A Model of Middlemen and Oligopolistic Market Makers

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January 14, 2009

We are grateful to a coeditor and two referees of the journal for comments and suggestions on an earlier draft. We also thank M. Baye, G. Hall, J. Morgan and D. Spulber and session participants at the 2005 Winter Meeting of the Econometric Society and the 2004 Fall Meeting of the Midwest Economics Association for comments on earlier versions of the paper.

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Abstract

This paper studies the endogenous structure of intermediation when heterogeneous intermediaries choose between becoming a middleman or a market maker, and the relation between the equilibrium market structure and price dispersion. We obtain three main results: First, middlemen and oligopolistic market makers can coexist in the market equilibrium. All market makers publicly post unique ask and bid prices. These prices serve as the high and low bounds respectively for the ask and bid prices of middlemen, when capacity cost is sufficiently large. Second, more efficient intermediaries choose to become market makers, while less efficient intermediaries choose to become middlemen. Third, if the fixed cost of capacity installation for market makers increases, the number of market makers declines, while the number of middlemen increases. As a result, both ask prices and bid prices become more dispersed.

Keywords: oligopolistic market makers; middlemen; intermediaries; price dispersion

JEL classifications: D43, L11, L13

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1. Introduction

The intermediation of trade in real goods is a significant feature of many markets. Typically, intermediation is facilitated by market makers and middlemen. Market makers facilitate trade by posting publicly observable bid and ask prices. On the other hand, middlemen stand ready to trade at bid and ask prices they quote on a private basis to consumers or producers who identify them via a costly search process. The purpose of this paper is to study the structure of an intermediated market when intermediaries *endogenously* choose between becoming a middleman or a market maker and the implications of the equilibrium choices for the price dispersion of the real good being traded. The endogeneity of the choice to become a market maker or a middleman has not been investigated in the literature despite the coexistence of these intermediary types in many real markets. Our study makes several new contributions to the literature. First, we derive conditions under which middlemen and oligopolistic market makers coexist in the market. In conjunction with this result we show that all market makers publicly post unique ask and bid prices. These prices serve as the high and low bounds respectively for the ask and bid prices of middlemen when capacity cost is sufficiently large. Second, more efficient intermediaries choose to become market makers, while less efficient intermediaries choose to become middlemen. Third, if the fixed cost of capacity installation for market makers increases, the number of market makers declines, while the number of middlemen increases. As a result, both ask prices and bid prices become more dispersed.

To put the setting we study in a modern context it is constructive to view intermediation as being accomplished through online and offline paths. Online intermediation has become the communication choice of market makers while offline intermediation, most often through telephonic communication, is the typical choice of middlemen. Market structures when both online market makers and offline middlemen are present have received surprisingly little attention. Notable and important exceptions are Rust and Hall (2003) who extend the price-setting middlemen model of Spulber (1996, 1999) by introducing a monopolist market maker. Baye and Morgan (2001, 2004) study a monopolist information gatekeeper, and Hendershott and Zhang (2006) examine a model with an upstream monopolist firm which can sell directly online as well as through middlemen to heterogeneous consumers.

Some markets however, such as the North American market for natural gas, have exhibited the coexistence of both middlemen as well as multiple market makers. The analysis of oligopolistic competition amongst market makers is limited. Rust and Hall (2003) suggest approaching the problem within a Bertrand-style price competition setting. Bertrand price competition among market makers will usually lead to a Walrasian equilibrium as shown by Stahl (1988). In such an equilibrium all middlemen will be driven out of the market. To explain the coexistence of oligopolistic competition among market makers and an active search market for middlemen, we allow non-degenerate imperfect competition among market makers. We formulate a two-stage game in which market makers first compete in capacity choice and then compete in bid and ask prices. Similar to the well known argument made by Kreps and Scheinkman (1983), we show that the outcome of our two-stage game for market makers is equivalent to the Cournot outcome when capacity cost is sufficiently large.

Intermediaries' capacity costs and the transaction costs they incur when buying and selling, play an important role in our model. The marginal cost of capacity installation is assumed to be the same for both middlemen and market makers. The costs of setting up an online marketplace are therefore expected to exceed the cost of telecommunications equipment. We therefore assume that the fixed cost of capacity installation for market makers is larger than it is for middlemen.[1] The population of potentially active intermediaries is represented by a uniform distribution of transaction costs. We show that the profit functions for middlemen and for market makers are decreasing functions of the transaction cost. The former, however, is flatter than the latter. The unique transaction cost at which the two profit functions intersect is a threshold below which the intermediaries choose to become market makers, and above which, intermediaries choose to become middlemen. As the fixed cost of capacity installation for market makers increases, the profit function of market makers shifts down. Correspondingly, the threshold transaction cost decreases so that fewer intermediaries

[1] Capacity is defined here as the upper limit of the quantity the intermediary has the ability to trade. The upper limit of capacity is determined by the physical limitation imposed by factors such as limits on quantities that can be transported for exchange. For example, an intermediary in the natural gas market first contracts with a pipeline owner to ship natural gas. Total pipeline capacity is fixed in the short run. The intermediary then contracts with buyers and sellers of natural gas to either buy or sell but is constrained by the pipeline capacity he has available.

choose to become market makers, and more intermediaries choose to become middlemen.

We show that as the marginal cost of capacity installation declines or the size of the market increases, profits for both market makers and middlemen increase, and the threshold transaction cost increases. This results in more intermediaries choosing to become market makers. The effect on the number of middlemen, however, is more complicated. On the one hand, the lower bound of the transaction cost for middlemen (the threshold transaction cost) increases. Higher profits however induce more intermediaries to enter the market so that the upper bound of the transaction cost for middlemen also increases. These two effects exactly cancel each other in our model leaving the number of middlemen on net unaffected by changes in the marginal cost of capacity installation and the size of the market.

Our study is related to the literature on the microstructure of market intermediation. However, to the best of our knowledge, our paper is the first to model the endogenous choices of intermediaries to become middlemen or market makers. Other than the work of Spulber (1996, 1999) and Rust and Hall (2003) which are the foundations upon which we build our model, Rubinstein and Wolinsky (1987) study a random matching model with buyers and sellers in which intermediaries act as dealers. Gehrig (1993) considers Bertrand competition between intermediaries who also compete within a search market. Gehrig shows that competition eliminates the bid-ask spread in his model. Yavas (1992, 1994) allows the consumer to choose between a market maker and a matchmaker and studies the effect of the matchmaker's presence on search intensity. Fingleton (1997) studies direct trade between buyers and sellers. Shevchenko (2004) studies a search setting in which intermediaries can hold inventories.

Our study is also related to the literature which focuses on capacity constrained oligopoly competition. Stahl (1988) studies a setting in which intermediaries bid to buy the capacities they will face when selling to consumers. Boccard and Wauthy (2004) extend Kreps and Scheinkman's duopoly result (1983) to the oligopoly setting. Loertscher (2008) introduces capacity constrained market makers in a symmetric oligopoly. An important difference between our model and these symmetric oligopolistic competition models, is that we allow different marginal costs amongst market makers. If capacity cost is sufficiently large, we show that a unique subgame perfect

Nash equilibrium (SPNE) is for market makers to choose Cournot capacities, accompanied by pure strategy market clearing ask and bid prices. If capacity cost is small, however, it is possible that market makers choose a mixed strategy price setting path.

We begin by laying out the model in Section 2. In Section 3 we study the equilibrium of the intermediary market when the numbers of middlemen and market makers are given. In Section 4 we examine intermediaries' endogenous choices to become a middlemen or a market makers and the implications of the equilibrium choices for the price dispersion of the real good being traded. Section 5 concludes the paper. We also present a mixed strategy equilibrium solution to the model in the Appendix where we show conditions under which the Cournot equilibrium solution is the unique subgame perfect Nash equilibrium. Table A1 presents a list of the variable symbols used in the paper along with a brief description of each variable.

2. The Model

There are three types of agents, intermediaries, consumers and producers. They trade in a homogeneous product market. Intermediaries begin by making a capacity choice which defines the limit of their ability to transact. Intermediaries then buy goods from producers and sell them to consumers. There are two types of intermediaries: market makers and middlemen. Market makers post the price at which they are willing to buy (bid price) and the price at which they are willing to sell (ask price) on the internet for all consumers and producers to freely see at no cost to the consumer or producer. In contrast the bid and ask prices of middlemen are private information that can only be discovered through a search process.

We use the superscript m to represent a market maker. The numbers of middlemen and market makers are given by N, and M. We allow N and M to be determined endogenously within the model. Let q_i and q_j^M be the capacity set and quantity sold by middlemen and market makers respectively. We show later that all intermediaries transact up to their capacity limit. We assume capacity installation costs are, $\theta q_i + F$ $(i = 1, \cdots, N)$, for middlemen, and $\theta q_j^m + F^m$ $(j = 1, \cdots, M)$, for market makers. The marginal cost of capacity installation, θ, is assumed to be identical for both middlemen and market makers. The fixed cost of capacity installation, however, differs. We assume that $F^m > F$. That is, the fixed cost of capacity installation for market makers is larger than for middlemen. To simplify the analysis, we let the fixed cost of capacity installation for middlemen henceforth equal to zero, i.e. $F = 0$.

Each intermediary incurs a transaction cost k per unit of the good purchased from any producer. The population of potentially active intermediaries is represented by a uniform distribution of transaction costs k on the interval $[\underline{k}, \overline{k}]$.

2.1. Consumers and Producers

The population of consumers is represented by a uniform distribution of willingness to pay levels on the interval $[\underline{v}, \overline{v}]$ (Spulber, 1996, 1999; Rust and Hall, 2003). A consumer of type v consumes one unit of the good if the price she pays is at most v. The unitary consumption assumption implies that a consumer will not make any subsequent transactions after her initial trade. Let p^m be the ask price and w^m the bid price posted by market makers. If the consumer buys from a market maker, she pays p^m and buys the good immediately.

If the consumer buys from middlemen, on the other hand, she first searches randomly across the N middlemen present in the market for an ask price that is less than or equal to the consumer's reservation price, to be defined shortly. Each middleman therefore faces an equal probability of making a trade. Consumers know the equilibrium distribution of ask prices, $F(p)$, offered by middlemen but not the particular middleman associated with each price.

Consumers remain in the market for a random length of time before permanently exiting from the market. Let $\lambda \in (0, 1)$ be the probability that a consumer exits the market and ρ be the time discount rate for period t. In each period a fraction λ of the population of consumers exits the market and is replaced by an equal fraction of new consumers. The optimal search strategy for a type v consumer takes the form of a reservation price rule: accept any ask price less than the reservation price $r_c(v)$. Following Rust and Hall (2003), the reservation price is implicitly defined by the unique solution to

$$v = r_c(v) + \frac{1}{\delta}\int_{\underline{p}}^{r_c(v)} F(p)dp, \tag{1}$$

where $\delta = 1/[\rho(1-\lambda)] - 1$ is the composite exit-adjusted discount rate per period.[2] The cost of searching in the model is the cost of waiting to transact implicit in the discount rate. The function $r_c(v)$ is strictly increasing in v on the interval $(v^*, \overline{v})$, where v^* is associated with the marginal consumer for whom the expected gain from searching is zero. It is easy to see that $v^* = r_c(v^*) = \underline{p}$, where $\underline{p}$ is the lowest ask price across all middlemen.

As in Rust and Hall (2003), consumers whose reservation price is above p^m will buy from a market maker, while those consumers whose reservation price is below p^m will search and buy from a middleman. Let $v_c(\underline{p}, p^m)$ be the willingness to pay of the marginal consumer with reservation price p^m. That is,

$$v_c(\underline{p}, p^m) = p^m + \frac{1}{\delta}\int_{\underline{p}}^{p^m} F(p)dp\ . \tag{2}$$

The consumer whose willingness to pay $v \in [\underline{v}, \underline{p})$ will not trade. If $v \in [\underline{p}, v_c(\underline{p}, p^m))$ then it is optimal for the consumer to search in the middlemen market. If $v \in [v_c(\underline{p}, p^m), \overline{v}]$ then it is optimal for the consumer to buy the good from a market maker.

Consumers and producers randomly search across the N middlemen. As a result each middleman will receive an equal share of the total searchers. Suppose that a middleman asks a price of p. Let $D_i(p)$ denote the mass of consumers who are among the initial population and who purchase the good in period i from a middleman. The total expected discounted demand for all middlemen, $D(p)$, is the expected discounted value of the stream of demands in all future periods by the initial population of consumers as well as the stream of demands from each succeeding generation of new consumers entering the market. That is,

$$D(p) = \sum_{i=0}^{\infty} \rho^i D_i(p) + \lambda \sum_{j=1}^{\infty} \rho^j \sum_{i=0}^{\infty} \rho^i D_i(p) = \frac{p^m - p}{N(1-\rho)}, \tag{3}$$

where N is the number of middlemen and p^m, the price charged by market makers, is the highest

[2]Rust and Hall (2003) assume that consumers have no memory.

reservation price of the consumers who search amongst middlemen.[3]

A producer of type c produces one unit of the good at a cost of c. The population of producers is represented by a uniform distribution of costs on the interval $[\underline{c}, \overline{c}]$. Producers know the equilibrium distribution of bid prices $G(w)$ from middlemen but not the particular middleman associated with each price w. Each producer searches randomly across middlemen and accepts any bid price w that exceeds the reservation price $r_p(c)$, which is given by the solution to

$$c = r_p(c) - \frac{1}{\delta} \int_{r_p(c)}^{\overline{w}} [1 - G(w)] \, dw. \tag{4}$$

The reservation price $r_p(c)$ is a strictly increasing function of c on the interval $(\underline{c}, c^*)$, where $c^* = r_p(c^*) = \overline{w}$ is the cost of the marginal producer for whom the expected gain from searching is zero. A producer whose reservation price is below w^m will buy from a market maker, while one whose reservation price is above w^m will search amongst middlemen. Let

$$v_p(\overline{w}, w^m) = w^m - \frac{1}{\delta} \int_{w^m}^{\overline{w}} [1 - G(w)] \, dw, \tag{5}$$

where $v_p(\overline{w}, w^m)$ is the value of the marginal producer with reservation price w^m. By reasoning similar to the articulation of demand, a middleman's total expected discounted supply function is therefore

$$S(w) = \frac{-w^m + w}{N(1-\rho)}, \tag{6}$$

3. The Intermediary Market

All intermediaries follow a three-stage decision process. In stage-one the number of middlemen and the number of market makers, N and M, are endogenously determined. In stage-two both middlemen and market makers make capacity decisions. In stage-three middlemen and market makers choose their bid and ask prices. The game is solved by backward induction. We begin by

[3] Rust and Hall (2003) present the development of the demand and supply functions shown in equations (3) and (6). A detailed derivation of the demand and supply functions is available from the authors upon request.

solving for the optimal ask and bid prices set by middlemen and market makers.taking as given the number of middlemen and market makers determined in stage-one, and the optimal capacity installed in stage-two.

3.1. Middlemen

Facing the demand and supply, $D(p)$ and $S(w)$, each middleman behaves like a monopolist. Thus, a middleman will select a capacity level equal to the amount of goods he purchases from producers. A middleman's present discounted value of trading profits is given by

$$\pi(p,w,k) = pD(p) - (w+k+\theta)S(w), \tag{7}$$

where the middleman buys at w and sells at p, incurring the transaction cost and capacity cost per unit of k and θ. Note that the marginal capacity cost is identical for all middlemen. Each middleman, indexed by the transaction cost k, solves the following problem

$$\max_{p,w} \pi(p,w,k) \text{ subject to } D(p) \leq S(w)\ . \tag{8}$$

The first order conditions associated with problem (8) yield optimal ask and bid prices, p and w respectively, for a middleman with transaction cost k

$$p(k) = \frac{3p^m + w^m + (k+\theta)}{4} \text{ and } w(k) = \frac{p^m + 3w^m - (k+\theta)}{4}\ . \tag{9}$$

A higher transaction cost increases the optimal ask price $p(k)$, but reduces the optimal bid price $w(k)$ conforming with intuition. These bid and ask prices also equate supply and demand in every period as has been shown by Spulber (1999, Ch. 6). Substitution of the optimal bid and ask prices into the profit function gives

$$\pi(p,w,k) = \frac{1}{8N(1-\rho)}\left(p^m - w^m - k - \theta\right)^2. \tag{10}$$

The value for k which solves $\pi(p,w,k) = 0$ gives the highest transaction cost that any middleman can incur while both serving the market and surviving. Denote this value as k^* where it follows that

$$k^* = p^m - w^m - \theta. \tag{11}$$

Any middleman with transaction cost $k > k^*$ will not enter the market.

Denote the lowest transaction cost for middlemen as $\underline{k}^d$.[4] Letting k in equation (9) equal first $\underline{k}^d$ and then k^*, we obtain the lower and upper bounds of the equilibrium distribution of ask and bid prices. Ask prices are uniformly distributed on the interval $[\underline{p}, p^m]$, and bid prices are uniformly distributed on the interval $[w^m, \overline{w}]$, where

$$\begin{aligned} \underline{p} &= \left(3p^m + w^m + \underline{k}^d + \theta\right)/4,\ \overline{p} = p^m \\ \underline{w} &= w^m,\ \overline{w} = \left(p^m + 3w^m - \underline{k}^d - \theta\right)/4. \end{aligned} \tag{12}$$

We restrict our attention to stationary pricing policies on the equilibrium path. Thus, the pair of equilibrium ask and bid prices derived above represent steady-state equilibrium prices in each period.

3.2. Oligopolistic Market Makers

Market maker $j (j = 1, \cdots, M)$ posts publicly observable ask and bid prices. There is no cost to a consumer or producer to view these prices. In terms of the three stages described above we begin by taking the number of market makers as having been determined in the first stage. Competition amongst market makers is then modeled as a two-stage game in which market makers compete in capacity in the second stage, and compete in bid and ask prices in the third stage.

3.2.1. Demand and Supply Faced by Market Makers

Market makers as a whole face total market demand, $X^m = \overline{v} - v_c(\underline{p}, p^m)$, and total market supply, $Y^m = v_p(\overline{w}, w^m) - \underline{c}$. Let $X^m = Y^m$, which implies that

[4] We derive $\underline{k}^d$ in Section 4.

$$p^m + w^m = \overline{v} + \underline{c} \ . \tag{13}$$

Using equations (2), (5), (12) and (13), we have:

$$X^m(p^m) = a_1 - a_2 p^m \text{ and } Y^m(w^m) = a_3 + a_4 w^m \ , \tag{14}$$

where $a_1 = \left[(8\delta + 1)\,\overline{v} + \underline{c} + \underline{k}^d + \theta\right] / (8\delta)$, $a_3 = -\left[(8\delta + 1)\,\underline{c} + \overline{v} - \underline{k}^d - \theta\right] / (8\delta)$, and $a_2 = a_4 = (4\delta + 1) / (4\delta)$. The inverse market demand and supply functions are therefore

$$p^m(X^m) = \frac{a_1}{a_2} - \frac{X^m}{a_2} \text{ and } w^m(Y^m) = -\frac{a_3}{a_4} + \frac{Y^m}{a_4}. \tag{15}$$

In equilibrium $X^m = Y^m = Q^m$. Then we have

$$p^m - w^m = \alpha - bQ^m \tag{16}$$

where

$$\alpha = \frac{a_1}{a_2} + \frac{a_3}{a_4} = \left[4\delta\,(\overline{v} - \underline{c}) + \underline{k}^d + \theta\right] / (4\delta + 1) > 0, \tag{17}$$

and

$$b = \frac{1}{a_2} + \frac{1}{a_4} = 8\delta / (4\delta + 1) > 0. \tag{18}$$

3.2.2. The Efficient-Rationing Rule

Let two market makers i and j post ask prices $p_i^m < p_j^m$. Let $\overline{q_i}^m$be the capacity constraint for market maker i. At p_j^m the demand faced by the two market makers together is $X^m(p_j^m) = \overline{v} - v_c(\underline{p}, p_j^m)$. If $\overline{q_i}^m \geq X^m(p_j^m)$, then all consumers in the interval $[v_c(\underline{p}, p_j^m), \overline{v}]$ prefer to buy the good from market maker i so that the residual demand for market maker j, $x_j^m(p_j^m)$, is zero. If $\overline{q_i}^m < X^m(p_j^m)$, we assume that the most eager consumers buy from i. Thus, the residual demand for j is

$$x_j^m(p_j^m) = \overline{v} - \overline{q_i}^m - v_c(\underline{p}, p_j^m) = a_1 - a_2 p_j^m - \overline{q_i}^m, \tag{19}$$

which implies an efficient-rationing rule when the market demand function is given by (14). This result can be easily generalized to the case of M market makers. Symmetrically, the efficient-rationing rule is also assumed for the supply side. Recall that an efficient-rationing rule is key to insuring the Cournot outcome emerges as an equilibrium of the two-stage, capacity-constrained price game.[5]

3.2.3. Two-Stage Market Makers' Competition

Let y_j^m and x_j^m denote the quantity bought and sold by market maker j, respectively. Assuming efficient rationing, market maker j will face a residual-demand function against other market makers. Similar to the well known argument made by Kreps and Scheinkman (1983), we will first construct the Cournot equilibrium in quantity competition, and then show that the solution of our two-stage game in which optimal bid and ask prices are established is equivalent to the Cournot solution. We focus on the pure strategy equilibrium in the text, and leave the analysis of a mixed strategy equilibrium to the Appendix. In the Appendix we prove that the Cournot solution is the unique subgame perfect Nash equilibrium in the two-stage game if the capacity cost is sufficiently large.

If $q_j^m = x_j^m = y_j^m$, market maker j's profit is

$$\pi_j^m = (p^m - w^m - k_j - \theta) q_j^m - F^m = \left[\alpha - t_j - b \sum_{i=1}^{M} q_i^m \right] q_j^m - F^m. \tag{20}$$

where $t_j = k_j + \theta$. Recall that $\theta q_j^m - F^m$ is the capacity cost for market maker j. The first order condition for profit maximization by a market maker is

$$\alpha - t_j - b \sum_{i=1}^{M} q_i^m - b q_j^m = 0 \text{ for } j = 1, \cdots, M\ . \tag{21}$$

[5] Readers are directed to Davidson and Deneckere (1986) for a thorough discussion of the efficient rationing rule in a Cournot equilibirum.

Market maker j's optimal reaction function is then

$$q_j^m = R_j(q_{-j}^m) = \frac{\alpha - t_j - b\sum_{i \neq j} q_i^m}{2b}, \tag{22}$$

where q_{-j} denotes the quantity transacted by all market makers other than j, and $R_j(\bullet)$ is the reaction function. Solving the M equations in (21) gives the Cournot equilibrium quantity

$$q_j^{m*} = \frac{\alpha + \sum_{i=1}^M t_i - (M+1)t_j}{(M+1)b}. \tag{23}$$

The total Cournot quantity is therefore

$$Q^{m*} = \sum_{j=1}^M q_j^{m*} = \frac{M\alpha - \sum_{i=1}^M t_i}{(M+1)b}. \tag{24}$$

Substituting (21) into (20), we obtain the equilibrium profit for market maker j

$$\pi_j^m = \frac{1}{b}(\alpha - t_j - bQ^{m*})^2 - F^m. \tag{25}$$

We now show that all market makers charge the same bid and ask prices in equilibrium. Suppose this was not true and suppose we let market maker j bid $w_j^m < \widetilde{w}$ where $\widetilde{w} = \max\{w_i^m(i = 1, \cdots, M)\}$, then market maker i with the highest bid could reduce its bid price and maintain its Cournot capacity; if i did not act to reduce its bid the result would be that i would buy the entire market supply at $\widetilde{w}$. Thus, market maker j buys nothing at w_j, whereas j could build up positive capacity by bidding $\widetilde{w} + \varepsilon$, for small ε, and make a positive profit. Similarly, let market maker j offer the ask price $p_j^m > \widetilde{p}$ where $\widetilde{p} = \min\{p_i^m(i = 1, \cdots, M)\}$. Then market maker i with the lowest ask price could increase its ask price and sell up to its equilibrium capacity. Conversely $\widetilde{p}$ is i's monopoly price and i will supply the entire market demand at $\widetilde{p}$. Thus, market maker j sells nothing at p_j, whereas j could make a positive profit by undercutting to $\widetilde{p} - \varepsilon$.

Given capacity $\overline{q_j}(j = 1, \cdots, M)$, we next show market makers buy and sell up to capacity. Suppose that $w_1^m = \cdots = w_M^m = w^m < w^m(\sum_{j=1}^M \overline{q_j}^m)$. Then the bid price is too low, in that at least some market maker j cannot realize its capacity. If $w_1^m = \cdots = w_M^m = w^m > w^m(\sum_{j=1}^M \overline{q_j}^m)$, all market makers strictly ration their suppliers. By a marginal reduction in its price, j would still be able to build up capacity and would earn a larger profit. Suppose that $p_1^m = \cdots = p_M^m = p^m > p^m(\sum_{j=1}^M \overline{q_j}^m)$. Then the ask price is too high, so that at least some market maker j cannot sell up to the capacity: $q_j^m < \overline{q_j}^m$. By charging $p^m - \varepsilon$, market maker j captures all of the market and can sell up to $\overline{q_j}^m$. For sufficiently small ε, we must have $(p^m - \varepsilon)\,\overline{q_j}^m > p^m q_j^m$, so that j would gain from undercutting. If $p_1^m = \cdots = p_M^m = p^m < p(\sum_{j=1}^M \overline{q_j}^m)$, all market makers strictly ration their customers. By raising its price, j would still be able to sell its capacity and would make more profit.

Now we show $\overline{q_j}^m = q_j^{m*}(j = 1, \cdots, M)$ is the Cournot Nash equilibrium. Suppose that market maker $i(i \neq j)$ buys Cournot quantity q_i^{m*}. The profit of market maker j, if it buys $q_j^m \neq q_j^{m*}$, equals the left hand side of the following equation

$$\begin{aligned} & \left[p^m(q_j^m + \sum_{i \neq j} q_i^{m*}) - w^m(q_j^m + \sum_{i \neq j} q_i^{m*}) - t_j\right] q_j^m \\ \leq & \left[p(q_j^{m*} + \sum_{i \neq j} q_i^{m*}) - w(q_j^{m*} + \sum_{i \neq j} q_i^{m*}) - t_j\right] q_j^{m*} \end{aligned} \tag{26}$$

using the result that profit measured at Cournot capacity q_j^{m*} exceeds profit measured at q_j^m when $q_j^m \neq q_j^{m*}$. Summarizing we have:

Proposition 1 *Assume market makers compete in capacity installed at the second stage and compete in ask and bid prices at the third stage. If the capacity installation cost is sufficiently large, all market makers install the Cournot capacity first, then ask the same price $p_1^m = \cdots = p_M^m = p^m = p^m(Q^{m*})$, and bid the same price $w_1^m = \cdots = w_M^m = w^m = w^m(Q^{m*})$. Each market maker j sells up to its Cournot capacity q_j^{m*}.*

4. The Choice to Become a Middleman or a Market Maker

We are now ready to study an intermediary's choice in the first stage to become either a middleman or a market maker. The choice is determined by the intermediary's transaction cost $k \in [\underline{k}, \overline{k}]$. We first show that more efficient intermediaries choose to become market makers, while less efficient intermediaries choose to become middlemen. We then solve for the numbers of middlemen and market makers, N and M. Using (16), we rewrite the profits for middlemen and market makers (10) and (25) as follows:

$$\pi(k) = \frac{1}{8N(1-\rho)} \left(\alpha - bQ^{m*} - k - \theta\right)^2 \tag{27}$$

$$\pi^m(k) = \frac{1}{b}(\alpha - bQ^{m*} - k - \theta)^2 - F^m \tag{28}$$

In Figure 1 we depict $\pi(k)$ and $\pi^m(k)$ as functions of k where the $\pi\pi$ curve represents the former and the $\pi^m\pi^m$ curve represents the latter. Both $\pi\pi$ and $\pi^m\pi^m$ are downward sloping. If $8N(1-\rho) < b$, then $\pi(k) > \pi^m(k)$ for any k so that no intermediary chooses to become a market maker. If $8N(1-\rho) > b$, $\pi^m\pi^m$ is steeper than $\pi\pi$. If $\pi^m\pi^m$ is always above $\pi\pi$, then all intermediaries choose to become market makers. If $\pi^m\pi^m$ and $\pi\pi$ intersect at E, then intermediary k chooses to become a market maker if $k < \underline{k}^d$, and chooses to become a middleman if $k \geq \underline{k}^d$. Those intermediaries whose transaction cost $k \geq k^*$ do not enter the market. In summary, more efficient intermediaries choose to become market makers.

To simplify the formal analysis, we discretize k. Let H be a sufficiently large, but predetermined number. We define

$$k_h = \underline{k} + \frac{\left(\overline{k} - \underline{k}\right)(h-1)}{H} \tag{29}$$

Thus, $k_1 = \underline{k}$ and $k_{H+1} = \overline{k}$. Since more efficient intermediaries choose to become market makers, the intermediaries $j = 1, \cdots, M$, characterized by a transaction cost less than or equal to k_j, are defined as market makers. The intermediaries $M + i = M + 1, \cdots, M + N$ are middlemen. Using (24) and (29), $\pi^m(k_j)$ and $\pi(k_{M+i})$ can be written as

$$\pi_j^m = A\left[\alpha-\theta-\underline{k}+\frac{\left(\overline{k}-\underline{k}\right)}{H}\left(\frac{M\left(M-1\right)}{2}-\left(j-1\right)\left(M+1\right)\right)\right]^2-F^m \quad (30)$$

$$\pi_{M+i} = B\left[\alpha-\theta-\underline{k}+\frac{\left(\overline{k}-\underline{k}\right)}{H}\left(\frac{M\left(M-1\right)}{2}-\left(M+i-1\right)\left(M+1\right)\right)\right]^2 \quad (31)$$

where $A=\frac{1}{b(M+1)^2}$ and $B=\frac{1}{8N(1-\rho)(M+1)^2}$.

The least efficient middleman earns zero profit. That is, $\pi_{M+N}=0$. On the other hand, the marginal market maker, M, must be indifferent between acting as a middlemen or a market maker. Hence, $\pi_M^m=\pi_M$. Therefore, the equilibrium numbers of middlemen and market makers, N and M, are determined by the two equations, $\pi_{M+N}=0$ and $\pi_M^m=\pi_M$, which are:

$$\alpha-\theta-\underline{k}+\frac{\left(\overline{k}-\underline{k}\right)}{H}\left(\frac{M\left(M-1\right)}{2}-\left(M+N-1\right)\left(M+1\right)\right) = 0$$

$$\left[\alpha-\theta-\underline{k}+\frac{\left(\overline{k}-\underline{k}\right)}{H}\left(\frac{M\left(M-1\right)}{2}-\left(M-1\right)\left(M+1\right)\right)\right]^2\left[\frac{1}{b}-\frac{1}{8N(1-\rho)}\right] = F^m\left(M+1\right)^2$$

Solving the above two equations, we obtain

$$N^* = \frac{1}{2}\left[\frac{b}{8(1-\rho)}+\left(\frac{b^2}{64(1-\rho)^2}+\frac{4bF^mH^2}{\left(\overline{k}-\underline{k}\right)^2}\right)^{\frac{1}{2}}\right] \quad (32)$$

$$\begin{aligned} M^* = & -\left(N^*+\frac{4\delta-1}{2\left(4\delta+1\right)}\right) \\ & +\left[\left(N^*+\frac{4\delta-1}{2\left(4\delta+1\right)}\right)^2+2\left(\frac{4\delta H\left(\overline{v}-\underline{c}-\theta-\underline{k}\right)}{\left(4\delta+1\right)\left(\overline{k}-\underline{k}\right)}+\frac{4\delta}{4\delta+1}-N^*\right)\right]^{\frac{1}{2}} \end{aligned} \quad (33)$$

where we have used (17) and (29) to obtain

$$\alpha = \left[4\delta\left(\overline{v}-\underline{c}\right)+\underline{k}^d+\theta\right]/\left(4\delta+1\right) \text{ and} \quad (34)$$

$$\underline{k}^d = k_M=\underline{k}+\frac{\left(\overline{k}-\underline{k}\right)\left(M-1\right)}{H} \quad (35)$$

to obtain (33).

We now consider the effect of changes in the marginal capacity cost, θ, the fixed capacity cost for market makers, F^m, and the size of the market, $(\overline{v} - \underline{c})$, on N^* and M^*. It is immediately seen from (32) that $\frac{\partial N^*}{\partial \theta} = 0$, $\frac{\partial N^*}{\partial(\overline{v}-\underline{c})} = 0$, and $\frac{\partial N^*}{\partial F^m} > 0$. Note that if $M^* > 0$, the second term on the right hand side of (33) must be greater than $N^* + \frac{4\delta-1}{2(4\delta+1)}$. Then it is easy to show that $\frac{\partial M^*}{\partial \theta} < 0$, $\frac{\partial M^*}{\partial(\overline{v}-\underline{c})} > 0$ and $\frac{\partial M^*}{\partial F^m} = \frac{\partial M^*}{\partial N^*}\frac{\partial N^*}{\partial F^m} < 0$. Summarizing we have:

Proposition 2 *More efficient intermediaries choose to become market makers. If the marginal cost of capacity installation declines or the size of the market increases, the number of market makers increases, while the number of middlemen remains constant. On the other hand, if the fixed cost of capacity installation for market makers increases, the number of market makers declines, while the number of middlemen increases.*

The intuition behind Proposition 2 can be seen in Figure 1. If the fixed cost of capacity installation for market makers, F^m, increases, then using (30), the profit curve for market makers, $\pi^m\pi^m$, shifts down to $\pi^{m\prime}\pi^{m\prime}$. Thus the lower bound of the transaction cost for middlemen, $\underline{k}^d$, declines. As a result, the number of market makers, M^*, declines, while the number of middlemen, N^*, increases. Note that as N^* increases, the profit curve for middlemen, $\pi\pi$, also becomes flatter. So the equilibrium moves from E to E', as illustrated in Figure 1.

If the marginal cost of capacity installation, θ, declines or the size of the market, $(\overline{v} - \underline{c})$, increases, then using (30) and (31), profits for both market makers and middlemen increase. The lower bound of the transaction cost for middlemen, $\underline{k}^d$, increases (not shown in Figure 1) and therefore the number of market makers, M^*, increases. On the other hand, the upper bound of the transaction cost for middlemen, k^*, also increases. The increases in $\underline{k}^d$ and k^* exactly cancel each other so that N^* is not affected.

5. Dispersion of Ask and Bid Prices

The results in Proposition 2 can be used to analyze the dispersion of ask and bid prices. Substituting (13) and (34) into (12), we have:

$$\underline{p} = \frac{p^m}{2} + \frac{1}{4}\left(\overline{v} + \underline{c} + \theta + \underline{k}^d\right), \ \overline{p} = p^m \tag{36}$$

$$\underline{w} = w^m, \ \overline{w} = \frac{w^m}{2} + \frac{1}{4}\left(\overline{v} + \underline{c} - \theta - \underline{k}^d\right) \tag{37}$$

Using (15), we obtain that the differences between the middlemen's lowest and highest ask prices equals the dispersion of bid prices. That is,

$$\overline{p} - \underline{p} = \overline{w} - \underline{w} = \frac{\delta}{(4\delta + 1)}\left[\overline{v} - \underline{c} - \theta - \underline{k}^d - 2Q^{m*}\right] \tag{38}$$

If the fixed cost of capacity installation for market makers, F^m, increases, the number of market makers, M, decreases. As a result, both the lower bound of the transaction cost for middlemen, $\underline{k}^d$, and total output served by market makers, Q^{m*}, decline. Thus, both $\left(\overline{p} - \underline{p}\right)$ and $(\overline{w} - \underline{w})$ become larger. The effect of a change in $(\overline{v} - \underline{c} - \theta)$ is more complicated. On the one hand, a larger $(\overline{v} - \underline{c} - \theta)$ directly increases $\left(\overline{p} - \underline{p}\right)$ and $(\overline{w} - \underline{w})$. On the other hand, a larger $(\overline{v} - \underline{c} - \theta)$ raises M and therefore increases both $\underline{k}^d$ and Q^{m*}, which reduces $\left(\overline{p} - \underline{p}\right)$ and $(\overline{w} - \underline{w})$. The overall effect of the change in $(\overline{v} - \underline{c} - \theta)$ on price dispersion, therefore, is ambiguous. Summarizing we have:

Proposition 3 *As the fixed cost of capacity installation for market makers increases, both ask prices and bid prices are more dispersed. On the other hand, when the marginal cost of capacity installation or the size of the market changes, its effect on price dispersion is ambiguous.*

The intuition behind the above proposition is straightforward. Since market makers charge the same prices, the price dispersion comes from the distribution of the prices charged by middlemen. As the number of middlemen, N, becomes larger, the distribution of middlemen becomes more dispersed. Using Proposition 2, if F^m increases, middlemen become more dispersed. However, if the marginal cost of capacity installation or the size of the market change, the distribution of middlemen is relatively unchanged.

6. Conclusions

Neoclassical economics leaves open the question of how actual markets attain equilibrium prices. The theory of intermediation and the microstructure of real markets developed by Spulber (1996, 1999) and Rust and Hall (2003), amongst others, has provided important insights about markets served by middlemen and a monopoly market maker. Many markets are however served by multiple but yet a limited number of market makers as well as middlemen. An extension of the models of Spulber and Rust and Hall is required to accommodate this market feature, and explain why some intermediaries choose to become middlemen, and others choose to become market makers. We fill this gap in the literature by developing a market microstructure model in which oligopolistic competition among market makers coexists with an active search market populated by middlemen, and intermediaries endogenously choose to become middlemen or market makers. A second result is that price dispersion of the good within the market is influenced by the numbers of market makers and middlemen which emerge in equilibrium.

A distinguishing feature of our model is the presence of capacity costs for all intermediaries. The presence of heterogeneous capacity costs allows us to simultaneously solve the problem of intermediaries' endogenous choices to become middlemen or market makers, and to derive the non-degenerate equilibrium of oligopolistic competition among market makers. We show that more efficient intermediaries choose to become market makers, while less efficient intermediaries choose to become middlemen. Market makers' intermediation behavior is decomposed into a two-stage process: capacity setting in the first stage and bid and ask price setting in the second. It is shown that the two-stage competition game among market makers is equivalent to Cournot competition when capacity cost is sufficiently large.

Our model is comprehensive yet sufficiently simple so that drawing out the intuition behind our results is not an onerous process. Importantly the model leads to nice intuitive conditions under which the numbers of middlemen and market makers are determined endogenously and in a closed form manner, along with the equilibrium ask and bid prices associated with these two types of intermediaries. The coexistence of both market makers and middlemen is a phenomenon that is observed in several real markets, such as the North American natural gas market. Conditions under which this phenomenon can emerge have not heretofore been analyzed in the literature.

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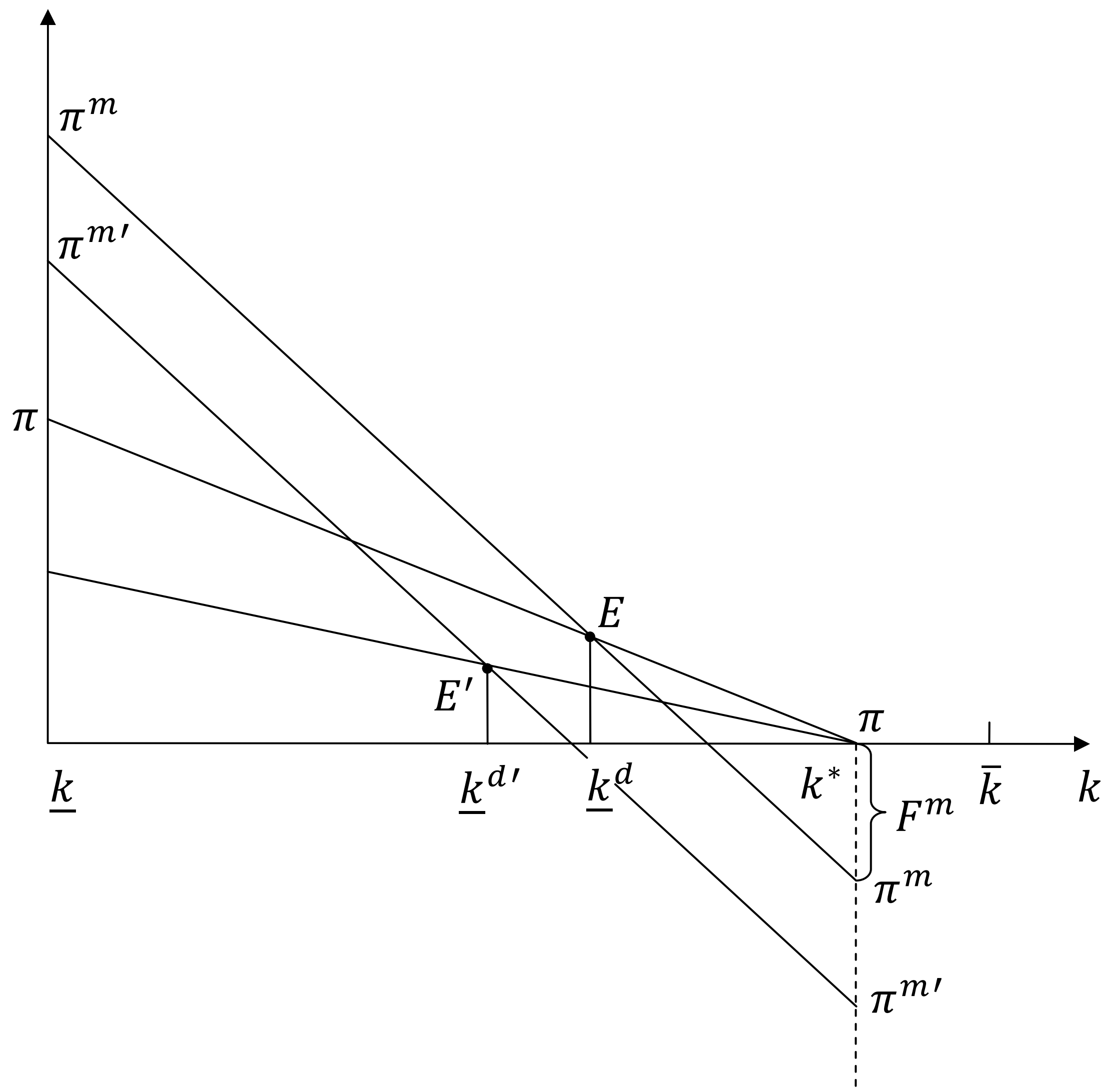

FIGURE 1: Profit Curves for Intermediaries. The figure illustrates the profit, π, of a middleman or a market maker in relation to transaction cost, k, and the fixed cost of capacity, F.

7. APPENDIX

In this appendix we analyze the mixed strategy equilibrium in the two-stage competition game for market makers. We prove that the Cournot outcome derived in the text is the unique subgame perfect Nash equilibrium if the marginal capacity cost is sufficiently large. As argued by Kreps and Scheinkman (1983), the mixed strategy occurs when some market maker j sets his capacity $\overline{q}_j > R_j(\overline{q}_{-j})$, and randomizes his price. We show that j's profit from choosing a mixed strategy is less then that of the Cournot outcome.

7.1. STAGE TWO PRICE COMPETITION

Let $\underline{p}^m$ and $\overline{w}^m$ be the lowest ask price and the highest bid price, respectively. When market makers randomize their prices, the quantity demanded by consumers who buy from market makers is at most $X(\underline{p}^m) = \overline{v} - v_c(\underline{p}, \underline{p}^m)$, and the quantity supplied by producers who sell to market makers is at most $Y(\overline{w}^m) = v_p(\overline{w}, \overline{w}^m) - \underline{c}$. Thus, if we show that for demand and supply functions $X(\underline{p}^m)$ and $Y(\overline{w}^m)$, market maker j's profit using the mixed strategy is less than that of the Cournot outcome, we have proved our result.

We first show that the demand for market makers, $X(p^m)$, is elastic at the Walrasian quantity Q^w. This result will be useful when we derive the SPNE of the game. Q^w is given by $p(Q^w) = w(Q^w)$. Using (15), the Walrasian quantity and prices are

$$Q^w = \frac{\alpha}{b} = \frac{4\delta\left(\overline{v} - \underline{c}\right) + \underline{k}^d + \theta}{8\delta}, \text{ and } p^w = w^w = \frac{\overline{v} + \underline{c}}{2} . \tag{A1}$$

The elasticity of demand at Q^w equals

$$\varepsilon = (dx/x) \,/\, (dp/p) = -\frac{(4\delta + 1)\left(\overline{v} + \underline{c}\right)}{4\delta\left(\overline{v} - \underline{c}\right) + \underline{k}^d + \theta} < -\frac{(4\delta + 1)\left(\overline{v} + \underline{c}\right)}{4\delta\left(\overline{v} - \underline{c}\right) + \overline{v} - \underline{c}} < -1 , \tag{A2}$$

where the fact that $\underline{k}^d + \theta < k^* + \theta = p^m - w^m \leq \overline{v} - \underline{c}$ has been used to derive the inequality. Thus, the demand function faced by market makers, $X(p^m)$ is elastic at $Q \leq Q^w$. Let p^s maximize sales revenue $p^m X(p^m)$. Sales revenue is maximized when $\varepsilon = -1$, which implies that $p^s < p^w$.

The total quantity bought by market makers, Q, is always less than the Walrasian quantity Q^w. To see this, note that if $Q > Q^w$, then the market maker i who bids the highest price must offer $w_i^m > w^w$. However, the highest ask price, $\overline{p}^m$ can be no more than the monopoly revenue maximizing price $p^s < p^w$, or the market clearing price $p^m(Q) < p^w = w^w$. So $\overline{p}^m < w^w$, which implies a negative profit for market maker i.

We now show that all market makers ask the market clearing price $p^m = p^m(Q)$. Given total quantity bought $Q \leq Q^w$, no market maker sets an ask price $p_j^m < p^m(Q)$, since otherwise j would sell the same quantity but at a lower price. Neither does any market maker ask $p_j^m > p^m(Q)$. If she does, there must be some market maker j selling a quantity less than the quantity bought. Consequently, reducing the ask price increases profit since demand is elastic. Therefore, all market makers ask the market clearing price $p^m = p^m(Q)$ in equilibrium. We are now ready to derive the mixed strategy equilibrium.

If two or more market makers set capacity at $\overline{q}_h \geq Q^w$ then we have unconstrained Bertrand competition in which all market makers who transact positive quantities set bid and ask prices $p^w = w^w = \frac{\overline{v}+c}{2}$ and earn zero profits. This outcome will not be the equilibrium in the full game since capacity is costly.

Let

$$q_j^* = R_j(\overline{q}_{-j}) = \frac{\alpha - k_j - b\sum_{i \neq j} \overline{q}_i}{2b} \tag{A3}$$

be the Cournot reaction function with transaction cost k_j (note that q_j^* differs from q_j^{m*} in (22) since the marginal capacity cost θ is not included here). Similar to the argument in Kreps and Scheinkman (1983), if $\overline{q}_{-h} < Q^w$ for at least one h and $\overline{q}_j > R_j(\overline{q}_{-j})$ for at least one j, there is no pure strategy equilibrium in the price setting stage.[6] The problem, however, is more complicated than in the symmetric oligopoly competition models of Kreps and Scheinkman (1983), Boccard and Wauthy (2004), and Loertscher (2008). The marginal costs k_j differ among market makers in our model. Therefore, the crucial result that the largest market maker sets the highest (lowest) ask

[6] If market makers bid the market clearing price $w = w^m\left(\overline{Q}\right)$, then j has an incentive to reduce the bid since $\overline{q}_j > R_j(\overline{q}_{-j})$. If market makers bid $w = w^m(Q) < w^m\left(\overline{Q}\right)$, then some market maker i who has not realized capacity has an incentive to increase the bid.

(bid) price in the symmetric case no longer holds. Nevertheless, the existence of a mixed strategy equilibrium for the game is guaranteed by the results of Dasgupta and Maskin (1986). We can nevertheless partially characterize equilibrium behavior.

We first show that at most one market maker bids the lowest price with positive probability. Suppose this were not true. Let $\underline{w}^m$ be the lowest bid price. Consider the market maker j with $\overline{q}_j > R_j(\overline{q}_{-j})$. If j buys up to capacity $\overline{q}_j$ at $\underline{w}^m$, j will prefer to bid a lower price since $\overline{q}_j > R_j(\overline{q}_{-j})$. If j buys less than $\overline{q}_j$ at $\underline{w}^m$, j will prefer to bid a slightly higher price since j's supply would then jump to $\overline{q}_j$ and profit would increase.

Let h be the market maker who bids the lowest price $\underline{w}^m$ with probability one. Ignoring the cost of capacity installation, her gross profit at $\underline{w}^m$ is

$$\begin{aligned}\pi_h^g &= [p^m(Y(\underline{w}^m)) - \underline{w}^m - k_h]\left[Y(\underline{w}^m) - \overline{q}_{-h}\right] \\ &= \left[p^m(q_h + \overline{q}_{-h}) - w^m\left(q_h + \overline{q}_{-h}\right) - k_h\right] q_h \ , \qquad \text{(A4)}\end{aligned}$$

where $q_h = Y(\underline{w}^m) - \overline{q}_{-h}$. Market maker h's profit must be maximized at $\underline{w}^m$. Hence maximizing profit implies $q_h = R_h(\overline{q}_{-h})$.

Now let $\overline{w}_h^m$ be the upper bound of market maker h's bid prices. We will show $\overline{w}_h^m = \overline{w}^m$ for all h and that each market maker buys up to its capacity at the upper bound. If $\overline{w}_i^m > \overline{w}_h^m$ for $i \neq h$, as we have previously argued, $\overline{w}_i^m$ must be i's monopoly price. Then h would earn zero profit but could earn a positive profit by bidding $\overline{w}_i^m + \varepsilon$. If h buys $q_h < \overline{q}_h$ at $\overline{w}^m$, then by bidding $\overline{w}^m + \varepsilon$, h could raise demand to utilize capacity $\overline{q}_h$. Summarizing we have

Lemma 1 *In the mixed strategy equilibrium, the upper bound of bid prices for all market makers is the same and each market maker buys up to its optimal capacity at the upper bound bid price. The market maker who bids the lowest price earns the gross profit,*

$$\pi_h^g = \left[p^m(R_h(\overline{q}_{-h}) + \overline{q}_{-h}) - w^m\left(R_h(\overline{q}_{-h}) + \overline{q}_{-h}\right) - k_h\right] R_h(\overline{q}_{-h}).$$

If a small capacity firm is outbid by a larger capacity firm, the small firm has a larger chance of not buying anything. The small firm may in turn adopt a more aggressive bidding strategy.

However, smaller firms are also less efficient firms in our model and are therefore more likely to bid lower prices due to higher transaction costs. Therefore, it is possible that a small firm may end up bidding the lowest price in the equilibrium. As a result the larger firm may earn a higher profit in the mixed strategy equilibrium than in the pure strategy equilibrium. This result differs from the classical result as described in Kreps and Scheinkman (1983).

7.2. Stage One Capacity Setting

We now show that the Cournot quantity is the unique equilibrium outcome if the marginal capacity cost, θ, is sufficiently large. Let all market makers other than j bid and ask the Cournot prices, $w = w^m(Q^*)$ and $p = p^m(Q^*)$. We want to show that j has no incentive to deviate from these prices. If she does, thereby buying quantity $q_j = q_j^* + \Delta q_j > q_j^*$, then we are in the region of a mixed strategy equilibrium. Let h be the market maker bidding the lowest price $\underline{w}^m$. If $j = h$, using Lemma 1 π_j^g is the same as the profit obtained at the Cournot capacity. Invoking Kreps and Scheinkman's argument, we can show that a deviation from the Cournot prices reduces net profit WHEN capacity is sufficiently costly. If $j \neq h$, according to the indifference property of a mixed strategy equilibrium, h's profits at $\underline{w}^m$ and $\overline{w}^m$ must be equal. Using Lemma 1 we have:

$$\left[p^m(R_h(\overline{q}_{-h}) + \overline{q}_{-h}) - w^m\left(R_h(\overline{q}_{-h}) + \overline{q}_{-h}\right) - k_h\right] R_h(\overline{q}_{-h}) = \left(\overline{z}^m - k_h\right) q_h^* \ , \tag{A5}$$

where $\overline{z}^m = p^m(Y(\overline{w}^m)) - \overline{w}^m$ is the bid-ask spread at $\overline{w}^m$. For market makers $h \neq j$, using (A3) we obtain

$$R_h(\overline{q}_{-h}) = q_h^* - \Delta q_j/2 \text{ and } R_h(\overline{q}_{-h}) + \overline{q}_{-h} = Q^* + \Delta q_j/2 \ . \tag{A6}$$

Using (A5) and (A6), we have $\overline{z}^m \leq p^m(Q^* + \Delta q_j/2) - w^m\left(Q^* + \Delta q_j/2\right)$. Now applying Lemma

1 we have the net profit of j, π_j

$$
\begin{aligned}
&= \left(\overline{z}^m - k_j\right)\left(q_j^* + \Delta q_j\right) - \theta\left(q_j^* + \Delta q_j\right) - F^m \\
&\leq \left(q_j^* + \Delta q_j\right)\left[p^m(Q^* + \Delta q_j/2) - w^m\left(Q^* + \Delta q_j/2\right) - k_j\right] - \theta\left(q_j^* + \Delta q_j\right) - F^m \\
&= \left[p^m(q_j^* + \Delta q_j/2 + q_{-j}^*) - w^m\left(q_j^* + \Delta q_j/2 + q_{-j}^*\right) - k_j\right]\left(q_j^* + \Delta q_j/2\right) \\
&\quad + \left[p^m(Q^* + \Delta q_j/2) - w^m\left(Q^* + \Delta q_j/2\right) - k_j\right]\Delta q_j/2 - \theta\left(q_j^* + \Delta q_j\right) - F^m \\
&\leq \pi_j^{Cournot} - \theta q_j^* - F^m + \{\left[p^m(Q^* + \Delta q_j/2) - w^m\left(Q^* + \Delta q_j/2\right) - k_j\right]/2 - \theta\}\Delta q_j \ , \quad \text{(A7)}
\end{aligned}
$$

where $\pi_j^{Cournot}$ represents j's profit at the Cournot equilibrium. The quantity $p^m(Q^* + \Delta q_j/2) - w^m\left(Q^* + \Delta q_j/2\right) - k_j$ is less than the maximum of market makers' bid-ask price spread. Thus, if the marginal installation cost θ is more than half of the maximum of market makers' bid-ask price spread, a choice to deviate from the Cournot prices leads to lower profit and therefore no market maker j will set $\overline{q}_j > R_j(\overline{q}_{-j})$. Given $\overline{q}_j \leq R_j(\overline{q}_{-j})$ for all market makers, the pure strategy equilibrium is the unique equilibrium. As we have argued in the text, therefore, the Cournot capacity and the corresponding market clearing prices $w^m\left(Q^{m*}\right)$ and $p^m(Q^{m*})$ define a unique SPNE of the game.

Proposition 4 *If marginal capacity cost is more than half of the maximum of market makers' bid-ask price spread, the Cournot outcome is the unique subgame perfect Nash equilibrium in the full game.*

TABLE A1.

NOTATION USED IN THE MAIN TEXT

Variable Symbol	Definition
$[\underline{\upsilon}, \overline{\upsilon}]$	Distribution of willingness to pay levels for the population of consumers
$\lambda \in (0, 1)$	Probability that a consumer or producer will exit the market in period t
$r_c(\upsilon)$	Reservation price for consumer υ
ρ	Time discount rate per period
δ	The exit-adjusted discount rate per period
$F(p)$	Equilibrium distribution of ask prices offered by middlemen
N	Number of middlemen
$[\underline{c}, \overline{c}]$	Distribution of production cost levels for the population of producers
$G(w)$	Equilibrium distribution of bid prices offered by middlemen
$r_p(c)$	Reservation price for producer c
$D(p)$	Expected discounted value of the stream of demands per middleman
$S(w)$	Expected discounted supply per middleman
$\left[\underline{k}, \overline{k}\right]$	Distribution of transaction cost levels of intermediaries
k^*	Zero profit transaction cost for a middleman
F, F^m	Fixed cost of capacity installation for a middleman, market maker

Table A1.

Notation Used in the Main Text

(continued)

Variable Symbol	**Definition**
M	Number of market makers
p^m	The ask price offered by market makers
$v_c(\underline{p}, p^m)$	The value of the marginal consumer with reservation price p^m
w^m	The bid price offered by market makers
$v_p(\overline{w}, w^m)$	The value of the marginal producer with reservation price w^m
a_1	The intercept of market demand faced by market makers
a_2	The slope of market demand faced by market makers
a_3	The intercept of market supply faced by market makers
a_4	The slope of market supply faced by market makers
q_j	The quantity traded by market maker j
q_j^*	Optimal Cournot quantity of market maker j
Q^m	The total quantity traded by market makers