiating equation (31), we have:

$$\frac{\partial L(.)}{\partial m}dm+\frac{\partial L(.)}{\partial \delta}d\delta+\frac{\partial L(.)}{\partial \theta}d\theta=d\left(\frac{k}{Y}\right).$$

Therefore:

$$\frac{\partial m}{\partial \delta}=-\frac{\partial L(.)}{\partial \delta}\bigg/\frac{\partial L(.)}{\partial m}>0,\quad \frac{\partial m}{\partial \theta}=-\frac{\partial L(.)}{\partial \theta}\bigg/\frac{\partial L(.)}{\partial m}<0,\quad \text{and } \frac{\partial m}{\partial (k/Y)}=1\bigg/\frac{\partial L(.)}{\partial m}<0.$$

The last inequality reveals that the equilibrium number of firms in the market will increase if the market capacity increases or the entry cost decreases. This is the same as the classical result in the monopolistic competition model. However, it is not clear whether m will asymptotically approach infinity if the market capacity tends to infinity.

The first inequality shows that the equilibrium number of firms in the market increases as the elasticity of substitution across a firm's own products increases. The second inequality shows that the equilibrium number of firms in the market decreases as the elasticity of substitution across different firms increases.

We now turn to the effects on the firm's proliferation level n. Differentiating equation (30) with respect to δ, we have

$$-\frac{Y\theta}{[(m-1)\theta+1]^2}\frac{\partial m}{\partial \delta}=f\frac{\partial n}{\partial \delta},$$

which implies that $\partial n/\partial \delta < 0$. Substituting (30) into (28), we have

$$\frac{(m-1)^2(\theta-1)\theta}{f(\delta-1)[\theta m(m-1)+\theta-1]}=\frac{n}{k+nf}\Leftrightarrow$$

$$\frac{f(\delta-1)}{(\theta-1)\theta}\left[\theta\left(1+\frac{1}{m-1}\right)+\frac{\theta-1}{(m-1)^2}\right]=f+\frac{k}{n}. \tag{35}$$

The left-hand side of equation (35) is decreasing in m and the right-hand side is decreasing in n. Thus, $\partial n/\partial Y$ has the same sign as $\partial m/\partial Y$, hence $\partial n/\partial Y > 0$. In the Appendix it is shown that $\partial n/\partial \theta > 0$. These results are summarized in the following proposition.

PROPOSITION 3. *If the elasticity of substitution across a firm's own products increases,* ceteris paribus*, the firm's proliferation level decreases and the number of firms in the market increases. On the contrary, if the elasticity of substitution across different firms increases,* ceteris paribus*, the firm's proliferation level increases and the number of firms in the market decreases. The firm proliferates more as the capacity of the market enlarges.*

Increase in own-product elasticity worsens the cannibalization effect within the firm's products, which drives down the firm's proliferation level. In a symmetric equilibrium, firms' profits are increased when all firms choose to produce fewer products, so more firms survive in the market. Increase in cross-firm elasticity makes the market less differentiated, therefore more competitive for the price competition in the second stage, which reduces the number of firms in the market. On the one hand, firms proliferate less in a less differentiated market; on the other hand, firms proliferate more with fewer firms competing in the market. We are able to show that the later effect dominates the former. Thus, firms proliferate more as cross-firm elasticity increases.

Higher entry cost reduces the firm's profit, which reduces the firm's proliferation level. However, the number of firms in the market is also reduced owing to higher entry cost, which drives firms to proliferate more. The total effect of entry cost on each firm's proliferation level is ambiguous.

4. Quality Improvement and Product Proliferation

It is useful to develop an alternative interpretation of this model. Let firms produce a single product and let the consumption of it be denoted by $x(i)$. Here, $x(i)$ consists of characteristics x_{ij}, $0 \le j \le n_i$. Correspondingly, q_i represents the hedonic price of the product. For this setup, a continuum version of the model will be used.[7]

The representative consumer maximizes

$$V=\left[\int_0^m x(i)^\alpha di\right]^{\frac{1}{\alpha}}, \tag{36}$$

subject to the budget constraint $\int_0^m q(i)x(i)di = Y$. The product $x(i)$ is a CES function of characteristics

$$x(i)=\left[\int_0^{n_i} x(ij)^\rho dj\right]^{\frac{1}{\rho}}, \tag{37}$$

and the price of the characteristic $x(ij)$ is p_{ij}. The hedonic price of the product $q(i) = [\int_0^{n_i} p(ij)^{1-\delta}dj]^{1/(1-\delta)}$ and the price index of the industry $q = [\int_0^m (i)^{1-\theta}di]^{1/(1-\theta)}$. In a symmetric equilibrium all characteristics would bear the same price and firms would employ equal quantities $x(ij) = x$ of each. Then (37) implies that $x(i) = n_i^{1/\rho}x$. We use $\chi = n_i x$ to measure the quantity consumed in the good. The quality of the good is given by $x(i)/\chi = n_i^{(1-\rho)/\rho}$. With $0 < \rho < 1$, we see that the quality of the product rises with the number of characteristics. This is the property of the quality to the improvements from increasing degrees of *specialization in consumer's satisfaction*. That is, when n_i grows, consumption consists of an ever larger number of finer satisfaction.[8]

Similar to the above sections, the demand function for x_{ij} is given by (5) and the profit of firm i is

$$\pi_i(P, H) = \int_0^{n_i} x(ij)(p(ij) - c)dj - n_i f, \tag{38}$$

where f now represents the marginal cost of quality improvement. All the analyses above go through. However, the focus now is the equilibrium quality n^*. We have the following proposition.

PROPOSITION 4. *Firms improve the quality of product when the market capacity increases, the marginal cost of quality improvement decreases, and the elasticity of substitution across different firms increases. Firms reduce the number of characteristics in the product, and thereby decrease the quality of the product when the characteristics of the product become more substitutable.*

The interesting part of this model is that both vertical innovation n and horizontal innovation m can be analyzed in one model. Multiple firms produce the same quality of differentiated products in the market equilibrium. Therefore, this model may be useful in studying both vertical and horizontal technological innovations.[9]

5. Conclusion and Discussion

I have constructed an oligopolistic competition model in this paper to study the reasons why firms innovate horizontally and vertically. The elasticity of substitution across firms' own products and the elasticity of substitution across different firms are allowed to differ. If the products produced by the same firm are more substitutable, it is shown that the firms will proliferate less and the number of firms in the market will increase. However, if the products produced by different firms are more substitutable, then fewer firms will survive in the market and each firm proliferates more.

In the Dixit–Stiglitz model, market expansion occurs solely through an increase in the number of firms, without any increase in the output of the product nor the firm's proliferation level. In the equilibrium of this model, the level of the firm's proliferation and the output of each product are functions of all parameters; the price of each product and the number of firms are functions of all parameters except the marginal innovation cost. Therefore, market expansion may occur through increases in the level of a firm's proliferation and the output of the product. This may well fit oligopolistic markets in reality.

I do not discuss whether the level of innovation produced in the market equilibrium is too low or too high. The problem can be studied in the second-best situation where a social planner chooses the number of products, N^s. Given N^s, firms will then choose the corresponding market equilibrium prices. The number of firms, m^s, is then determined by the zero-profit condition, with each firm producing N^s/m^s products. However, it is not possible to have a clean comparison between social optimal proliferation and market equilibrium proliferation, owing to complicated solution formulas. Using numerical solutions for wide ranges of parameter values, we may be able to study this issue in the future.

The firms' strategic interactions in both prices and innovations are fully studied, while I believe that the model is still tractable. Marginal innovation cost f and the entry cost k can be connected to the usage of human capital, labor, and investment. The model can then be applied to international trade, growth theory, and macroeconomics for further research.

Appendix

The Consumer's Utility-Maximization Problem

The objective is to prove that the consumer's utility maximization problem can be decomposed into two steps. In step one, the consumer maximizes x_i subject to the expenditure constraint on the products of firm i. Hence:

$$\max_{x_{ij}} x_i = \left[\sum_{j=1}^{n_i} x_{ij}^{\rho}\right]^{\frac{1}{\rho}} \tag{A1}$$

$$\text{s.t.} \sum_{j=1}^{n_i} p_{ij} x_{ij} \le Y_i, \tag{A2}$$

where $\Sigma_{i=1}^{m} Y_i = Y$. In step two, the consumer then maximizes V^C subject to the budget constraint on composite goods with corresponding price indexes. Hence:

$$\max_{x_i} V^C(x_1, \ldots, x_m) = \left[\sum_{i=1}^{m} x_i^{\alpha}\right]^{\frac{1}{\alpha}} \tag{A3}$$

$$\text{s.t.} \sum_{i=1}^{m} q_i x_i \le Y. \tag{A4}$$

We want to show that x_{ij}^* is the solution to maximize *compound* CES utility function (3) subject to budget constraint (4) if and only if x_{ij}^* is the solution to maximize the sub-CES function (A1) subject to (A2) and $x_{ij}^* = [\Sigma_{j=1}^{n_i}(x_{ij}^*)^{\rho}]^{1/\rho}$ is the solution to maximize the sub-CES function (A3) subject to (A4).

Denote V^* as the maximized value of *compound* CES function (3) and V^{C*} as the maximized value of (A3). Let x_{ij}^* solve for (A1) and (A2), while x_i^C solves for (A3) and (A4). It is straightforward to show that $x_{ij}^* = Y_i/(p_{ij}^{\delta} q_i^{1-\delta})$ and $q_i x_i^* = Y_i$. Similarly, $x_i^C = Y/(q_i^{\theta} q^{1-\theta})$ and $qV^{C*} = Y$.

Let $x_i^* = x_i^C$, which requires that

$$Y_i/q_i = Y/(q_i^{\theta} q^{1-\theta}). \tag{A5}$$

Note that x_{ij}^* ($i = 1, \ldots, m$ and $j = 1, \ldots, n_i$) satisfies (4), so we must have $V^{C*} = V(x_{11}^*, \ldots, x_N^*) \le V^*$.

Now let x_{ij}^* be the solution to maximize (3). Let $Y_i = \Sigma_{j=1}^{n_i} p_{ij} x_{ij}^*$. Then x_{ij}^* must be the solution to maximize (A1) subject to (A2). Therefore, $x_{ij}^* = Y_i/(p_{ij}^{\delta} q_i^{1-\delta})$. It is easy to see that $(x_1^*, \ldots, x_m^*)$ satisfies (A4), so we must have $V^* = V^C(x_1^*, \ldots, x_m^*) \le V^{C*}$. Therefore $V^* = V^{C*}$, which proves the result.

To Prove that $\partial n/\partial\theta > 0$

Differentiating equality (31) subject to θ and after some computations, we have $-C - D = (A + B)(\partial m/\partial\theta)$, where $D = [k(m-1)]/Y$, $B = (k\theta)/Y$, and

$$C = \frac{(m-1)^2\left[(\theta-1)^2 + \theta^2(m-1)m\right]}{(\delta-1)[\theta m(m-1)+\theta-1]^2},$$

$$A = \frac{\theta(\theta-1)(m-1)[2(\theta-1)+\theta(m-1)]}{(\delta-1)[\theta m(m-1)+\theta-1]^2}.$$

Differentiating (32) subject to θ gives

$$\frac{\partial n}{\partial\theta} = -\frac{Y}{f[(m-1)\theta+1]^2}\left[(m-1)+\theta\frac{\partial m}{\partial\theta}\right]$$

$$= -\frac{Y}{f[(m-1)\theta+1]^2}\left[(m-1)-\theta\frac{(C+D)}{(A+B)}\right]$$

and

$$(m-1)-\theta\frac{(C+D)}{(A+B)} = \frac{[A(m-1)-\theta C]}{(A+B)},$$

since $B(m-1) - \theta D = 0$. Computations reveal that $[A(m-1) - \theta C]$ has the same sign as $(\theta-1)^2 - (m-1)\theta[(m-1)\theta+1] < 0$, since $\theta - 1 < (m-1)\theta$ when $m \geq 2$. Therefore, $\partial n/\partial\theta > 0$.

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Notes

1. In "new trade theory," the Dixit–Stiglitz (D–S) model is used by Helpman and Krugman (1985) and many others to provide a scale economy view of intraindustry trade. Grossman and Helpman (1991) use the D–S model in their growth theory studies where the number of products is used to represent the level of innovation. A popular macroeconomic application is Blanchard and Kiyotaki (1987), where the D–S model is used to examine the effects of aggregate demand on economic activity.
2. Note that if $\alpha = \rho$, the *compound* CES function is reduced to the usual CES function.
3. In the Dixit–Stiglitz model, the consumer's utility $V = (\Sigma_{i=1}^{n} x_i^{\alpha})^{1/\alpha}$ and the logarithms of demand for product i, $\ln x_i = \ln Y - \delta \ln p_i - (1 - \delta)\ln q$. Monopolistic assumption implies that $\partial \ln x_i / \partial \ln p_j = 0$. Therefore, any price other than product i's own price has no effect on x_i. In other words, the interactions among products are removed by the monopolistic assumption.
4. See Hart (1985) for a formal discussion. A continuum version of the model which cuts through the debate on approximated solutions to the discrete model will be used in section 4.
5. Asymmetric equilibria may exist. In general, the elasticity of substitution and the marginal cost of proliferation are different for different firms. Asymmetric equilibria could be used to study firms' more sophisticated behavior.
6. I am grateful to Ping Wang for discussion on this point.
7. Note that firms' strategic interactions on number of products disappear in this section since each firm is restricted to produce a single product.
8. Ethier (1982) uses a similar model. The difference between his model and ours is the interpretation of x_{ij}. We use x_{ij} to represent the product characteristics, while Ethier uses x_{ij} to represent intermediate inputs.
9. For discussions on vertical innovation models, readers are guided to Grossman and Helpman (1991) and Glass (1998).