Price Dispersion In a Model With Middlemen And Oligopolistic Market Makers

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Abstract

When Enron exited the North American natural gas market in late 2001 the dispersion of natural gas prices in North America jumped by a factor of 4 only to settle back to its pre-Enron exit level within roughly one month. One explanation for this observation is that the dispersion of bid and ask prices for natural gas sharply widened following Enron's exit. We develop a microstructure model of a market for a homogeneous good in which oligopolistic market makers coexist with an active search market and show that the exit of a market maker within this setting leads to an increase in the dispersion of bid and ask prices for the good. We also determine conditions under which the increase in dispersion is only temporary, which turn out to be only mildly restrictive. Market makers in the model post bid and ask prices which are freely observable. Middlemen stand ready to trade at bid and ask prices they quote on a private basis to consumers or producers who identify these intermediaries via a costly search process. We model competition between market makers as a two-stage game: capacity setting in the first stage and bid and ask price setting in the second stage. We characterize the equilibrium market structure of intermediaries and the distribution of prices in equilibrium and prove existence. We then use the model to study the effect on prices that emerges following a change in the market structure of intermediation, specifically through the exit of a market maker. Exit of a market maker initially results in a shift of trade from market makers as a whole to middlemen resulting in an increase in price dispersion. Following transition to the new market structure with fewer market makers, the volume served by all market makers declines and price dispersion increases.

Keywords: oligopolistic market makers; middlemen; intermediaries; price dispersion; North American natural gas market

JEL classifications: D43, L11, L13

1. Introduction

Price dispersion and changes in dispersion in real markets are a common phenomenon. There are at least two possible explanations for why one might observe a change in price dispersion in a real market. The first explanation is that the effective bid-ask spread for the good widens resulting in an increase in the spread between the highest and lowest transaction price. The second explanation is that the bid-ask spread does not change but the *levels* of the bid and ask prices jointly shift, that is, prices become more volatile. In this study we focus our attention on the first explanation. We study the dispersion of prices in a market composed of both middlemen and oligopolistic market makers and how dispersion is influenced by a change in the market structure of trade intermediation. The intermediation of trade in real goods is a significant feature of many markets. Often, intermediation is facilitated by online as well as offline intermediaries. We use the terminology 'market maker' to denote an online intermediary, and the terminology 'middleman' to denote an offline intermediary. Most market makers today facilitate trade by posting publicly observable bid and ask prices on the internet. On the other hand, middlemen stand ready to trade at bid and ask prices they quote on a private basis to customers who communicate with them largely via cellphone or telephone. The implications of such trading structures when online market makers are present have received surprisingly little attention, notable and important exceptions being Rust and Hall (2003) who extend the price-setting middlemen model of Spulber (1996, 1999) by introducing a monopolist market maker, and Baye and Morgan (2001) who study a monopolist information gatekeeper. Some markets however, such as the North American market for natural gas for which we present empirical evidence in the next section, have exhibited the joint existence of both middlemen as well as multiple market makers. The analysis of such an economic setting requires an alternative formulation of the model which is the centerpiece of our analysis.

Our study makes several new contributions to the literature on intermediation in real markets. First, the theoretical model developed in this study is new to the literature. The model features both oligopolistic market makers and competitive middlemen which, consistent with real markets, face different costs when executing transactions. Our framework extends the work of Spulber (1996,

1999) and Rust and Hall (2003). Further, the presence of asymmetric costs in the model differs from the symmetric oligopolistic competition model developed in Loertscher (2005) and allows us to examine a facet of intermediated markets for real goods that has not heretofore been investigated. Finally, our focus is on the dispersion of prices in an equilibrium in which market makers and middlemen coexist, and, on how the dispersion of prices changes when there is a change in the market structure of intermediated trade.

The study of oligopolistic competition amongst market makers is limited. Rust and Hall (2003) suggest approaching the problem within a Bertrand-style price competition setting. The problem, however, is that Bertrand price competition among market makers will usually lead to a Walrasian equilibrium as shown by Stahl (1988). In such an equilibrium all middlemen will be driven out of the market. To explain the coexistence of oligopolistic competition among market makers within an active search market, we introduce non-degenerate imperfect competition among market makers.

The demand and supply faced by market makers can be expressed as linear functions of price within the search setting in which market makers and middlemen coexist. We formulate a two-stage game in which market makers compete in capacity choice at the first stage and in bid and ask prices in the second stage. Similar to the well known argument made by Kreps and Scheinkman (1983), we show that the outcome of our two-stage game is equivalent to the Cournot outcome when capacity cost is sufficiently large.

We use the model to analyze the dispersion of prices in the market, and in particular how this dispersion changes when the market structure of intermediation changes. Price dispersion can consist of offline price dispersion in the search market populated by middlemen as well as online price dispersion that might for instance result from an equilibrium mixed price quoting strategy followed by market makers such as in the model of Baye and Morgan (2001, 2004). Our focus is on offline price dispersion. In our model middlemen are heterogenous in the transaction costs they incur when intermediating trade. A higher transaction cost increases the optimal ask price, but reduces the optimal bid price offered by a middleman. When capacity cost is sufficiently large, all market makers publicly post a unique ask (bid) price, which serves as the high (low) bound for the ask (bid) prices of middlemen. The consumer (producer) who has a reservation price lower (higher) than the price posted by the market makers transacts through middlemen. In this setting the spread between the highest and lowest ask (bid) price, what we refer to as dispersion, equals the spread between the most efficient middleman's ask (bid) price and the ask (bid) price posted by market makers.

As market makers enter or exit the market, the ask and bid prices posted by all market makers will change, and therefore the dispersion of ask and bid prices will also change. We show that the exit of a market maker initially results in a shift of trade from market makers as a whole to middlemen resulting in an increase in price dispersion. Following transition to the new market structure with fewer market makers, the volume served by all market makers declines and price dispersion increases.

Our study is related to the literature on the microstructure of market intermediation. Other than the work of Spulber (1996) and Rust and Hall (2003) which are the foundations upon which we build our model, Rubinstein and Wolinsky (1987) study a random matching model with buyers and sellers in which intermediaries act as dealers. Gehrig (1993) considers Bertrand competition between intermediaries who also compete within a search market and shows that competition eliminates the bid-ask spread. Yavas (1992, 1994) allows the consumer to choose between a market maker and a matchmaker and studies the effect of the matchmaker's presence on search intensity. Fingleton (1997) studies direct trade between buyers and sellers. Shevchenko (2004) studies a search setting in which intermediaries can hold inventories.

Our study is also related to the literature which focuses on capacity constrained oligopoly competition. Stahl (1988) studies a setting in which intermediaries bid to buy the capacities they will face when selling to consumers. Boccard and Wauthy (2004) extend Kreps and Scheinkman's duopoly result (1983) to the oligopoly setting. Loertscher (2005) introduces capacity constrained market makers in a symmetric oligopoly. An important difference between our model and these symmetric oligopolistic competition models, is that we allow different marginal costs amongst market makers. If the capacity cost is sufficiently large, we show that a unique subgame perfect Nash equilibrium (SPNE) is for market makers to choose Cournot capacities, accompanied by pure strategy market clearing ask and bid prices. If the capacity cost is small, however, it is possible that market makers choose a mixed strategy price setting path.

We begin by setting the stage with a brief review of events in the North American natural gas market during the latter part of 2001. A significant change in the intermediation of trade in the natural gas market occurred during late 2001. This was also a time during which there was an uprecedented increase in the dispersion of natural gas prices. Section 3 presents the framework for the model within a market setting that includes middlemen but no market makers. We then introduce a fixed number of oligopolistic market makers in section 4. In both sections 3 and 4 our ultimate focus is on the dispersion of prices in equilibrium. In Section 5 we turn to the effect on the dispersion of prices when a market maker exits. Section 6 concludes the paper. Table A1 presents a list of the variable symbols used in the paper along with a brief description of each variable.

2. The North American Natural Gas Market

The North American natural gas industry is composed of a distinct chain of businesses and exhibits little vertical integration. There is an active market for physical gas as well as an active market for gas futures contracts.[1] The parties involved include producers, gatherers, processors, pipelines, marketers (both market makers and middlemen as we have defined them), distributors and end users. In late 2001 the intermediation of trade in this market experienced an abrubt change when Enron, the largest market intermediary in terms of volume ceased providing this service and exited.

Figure 1 displays the relative dispersion of North American natural gas prices for the period 5/01/2001 through 5/24/2002. We measure dispersion using a relative range statistic for each day of the sample period equal to $Rg_t =$ (Daily High Price - Daily Low Price)/Daily Midpoint Price, where we approximate the Midpoint Price with the average of the high and low prices. We obtained the daily high and low spot prices of natural gas, stated in MMCF (millions of cubic feet), for gas delivered at the Henry Hub for the period 1/2/2001 through 5/24/2002 from records maintained by Platts. Platts is a leading industry monitor and publisher of energy industry information. The

[1] The primary market for natural gas futures contracts is the New York Mercantile Exchange (http://www.nymex.com). For an insightful analysis of the evolution of the natural gas market see Doane and Spulber (1994).

Henry Hub is a major delivery point for natural gas, is located in Louisiana, and is the designated delivery point for the natural gas futures contract traded on the New York Mercantile Exchange (NYMEX). The graph shows that not only did price dispersion for this market change during the time period illustrated but that there was roughly a 4-fold increase in price dispersion during late November and early December of 2001. It happens that the period late November through early December 2001 corresponds to a period in which there was a significant change in the market structure of intermediation in the North American natural gas market. Specifically, that period surrounds the exit of Enron, a major intermediary at the time, from this market.[2]

Table 1 presents regression estimation results which reveal that the increase in dispersion during late 2001 was not due to other underlying fundamental supply and demand effects such as changes in the amount of gas in storage which are correlated with supply changes, changes in the weather, which are correlated with demand changes, or statistical artifacts such as lagged effects or calendar time effects. Two dummy variables are included in the estimated model. Specifically, $D1$ takes the value 1 during the period 11/16/2001 to 12/10/2001 and 0 otherwise, and, $D2$ takes the value 1 during the period following 12/10/2001. The period selected in defining D_1 is motivated by the fact that this was the period during which Enron's participation in this market ceased. The model estimated is given by

$$\begin{aligned} Rg_t = {} & \beta_0 + \beta_1 Rg_{t-1} + \beta_2 D1_t + \beta_3 D2_t + \beta_4 T_t + \beta_5 W_t + \beta_6 Th_t + \beta_7 F_t + \beta_8 PS_t + \beta_9 PS_{t-1} \\ & + \beta_{10} NS_t + \beta_{11} NS_{t-1} + \beta_{12} \Delta CDD_{t+6} + \beta_{13} \Delta CDD_t + \beta_{14} \Delta HDD_{t+6} + \beta_{15} \Delta HDD_t + \varepsilon_t \end{aligned}$$

where Rg_t is the relative price range, T_t takes the value 1 if day t is a Tuesday and 0 otherwise, and corresponding dummies are identified for W (Wednesday), Th (Thursday) and F (Friday). It is possible that the spike in dispersion is due to a shift in net demand factors. We control for such effects by introducing supply and demand factors known to be important determinants of the behavior of natural gas prices. The first factor we introduce is news about the change in the supply of natural gas in storage. Natural gas is a storable commodity and storage is used to

[2] During the 3rd Quarter of 2001 Enron handled roughly 12% of the wholesale trades in North American natural gas. The amount handled by the company effectively dropped to 0 during the 4th Quarter of 2001.

carry forward gas to satisfy future demand. During the period of time we study the American Gas Association compiled and distributed a weekly report on the amount of natural gas in storage in North America. In addition, during this period Bloomberg compiled and reported consensus forecasts of the amount of gas expected to be reported in storage. We define two variables, the first, PS, equals the difference between the AGA storage report of gas in storage and the forecast of gas in storage reported by Bloomberg, whenever the difference is greater than 4 Bcf.[3] We define a corresponding variable, NS, as the negative storage shock, which equals the difference as defined above whenever the AGA report is 4 Bcf less than the Bloomberg forecast. The second factor introduced reflects the demand side. Here we use weather related variables to proxy for demand effects. There are 13 major cities serviced by gas pipelines that are interconnected through the Henry Hub.[4] We compute a weather index based upon daily temperatures observed at each of these cities.[5] We then compute two weather-related measures, ΔCDD and ΔHDD, where ΔCDD represents the difference between the temperature for the day and the 30-year average temperature, whenever the actual temperature is greater than 60° F, and ΔHDD is the corresponding difference whenever the actual temperature is less than 60° F. Therefore these variables measure unusual temperature variation and hence unusual demand. The subscript notation $t + 6$ indicates the quantity is a proxy for the 6-day ahead weather forecast. We estimate the model using the total time series spanning the period 1/2/2001-5/24/2002. The estimation results are presented in Table 1.[6] The sign of the coefficient β_2 is positive and the point estimate equals 12.443 (t-statistic: 3.78). The point estimate of β_3 is 0.028 (t-statistic: 0.10). The interpretation of the results reported in Table 1 is therefore that price dispersion in this market was significantly greater during late 2001 but had returned to its pre-November 2001 level by late December. As mentioned earlier, there are at least two possible explanations for why price dispersion may have increased in this market. One explanation is that the effective bid-ask spread for natural gas widened resulting in

[3] Bcf: billion cubic feet.

[4] Dallas, BatonRouge, Chicago, Phoenix, Los Angeles, Atlanta, Saint Louis, New York, Philadelphia, Little Rock, Oklahoma City, Denver and Cleveland.

[5] These data are obtained from numerous regional offices of the national weather service or from the relevant offices of the states in which the cities are located.

[6] Coefficient estimates reported in Table 1 are actual values times 10^2. The t-statistics reported in Table 1 are based upon standard errors corrected for heteroscedasticity and autocorrelation of the disturbances following Newey and West (1987).

an increase in the spread between the highest and lowest transaction price possibly due to some change in the market structure of trading. The remainder of the paper develops a model of market intermediation in which we show that under plausible conditions, the exit of an intermediary does indeed lead to an increase in price dispersion which can under certain conditions be followed by a reversion to the level observed prior to the structural change. We now turn to the backbone of the model, a setting in which middlemen are present but no market makers.

3. The Market With Middlemen But No Market Maker

Market makers post the price at which they are willing to buy (bid price) and the price at which they are willing to sell (ask price) for all consumers and producers to freely see at no cost to the consumer or producer. In contrast the bid and ask prices of middlemen are private information. We assume a consumer or producer can discover the bid and ask prices of a middleman through a costly search process. We begin our discussion with the development of the search market in which only middlemen are present.

3.1. Consumers And Producers

The population of consumers is represented by a uniform distribution of willingness to pay levels on the interval $[\underline{v}, \overline{v}]$ similar to Spulber (1996, 1999) and Rust and Hall (2003). We assume any particular consumer searches randomly across the N middlemen present in the market. Each middleman therefore faces an equal probability of making a trade. Consumers know the equilibrium distribution of ask prices, p, offered by middlemen $F(p)$ but not the particular middleman associated with each price. A consumer of type v consumes one unit of the good if the price she pays is at most v. The unitary consumption assumption implies that these consumers will not make any subsequent transactions after their initial trade. Consumers remain in the market for a random length of time before permanently exiting from the market. Let $\lambda \in (0, 1)$ be the probability that a consumer exits the market and ρ be the time discount rate for period t. In each period a fraction

λ of the population of consumers exits the market and is replaced by an equal fraction of new consumers. The optimal search strategy for a type v consumer takes the form of a reservation price rule: accept any ask price less than the reservation price $r_c(v)$. The reservation price is defined by the standard recursive equation

$$v - r_c(v) = \frac{1}{1+\delta}\left[\int_{\underline{p}}^{r_c(v)} (v-p)\,dF(p) + \int_{r_c(v)}^{\infty} (v - r_c(v))\,dF(p)\right] ,$$

which can be simplified as

$$v = r_c(v) + \frac{1}{\delta}\int_{\underline{p}}^{r_c(v)} F(p)dp , \tag{1}$$

where $\delta = 1/[\rho(1-\lambda)] - 1$ is the composite exit-adjusted discount rate per period. The cost of searching in the model is the cost of waiting to transact implicit in the discount rate. The function $r_c(v)$ is strictly increasing in v on the interval $(v^*, \overline{v})$, where v^* is the marginal consumer for whom the expected gain from searching is zero. It is easy to see that $v^* = r_c(v^*) = \underline{p}$, and $\underline{p}$ is the lowest ask price across all middlemen.

Consumers and producers randomly search across the N middlemen. As a result each middleman will receive an equal share of the total searchers. Suppose that a middleman asks a price of p. Let $D_i(p)$ denote the mass of consumers who are among the initial population and who purchase the good in period i. Total expected discounted demand, $D(p)$, is the expected discounted value of the stream of demands in all future periods by the initial population of consumers as well as the stream of demands from each succeeding generation of new consumers entering the market. That is,

$$D(p) = \sum_{i=0}^{\infty} \rho^i D_i(p) + \lambda \sum_{j=1}^{\infty} \rho^j \sum_{i=0}^{\infty} \rho^i D_i(p) = \frac{\overline{r_c} - p}{N\,(1-\rho)} , \tag{2}$$

where N is the number of middlemen and $\overline{r_c}$ is the highest reservation value of the population of consumers.[7] Total discounted demand, conditional on p, depends upon the marginal gain from trade assessed by the consumer with the highest personal per unit value $\overline{v}$ (highest reservation price

[7] Spulber (1996) and Rust and Hall (2003) present the development of the demand and supply functions shown in equations (2) and (4). A detailed derivation of the demand and supply functions is available from us upon request.

$\overline{r_c}(\overline{v})$).

A producer of type c produces one unit of the good at a cost of c. The population of producers is represented by a uniform distribution of costs on the interval $[\underline{c}, \overline{c}]$. Producers know the equilibrium distribution of bid prices $G(w)$ but not the particular middleman associated with each price w. Each producer searches randomly across middlemen and accepts any bid price w that exceeds the reservation price $r_p(c)$, which is given by the solution to

$$c = r_p(c) - \frac{1}{\delta} \int_{r_p(c)}^{\overline{w}} [1 - G(w)]\, dw \ . \tag{3}$$

The reservation value $r_p(c)$ is a strictly increasing function of c on the interval $(\underline{c}, c^*)$, where $c^* = r_p(c^*) = \overline{w}$ is the cost of the marginal producer for whom the expected gain from searching is zero (i.e. the producer with the largest reservation selling price). Producers also exit the market each period with probability $\lambda \in (0, 1)$ and are replaced accordingly with new producers. By reasoning similar to the articulation of demand, a middleman's total expected discounted supply function is therefore

$$S(w) = \frac{-\underline{r_p} + w}{N(1-\rho)} \ , \tag{4}$$

where $\underline{r_p}$ is the lowest reservation value of the population of producers.

The equilibrium demand of consumers who participate in the market is given by $(\overline{r_c} - p)/(1-\rho) = ND(p)$, which is equal to the equilibrium supply of producers who participate in the market, $(-\underline{r_p} + w)/(1-\rho) = NS(w)$.

3.2. MIDDLEMEN

Middlemen incur a transaction cost k per unit of the good purchased from any producer. The population of potentially active middlemen is represented by a uniform distribution of transaction costs k on the interval $[\underline{k}, \overline{k}]$. A middleman's present discounted value of trading profits is given by

$$\pi(p, w, k) = pD(p) - (w + k)S(w) \ , \tag{5}$$

where the middleman buys at w, incurs the cost per unit of k and sells at p. Each middleman, indexed by k, solves the following problem

$$\max_{p,w} \pi(p, w, k) \text{ subject to } D(p) \leq S(w) \ . \tag{6}$$

The first order conditions associated with problem (6) yield optimal ask and bid prices, p and w respectively, for a middleman with transaction cost k

$$p(k) = \frac{3\overline{r_c} + \underline{r_p} + k}{4} \text{ and } w(k) = \frac{\overline{r_c} + 3\underline{r_p} - k}{4} \ . \tag{7}$$

A higher transaction cost increases the optimal ask price $p(k)$, but reduces the optimal bid price $w(k)$ conforming with intuition. These bid and ask prices also equate supply and demand in every period as has been shown by Spulber (1999, Ch. 6). Substitution of the optimal bid and ask prices into the profit function gives

$$\pi(p, w, k) = \frac{1}{8N(1-\rho)} \left(\overline{r_c} - \underline{r_p} - k\right)^2 \ . \tag{8}$$

The value for k which solves $\pi(p, w, k) = 0$ gives the highest transaction cost that any middleman can incur while both serving the market and surviving. Denote this value as k^* where it follows that

$$k^* = \overline{r_c} - \underline{r_p} \ . \tag{9}$$

Any middleman with transaction cost $k > k^*$ will not enter the market. If $k^* < \underline{k}$, the market is inactive since no middleman has a transaction cost less than k^* and hence none could survive. We are interested in active markets and so focus on the case $k^* > \underline{k}$. We will also assume that $\overline{k}$ is sufficiently large so that $k^* < \overline{k}$. Thus, the number of middlemen operating in equilibrium is represented by $N = k^* - \underline{k}$.

Letting k in equation (7) equal first $\underline{k}$ and then k^*, we obtain the lower and upper bounds of the equilibrium distribution of ask and bid prices. Ask prices are uniformly distributed on the interval

$[\underline{p}, \overline{p}]$, and bid prices are uniformly distributed on the interval $[\underline{w}, \overline{w}]$, where

$$\begin{aligned} \underline{p} &= \left(3\overline{r_c} + \underline{r_p} + \underline{k}\right)/4,\ \overline{p} = \overline{r_c} \\ \underline{w} &= \underline{r_p},\ \overline{w} = \left(\overline{r_c} + 3\underline{r_p} - \underline{k}\right)/4\ . \end{aligned} \tag{10}$$

The maximum ask price is equal to the highest reservation value of the population of consumers and the minimum bid price is equal to the lowest reservation value of the population of producers. We restrict our attention to stationary pricing policies on the equilibrium path. Thus, the pair of equilibrium ask and bid prices derived above represent steady-state equilibrium prices in each period. Substituting the uniform distribution $F(p)$ and expression (10) into equation (1) for the consumer with the highest reservation price $\overline{r_c} = r_c(\overline{v})$, and similarly, substituting the uniform distribution $G(w)$ and expression (10) into equation (3) for the producer with the lowest reservation price $\underline{r_p} = r_p(\underline{c})$, we solve for $\overline{r_c}$ and $\underline{r_p}$. The operation gives $\overline{r_c} + \underline{r_p} = \overline{v} + \underline{c}$. Using this result, expression (10) can be rewritten as:

$$\begin{aligned} \underline{p} &= \frac{\overline{r_c}}{2} + \frac{\overline{v} + \underline{c} + \underline{k}}{4},\ \overline{p} = \overline{r_c} \\ \underline{w} &= \underline{r_p},\ \overline{w} = \frac{\underline{r_p}}{2} + \frac{\overline{v} + \underline{c} - \underline{k}}{4}\ . \end{aligned} \tag{11}$$

4. The Market With Oligopolistic Market Makers

We now extend Rust and Hall's (2003) model by introducing oligopolistic market makers, who with $N^{\hat{}}$ middlemen, serve the market. We begin by assuming there are M market makers. Market maker $j(j = 1, \cdots, M)$ posts publicly observable ask and bid prices $\left(p_j^m, w_j^m\right)$. There is no cost to a consumer or producer to view these prices. Competition amongst market makers is modeled as a two-stage game in which market makers compete in capacity in the first stage, and compete in bid and ask prices in the second stage.

4.1. Demand And Supply Faced By Market Makers

Consider an economy in which both middlemen and multiple market makers serve the market. Let $\hat{\underline{p}}$ be the lowest ask price and $\hat{\overline{w}}$ the highest bid price presented by middlemen. Let p^m be the (lowest) ask price and w^m the (highest) bid price posted by market makers. We will first derive the demand and supply functions faced by market makers as a whole. Following Rust and Hall (2003), let $v_c(\hat{\underline{p}}, p^m)$ be the value of the marginal consumer with reservation value p^m. That is,

$$v_c(\hat{\underline{p}}, p^m) = p^m + \frac{1}{\delta} \int_{\hat{\underline{p}}}^{p^m} F(p)dp \ . \tag{12}$$

The consumer who has the reservation value $v \in [\underline{v}, \hat{\underline{p}})$ will not trade. If $v \in [\hat{\underline{p}}, v_c(\hat{\underline{p}}, p^m))$, it is optimal for consumers to search in the middlemen market. If $v \in [v_c(\hat{\underline{p}}, p^m), \overline{v}]$, then it is optimal for consumers to buy the good from a market maker. A symmetric argument gives

$$v_p(\hat{\overline{w}}, w^m) = w^m - \frac{1}{\delta} \int_{w^m}^{\hat{\overline{w}}} [1 - G(w)]\, dw \ , \tag{13}$$

where $v_p(\hat{\overline{w}}, w^m)$ is the value of the marginal producer with reservation value w^m. Applying the results from Section 2, it can be shown that the distribution functions of the ask and bid prices presented by middlemen, $F(p)$ and $G(w)$, are still uniform with supports

$$\begin{aligned} \hat{\underline{p}} &= (3p^m + w^m + \underline{k})/4, \ \hat{\overline{p}} = p^m \\ \hat{\underline{w}} &= w^m, \ \hat{\overline{w}} = (p^m + 3w^m - \underline{k})/4 \ . \end{aligned} \tag{14}$$

Market makers as a whole face total market demand, $X = \overline{v} - v_c(\hat{\underline{p}}, p^m)$, and total market supply, $Y = v_p(\hat{\overline{w}}, w^m) - \underline{c}$. Let $X = Y$, which implies that

$$p^m + w^m = \overline{v} + \underline{c} \ . \tag{15}$$

Using equations (12), (13), (14) and (15), we have:

$$X(p^m) = \hat{a_1} - \hat{a_2} p^m \text{ and } Y(w^m) = \hat{a_3} + \hat{a_4} w^m \ , \tag{16}$$

where $\hat{a_1} = [(8\delta+1)\,\overline{v} + \underline{c} + \underline{k}]\,/\,(8\delta)\,,\ \hat{a_3} = -\,[(8\delta+1)\,\underline{c} + \overline{v} - \underline{k}]\,/\,(8\delta)\,,$ and $\hat{a_2} = \hat{a_4} = (4\delta+1)\,/\,(4\delta)\,.$ The inverse market demand and supply functions are therefore

$$p^m(X) = \frac{\hat{a_1}}{\hat{a_2}} - \frac{X}{\hat{a_2}} \text{ and } w^m(Y) = -\frac{\hat{a_3}}{\hat{a_4}} + \frac{Y}{\hat{a_4}} \ . \tag{17}$$

The Walrasian quantity Q^w is given by $p(Q^w) = w(Q^w)$. Let $\alpha = \frac{\hat{a_1}}{\hat{a_2}} + \frac{\hat{a_3}}{\hat{a_4}} = [4\delta\,(\overline{v} - \underline{c}) + \underline{k}]\,/\,(4\delta+1) > 0$ and $b = \frac{1}{\hat{a_2}} + \frac{1}{\hat{a_4}} = 8\delta/\,(4\delta+1) > 0$. The Walrasian quantity and prices are

$$Q^w = \frac{\alpha}{b} = \frac{4\delta\,(\overline{v} - \underline{c}) + \underline{k}}{8\delta}, \text{ and } p^w = w^w = \frac{\overline{v} + \underline{c}}{2} \ . \tag{18}$$

The elasticity of demand at Q^w equals

$$\varepsilon = (dx/x)\,/\,(dp/p) = -\frac{(4\delta+1)\,(\overline{v} + \underline{c})}{4\delta\,(\overline{v} - \underline{c}) + \underline{k}} < -\frac{(4\delta+1)\,(\overline{v} + \underline{c})}{4\delta\,(\overline{v} - \underline{c}) + \overline{v} - \underline{c}} < -1 \ ,$$

where the fact that $\underline{k} < k^* = \overline{r_c} - \underline{r_p} \leq \overline{v} - \underline{c}$ has been used to derive the inequality. Thus, the demand function faced by market makers, $X(p^m)$ is elastic at $Q \leq Q^w$. Let p^s maximize sales revenue $p^m X(p^m)$. Sales revenue is maximized when $\varepsilon = -1$, which implies that $p^s < p^w$. These results will be useful when we derive the SPNE of the game.

4.2. The Efficient-Rationing Rule

Let two market makers i and j post ask prices $p_i^m < p_j^m$. Assume market maker i self imposes a capacity constraint $\overline{q_i}$. At p_j, the demand faced by the two market makers together is $X(p_j^m) = \overline{v} - v_c(\hat{\underline{p}}, p_j^m)$. If $\overline{q_i} \geq X(p_j^m)$, then all consumers in the interval $[v_c(\hat{\underline{p}}, p_j^m), \overline{v}]$ prefer to buy the good from i so that the residual demand for market maker j, $x_j(p_j^m)$, is zero. If $\overline{q_i} < X(p_j^m)$, we assume that the most eager consumers buy from i. Thus, the residual demand for j is

$$x_j(p_j^m) = \overline{v} - \overline{q_i} - v_c(\hat{\underline{p}}, p_j^m) = \hat{a_1} - \hat{a_2} p_j^m - \overline{q_i} \ , \tag{19}$$

which implies an efficient-rationing rule when the market demand function is given by (16). This result can be easily generalized to the case of M market makers. Symmetrically, the efficient-rationing rule is also assumed for the supply side. Recall that an efficient-rationing rule is key to insuring the Cournot outcome emerges as an equilibrium of the two-stage, capacity-constrained price game.[8]

4.3. Stage-Two Simultaneous Ask And Bid Price Setting

Let y_j and x_j denote the quantity bought and sold by market maker j, respectively. Market maker j incurs a per unit cost t_j for each transaction processed. We order market makers so that t_j is nondecreasing in j. Therefore, market maker 1 is the most efficient in terms of having the lowest processing cost while market maker M is the least efficient. Assuming efficient rationing, market maker j will face a residual-demand function against other market makers. We first construct the Cournot equilibrium in quantity competition, and then discuss ask and bid price setting in the stage-two subgame.

If $q_j = x_j = y_j$, market maker j's profit is

$$\pi_j = (p^m - w^m - t_j)q_j = \left[\alpha - t_j - b\sum_{i=1}^{M} q_i\right] q_j \ . \tag{20}$$

Recall that we are working with expected discounted demand and supply and that we restrict our attention to stationary pricing policies on the equilibrium path. The first order condition for profit maximization by a market maker is

$$\alpha - t_j - b\sum_{i=1}^{M} q_i - bq_j = 0 \text{ for } j = 1, \cdots, M \ , \tag{21}$$

which gives market maker j's optimal reaction function

$$q_j = R_j(q_{-j}) = \frac{\alpha - t_j - b\sum_{i\neq j} q_i}{2b} \ , \tag{22}$$

where q_{-j} denotes the quantity transacted by all other market makers other than j ,and $R_j(\bullet)$ is

[8] Readers are directed to Davidson and Deneckere (1986) for a thorough discussion of the efficient rationing rule in a Cournot equilibirum.

the reaction function. Solving the m equations in (21) gives the Cournot equilibrium quantity

$$q_j^* = \frac{\alpha + \sum_{i=1}^{M} t_i - (M+1)r_j}{(M+1)b} \ . \tag{23}$$

It is easy to see $q_1^* > q_2^* > \cdots > q_M^*$. We assume the processing cost t_j for any market maker is sufficiently small so that $q_M^* > 0$. The total Cournot quantity is therefore

$$Q^* = \sum_{j=1}^{M} q_j^* = \frac{M\alpha - \sum_{i=1}^{M} t_i}{(M+1)b} \ . \tag{24}$$

We now show that all market makers ask the market clearing price $p^m = p^m(Q)$ in equilibrium. The total quantity bought by market makers, Q, is always less than the Walrasian quantity Q^w. To see this, note that if $Q > Q^w$, then the market maker i who bids the highest price must offer $w_i^m > w^w$. However, the highest ask price, $\overline{p}^m$, can be no more than the monopoly revenue maximizing price $p^s < p^w$, or the market clearing price $p^m(Q) < p^w = w^w$. So $\overline{p}^m < w^w$, which implies a negative profit for market maker i.

Given total quantity bought $Q \leq Q^w$, no market maker sets an ask price $p_j^m \leq p^m(Q)$, since otherwise j would sell at the same quantity but a lower price. Neither does any market maker ask $p_j^m > p^m(Q)$. If she does, there must be some market maker j selling a quantity less than the quantity bought. Then reducing the ask price increases profit since the demand is elastic. Therefore, all market makers ask the market clearing price $p^m = p^m(Q)$ in equilibrium.

Let $\overline{q}_j$ be the capacity set by market maker j in the first stage. We now are ready to derive the equilibrium of the price setting subgame. We consider two cases: Case I, all market makers set their capacities at or below the optimum, $\overline{q}_j \leq R_j(\overline{q}_{-j})$ for all j, and Case II, at least one market maker sets capacity above the optimum, $\overline{q}_j > R_j(\overline{q}_{-j})$ for at least one j.

4.3.1. Case I: Pure Strategy Equilibrium, $\overline{q}_j \leq R_j\left(\overline{q}_{-j}\right)$ for all j

In this case, the total capacity of market makers equals,

$$\overline{Q} = \sum_{j=1}^{M} \overline{q}_j \leq \sum_{j=1}^{M} R_j(\overline{q}_{-j}) = \frac{M\alpha - \sum_{i=1}^{M} t_i - b(M-1)\overline{Q}}{2b} \ , \tag{25}$$

which implies that $\overline{Q} \leq Q^* < Q^w$.

We first show that $w^m = w^m\left(\overline{Q}\right)$ is an equilibrium. No market maker j bids a price higher than $w^m(\overline{Q})$ since otherwise j would buy at the same quantity but a higher price. If all market makers set $w^m = w^m\left(\overline{Q}\right)$, then j has no incentive to reduce her bid price. If she does, noting that we have shown above that market makers always set the market clearing ask price and $\overline{Q} < Q^w$, then her profit becomes

$$\pi_j(q_j) = \left[p^m(q_j + \overline{q}_{-j}) - w^m\left(q_j + \overline{q}_{-j}\right) - t_j\right] q_j \ ,$$

and $q_j < \overline{q}_j$. Then $\pi_j(q_j) < \pi_j(\overline{q}_j)$ since $\overline{q}_j < R_j(\overline{q}_{-j})$.

We now prove the uniqueness of the equilibrium. All market makers bid the same price in equilibrium. Suppose this were not true and suppose we let market maker j bid $w_j^m < \overline{w}^m$ where $\overline{w}^m = \max\{w_i^m(i = 1, \cdots, j-1, j+1, \cdots, M)\}$, then market maker i with the highest bid could reduce her bid price and maintain her capacity; if i did not act to reduce her bid price the result would be that i would buy the entire market supply at $\overline{w}^m$. In this case j buys nothing at w_j^m, whereas j could capture demand (build up a positive capacity) by bidding $\overline{w}^m + \varepsilon$ for small ε and earn a positive profit.

Now suppose that $w_1^m = \cdots = w_M^m = w^m < w^m\left(\overline{Q}\right)$ is the equilibrium bid price. Then at least one market maker j could not realize its capacity. By charging $w^m + \varepsilon$, j could capture the entire market and would buy and sell up to capacity, which would increase her profit for a sufficiently small $\varepsilon > 0$. Thus, $w^m < w^m\left(\overline{Q}\right)$ cannot be the equilibrium bid price. Summarizing the results, we have:

Lemma 1 *Suppose capacities are $\overline{q}_j \leq R_j(\overline{q}_{-j})$ for all $j = 1, \cdots, M$. There exists a unique Nash equilibrium in the price setting subgame in which all market makers offer the market clearing bid and ask prices, $w = w^m\left(\overline{Q}\right)$ and $p = p^m(\overline{Q})$.*

4.3.2. CASE II: MIXED STRATEGY EQUILIBRIUM, $\overline{q}_j > R_j\left(\overline{q}_{-j}\right)$ FOR AT LEAST ONE j

If two or more market makers set $\overline{q}_j \geq Q^w$ then we have unconstrained Bertrand competition in which all market makers who transact positive quantities set bid and ask prices $p^w = w^w = \frac{\overline{v}+c}{2}$ and earn zero profits. This outcome will not be the equilibrium in the full game since capacity is costly.

Similar to the argument in Kreps and Scheinkman (1983), if $\overline{q}_{-j} < Q^w$ for at least one j and $\overline{q}_k > R_k(\overline{q}_{-k})$ for at least one k, there is no pure strategy equilibrium.[9] The problem, however, is more complicated than in the symmetric oligopoly competition models of Kreps and Scheinkman (1983), Boccard and Wauthy (2004), and Leortscher (2005). The marginal costs t_j differ among market makers in our model. Therefore, the crucial result that the largest market maker sets the highest (lowest) ask (bid) price in the symmetric case no longer holds. Nevertheless, the existence of a mixed strategy equilibrium for the game is guaranteed by the results of Dasgupta and Maskin (1986). We can still partially characterize equilibrium behavior which will be used to analyze price dispersions in a later section.

We first show that at most one market maker bids (asks) the lowest (highest) price with positive probability. Suppose this were not true. Let $\underline{w}^m$ be the lowest bid price. Consider the market maker j with $\overline{q}_j > R_j(\overline{q}_{-j})$. If j buys up to capacity $\overline{q}_j$ at $\underline{w}^m$, j will prefer to bid a lower price since $\overline{q}_j > R_j(\overline{q}_{-j})$. If j buys less than $\overline{q}_j$ at $\underline{w}^m$, j will prefer to bid a slightly higher price since j's supply would then jump to $\overline{q}_j$ and profit would increase.

Let k be the market maker who bids less than $\underline{w}^m$ with probability one. Her profit at $\underline{w}^m$ is

$$\begin{aligned}\pi_k &= \left[p^m(Y(\underline{w}^m)) - \underline{w}^m - t_k\right]\left[Y(\underline{w}^m) - \overline{q}_{-k}\right] \\ &= \left[p^m(q_k + \overline{q}_{-k}) - w^m\left(q_k + \overline{q}_{-k}\right) - t_k\right] q_k\ ,\end{aligned}$$

where $q_k = Y(\underline{w}^m) - \overline{q}_{-k}$. Market maker k's profit must be maximized at $\underline{w}^m$. Hence maximizing

[9] If market makers bid the market clearing price $w = w^m\left(\overline{Q}\right)$, then k has an incentive to reduce the bid since $\overline{q}_k > R_k(\overline{q}_{-k})$. If market makers bid $w = w^m(Q) < w^m\left(\overline{Q}\right)$, then some market maker j who has not realized capacity has an incentive to increase the bid.

profit implies $q_k = R_k(\overline{q}_{-k})$.

Now let $\overline{w}^m_j$ be the upper bound of market maker j's bid prices. We will show $\overline{w}^m_j = \overline{w}^m$ for all j and that each market maker buys up to its capacity at the upper bound. If $\overline{w}^m_i > \overline{w}^m_j$ for $j \neq i$, as we have previously argued, $\overline{w}^m_i$ must be i's monopoly price. Then j would earn zero profit but could earn a positive profit by bidding $\overline{w}^m_i + \varepsilon$. If j buys $q_j < \overline{q}_j$ at $\overline{w}^m$, then by bidding $\overline{w}^m + \varepsilon$, j could raise demand to utilize capacity $\overline{q}_j$. Summarizing we have

Lemma 2 *In the mixed strategy equilibrium, the upper bound of bid prices for all market makers is the same and each market maker buys up to its optimal capacity at the upper bound bid price. The market maker who bids the lowest price earns the profit,*

$$\pi_k = \left[p^m(R_k(\overline{q}_{-k}) + \overline{q}_{-k}) - w^m\left(R_k(\overline{q}_{-k}) + \overline{q}_{-k}\right) - t_k\right] R_k(\overline{q}_{-k}).$$

If a small firm is outbid by a larger firm, the small firm has a larger chance of not buying anything. The small firm may in turn adopt a more aggressive bidding strategy. However, smaller firms are also less efficient firms in our model and are therefore more likely to bid lower prices due to higher transaction costs. Therefore, it is possible that a small firm may end up bidding the lowest price in the equilibrium. As a result the larger firm may earn a higher profit in the mixed strategy equilibrium than in the pure strategy equilibrium. This result differs from the classical result as described in Kreps and Scheinkman (1983) in which the larger firm earns the same profit both in the mixed strategy equilibrium and in the pure strategy equilibrium.

4.4. Stage-One Capacity Setting

Let the capacity installation cost be f, which is identical for all market makers. We show that the Cournot quantity is the unique equilibrium outcome if f is sufficiently large. Let all market makers other than j bid and ask the Cournot prices, $w = w^m(Q^*)$ and $p = p^m(Q^*)$. We want to show that j has no incentive to deviate from these prices. If she does, thereby buying quantity $q_j = q^*_j + \Delta q_j > q^*_j$, then we are in the region of a mixed strategy equilibrium. Let k be the market maker bidding the lowest price $\underline{w}^m$. If $j = k$, using Lemma 2 π_j is the same as the profit obtained at the Cournot capacity. Invoking Kreps and Scheinkman's argument, the deviation reduces net profit since the capacity is costly. If $j \neq k$, according to the indifference property of a mixed strategy equilibrium, k's profits at $\underline{w}^m$ and $\overline{w}^m$ must be equal. Using Lemma 2 we have:

$$\left[p^m(R_k(\overline{q}_{-k}) + \overline{q}_{-k}) - w^m\left(R_k(\overline{q}_{-k}) + \overline{q}_{-k}\right) - t_k\right] R_k(\overline{q}_{-k}) = (\overline{z}^m - t_k)\, q_k^* \ , \tag{26}$$

where $\overline{z}^m = p^m(Y(\overline{w}^m)) - \overline{w}^m$ is the bid-ask spread at $\overline{w}^m$. For market makers $k \neq j$, using (22) we obtain

$$R_k(\overline{q}_{-k}) = q_k^* - \Delta q_j/2 \text{ and } R_k(\overline{q}_{-k}) + \overline{q}_{-k} = Q^* + \Delta q_j/2 \ . \tag{27}$$

Using (26) and (27), we have $\overline{z}^m \leq p^m(Q^* + \Delta q_j/2) - w^m\,(Q^* + \Delta q_j/2)$. Now applying Lemma 2 we have the net profit of j, $\pi_j - f\left(q_j^* + \Delta q_j\right)$

$$\begin{aligned}
&= (\overline{z}^m - t_j - f)\left(q_j^* + \Delta q_j\right) \\
&\leq \left(q_j^* + \Delta q_j\right)\left[p^m(Q^* + \Delta q_j/2) - w^m\,(Q^* + \Delta q_j/2) - t_j - f\right] \\
&= \left[p^m(q_j^* + \Delta q_j/2 + q_{-j}^*) - w^m\left(q_j^* + \Delta q_j/2 + q_{-j}^*\right) - t_j\right]\left(q_j^* + \Delta q_j/2\right) \\
&\quad + \left[p^m(Q^* + \Delta q_j/2) - w^m\,(Q^* + \Delta q_j/2) - t_j\right]\Delta q_j/2 - f\left(q_j^* + \Delta q_j\right) \\
&\leq \pi_j^{Cournot} - f q_j^* + \{\left[p^m(Q^* + \Delta q_j/2) - w^m\,(Q^* + \Delta q_j/2) - t_j\right]/2 - f\}\Delta q_j \ ,
\end{aligned}$$

where $\pi_j^{Cournot}$ represents j's profit at the Cournot equilibrium. The quantity $p^m(Q^* + \Delta q_j/2) - w^m\,(Q^* + \Delta q_j/2) - t_j$ is less than the maximum of market makers' bid-ask price spread. Thus, if the installation cost f is more than half of the maximum of market makers' bid-ask price spread, deviation does not pay for j, and therefore Cournot capacity and the corresponding market clearing prices $w^m\,(Q^*)$ and $p^m(Q^*)$ define a unique SPNE of the game.

Proposition 1 *If the capacity cost is more than half of the maximum of market makers' bid-ask price spread, the Cournot outcome is the unique subgame perfect Nash equilibrium in the full game.*

If f is small, then market makers may choose to follow a mixed strategy price setting behavior.

The bid prices offered by market makers will distribute from $\underline{w}^m$ to $\overline{w}^m$, and ask prices correspondingly will distribute from $\overline{p}^m$ to $\underline{p}^m$.

4.5. Dispersion Of Ask And Bid Prices

We now assume that f is large in which case market makers behave as in the pure strategy price setting case (Case 1). All market makers charge the same price. The consumer who has a reservation value lower than p^m and the producer who has a reservation value higher than w^m transact through middlemen rather than through a market maker. If p^m is greater than $\overline{r_c}$, the highest reservation value of the population of consumers, and correspondingly if w^m is less than $\underline{r_p}$, the lowest reservation value of the population of producers, then no consumer (producer) will be served by market makers and the market is reduced to the market with middlemen but no market maker.

If $p^m < \overline{r_c}$ and correspondingly, $w^m > \underline{r_p}$, both middlemen and market makers coexist in the market. The distribution of ask prices, $F(p)$, and the distribution of bid prices, $G(w)$, are uniform distributions with supports given by (14). All of our analysis of the behavior of middlemen developed in Section 2 applies here as well, except that the highest reservation value of consumers and the lowest reservation value of producers who transact through middlemen are replaced by $\hat{\overline{r_c}} = p^m$ and $\hat{\underline{r_p}} = w^m$, respectively. The highest transaction cost a middleman can bear and also survive equals $\hat{k^*} = \hat{\overline{r_c}} - \hat{\underline{r_p}} = p^m - w^m$, and the mass of active middlemen equals $\hat{N} = \hat{k^*} - \underline{k} = p^m - w^m - \underline{k}$. Substituting (15) into (14), we have:

$$
\begin{aligned}
\hat{\underline{p}} &= \frac{p^m}{2} + \frac{\overline{v} + \underline{c} + \underline{k}}{4}, \ \hat{\overline{p}} = p^m \\
\hat{\underline{w}} &= w^m, \ \hat{\overline{w}} = \frac{w^m}{2} + \frac{\overline{v} + \underline{c} - \underline{k}}{4} \ .
\end{aligned} \tag{28}
$$

The difference between the middlemen's lowest and highest ask prices equals

$$
\hat{\overline{p}} - \hat{\underline{p}} = \frac{p^m}{2} - \frac{\overline{v} + \underline{c} + \underline{k}}{4} \ ,
$$

which decreases as p^m decreases. The difference between the middlemen's lowest and highest bid prices is given by

$$\overline{w}^{\hat{}} - \underline{w}^{\hat{}} = \frac{\overline{v} + \underline{c} - \underline{k}}{4} - \frac{w^m}{2} ,$$

which decreases as w^m increases. In this case when the total volume served by market makers Q increases, p^m decreases and w^m increases. We summarize these results as follows.

PROPOSITION 2 *Suppose that the capacity cost is more than half of the maximum of market makers' bid-ask price spread. There is no online price dispersion. All price dispersion arises from the search market. As the total volume Q served by the online market makers increases, both ask price dispersion and bid price dispersion decline.*

The market makers' online ask price imposes an upper bound on the middlemen's ask price. If p^m decreases, both the upper bound of the ask price of middlemen, $\overline{p}^{\hat{}}$, and the lower bound, $\underline{p}^{\hat{}}$, decline. However, the lower bound decreases less than the upper bound. Thus offline ask price dispersion is reduced as p^m decreases. The same result applies to offline bid prices.

If the capacity cost f is small and market makers choose the mixed strategy price setting, there will be online price dispersion as well. A lower value for p^m reduces offline price dispersion, but online price dispersion may increase. Price dispersion that arises from both online and offline dispersion is a complicated but interesting issue, which we leave for future research. We focus on offline price dispersion in all subsequent analysis.

4.6. AN EXAMPLE

We modify an example presented in Rust and Hall (2003) to illustrate our results up to this point. Assume the capacity cost is sufficiently large so that market makers behave as in the pure strategy price setting case. Figure 2 depicts the example. Let $\underline{c} = \underline{v} = 0$ and $\overline{c} = \overline{v} = 1$. Along the horizontal axis, buyers' valuations are plotted from high to low, whereas sellers' costs are plotted from low to high. Prices are represented on the vertical axis. The market with middlemen but no market makers is depicted as the benchmark case. The consumer with the highest value $\overline{v} = 1$ (located at O) has the reservation price $\overline{r_c}$. The marginal consumer with the value v^* (located at S) has the reservation price $r_c(v^*)$. In this benchmark case the reservation price curve is represented by AB. Ask prices are spread between $\underline{p} = v^*$ and $\overline{p} = \overline{r_c}$.

Suppose transaction costs of market makers are sufficiently small so that $p^m < \overline{p}$. Middlemen and oligopolistic market makers coexist in the market. The consumer who is indifferent between trading with market makers and trading with middlemen has the value $v_c(\underline{p}^{\wedge}, p^m)$. All consumers with values above $v_c(\underline{p}^{\wedge}, p^m)$ trade with market makers. The highest reservation price for consumers trading with middlemen is now reduced to p^m, and the lowest reservation price for consumers trading with middlemen is reduced to $\underline{p}^{\wedge}$. Ask prices are now spread between $\underline{p}^{\wedge}$ and p^m. Using expressions (11) and (28), the distance between $\underline{p}^{\wedge}$ and $\underline{p}$ is equal to $\left(\overline{r_c} - p^m\right)/2$, half of the distance between p^m and $\overline{p}$. Therefore, the ask price dispersion in the market with middlemen and oligopolistic market makers, $p^m - \underline{p}^{\wedge}$, is less than that in the benchmark case where only middlemen exist, $\overline{p} - \underline{p}$, by the amount $(1/2)\left(\overline{p} - p^m\right)$. Comparing the lower bounds of the integrals in expressions (1) and (12), because $\underline{p}^{\wedge} < \underline{p}$, we see that the reservation prices of consumers transacting through middlemen in this case are lower than in the benchmark case. Thus, the reservation price curve $A^m B^m$ lies under the curve AB. When p^m shifts down to $p^{m\prime}$, the reservation price curve of consumers transacting through middlemen shifts further down to $A^{m\prime} B^{m\prime}$. The ask price dispersion is further reduced by the amount $(1/2)\left(p^m - p^{m\prime}\right)$. A symmetric analysis applies to bid prices which is illustrated in the lower half of Figure 2.

5. The Effect Of A Change In Market Structure On Price Dispersion

We now consider how a change in market structure represented by a change in the number of market makers serving the market influences price dispersion. We study the case in which a single market maker j exits from the market. Immediately after the exit of the market maker consumers and producers who would have otherwise traded with that party are forced to search for a new counterparty for their trades. We define a transition period after the exit of market maker j during which the remaining market makers and active middlemen do not adjust their capacities. One way to motivate a transition period is to assume that short-run adjustment costs are a convex function of capacity. Therefore, market makers who handle a large volume of transactions choose not to adjust their capacity, whereas middlemen who handle a small volume of transactions choose to respond. Potential middlemen with costs in the interval $[k^{*\hat{}}, \overline{k}]$ will enter into the market and the consumers and producers who would have transacted with market maker j must implement a search across these newly entered middlemen to identify trading partners. Consumers and producers who had already planned to transact with the remaining $M-1$ market makers and $N^{\hat{}}$ middlemen do not deviate from their original optimal choices (prices and quantities). Following the transition period the displaced consumers and producers settle into trading relations with the remaining $M-1$ market makers if their reservation values are respectively higher than the ask price and lower than the bid price offered by those market makers, otherwise they are served by middlemen.

5.1. Price Dispersion During The Transition From M To $M-1$ Market Makers

We assume that consumers who would have been served by the exiting market maker j have values that are evenly spread from $v_c(\underline{p}^{\hat{}}, p^m)$ to $\overline{v}$. Therefore, during the transition period, $\frac{q_j^*}{Q^*}$ share of consumers in the interval of $[v_c(\underline{p}^{\hat{}}, p^m), \overline{v}]$ will be served by newly entered middlemen.

The capacity set by any market maker equals $q_i^* = R_i(q_{-i}^*)$ for all i before j's exit. $R_i(q_{-i}^*)$ increases after j's exit for $i \neq j$. Thus, $q_i^* < R_i(q_{-i}^*)$ during the transition period. The remaining $M-1$ market makers choose the pure strategy equilibrium price setting in which $p^{m\prime} = p^m(\sum_{i\neq j} q_i^*) > p^m$. Consumers in the interval $[v_c(\underline{p}^{\hat{}}, p^m), v_c(\underline{p}^{\hat{}}, p^{m\prime})]$ will also be served by newly entered middlemen.

Using the same argument as in Section 2, the highest price asked by newly entered middlemen is $\overline{p}^T = \overline{r_c}^T > \overline{r_c}$, and the lowest price asked by newly entered middlemen is $\underline{p}^T = \overline{r_c}^T/2 + \left(\overline{v} + \underline{c} + k^{*\hat{}}\right)/4$.[10] The reservation price curve for consumers served by newly entered middlemen is represented by curve $A^T B^T$ in Figure 1. Note that since $\underline{p}^T > \underline{p}$, the curve $A^T B^T$ is above the curve AB. Thus, the difference between the lowest and the highest prices asked by all active middlemen

[10] Using equations (1), (3) and (10) it can be shown that $\overline{r_c}^T = \frac{\left[(8\delta+1)\overline{v}+\underline{c}+k^{*\hat{}}\right]}{8\delta+2}$ which is greater than $\overline{r_c}$ since $k^{*\hat{}} > \underline{k}$ (where $\delta = 1/[\rho(1-\lambda)] - 1$ is the composite exit-adjusted discount rate per period).

and market makers becomes

$$\overline{p}^T - \underline{p}^{\hat{}} > p^m - \underline{p}^{\hat{}} \ .$$

Similarly, $\frac{q_j^*}{Q^*}$ share of producers in the interval $[\underline{c}, v_p(\overline{w}^{\hat{}}, w^m)]$ and producers in the interval $[v_p(\overline{w}^{\hat{}}, w^{m\prime}), v_p(\overline{w}^{\hat{}}, w^m)]$ will be served by newly entered middlemen. The lowest price bid by any middleman is $\underline{w}^T < \underline{w}$. Thus, the difference between the lowest and the highest bid prices in the market becomes

$$\overline{w}^{\hat{}} - \underline{w}^T > \overline{w}^{\hat{}} - w^m \ .$$

Summarizing we have:

Proposition 3 *The differences between the lowest and the highest bid prices and the lowest and highest ask prices increase during the transition from M to $M-1$ market makers. In other words, both bid and ask prices become more dispersed during the transition period.*

At this stage it will be beneficial to articulate the connection between the dispersion of bid and ask prices and the dispersion of the prices at which transactions occur. As Figure 2 indicates, the ask price is always greater than the bid price. Let ask prices be spread between p_1 and $p_2 (p_2 > p_1)$ and bid prices be spread between w_1 and $w_2 (w_2 > w_1)$. Therefore, because transactions occur at either an ask or a bid price, transaction prices will be spread between w_1 and p_2, which are the lower bound of bid prices and the upper bound of ask prices. The following proposition follows.

Proposition 4 *An increase in the spread of ask prices and an increase in the spread of bid prices results in an increase in the spread of transaction prices. During the transition period from M to $M-1$ market makers, the dispersion of transaction prices increases from (w^m, p^m) to $(\underline{w}^T, \overline{p}^T)$.*

5.2. Price Dispersion Following The Transition Period

The new structure of the market following the transition period is characterized by $m-1$ market makers. Let Q^h be the Cournot quantity chosen by h market makers in total. It is easy to show that $Q^{m-1} < Q^m$, which implies that $p^{m-1} > p^m$ and $w^{m-1} < w^m$. With $m-1$ market makers serving the market, the amount of consumer and producer trade volume served by all market makers decreases.[11] Further the differences between the lowest and the highest bid prices and the lowest and highest ask prices increase. Note that the change in price dispersion is determined by the change in Q^m, the total trade volume served by market makers. If there is entry by new market makers and/or if there is a change in the transaction cost t_i, possibly due to market reorganization and consolidation, then it is possible that the total quantity served by all market makers after the transition period will return to the pre-exit level Q^m.[12] If a change in market structure does not significantly affect the total trade volume served by all market makers, the prediction is neither will it significantly affect price dispersion. Therefore, if the total trade volume serviced by market makers following the transition period recovers to the pre-exit level, then price dispersion will also return to its pre-exit level. The empirical behavior of price dispersion following Enron's exit from the North American natural gas market documented in section 2 is consistent with this explanation.

6. Conclusions

Neoclassical economics leaves open the question of how actual markets attain equilibrium prices. The theory of intermediation and the microstructure of markets developed by Spulber (1996, 1999) and Rust and Hall (2003), amongst others, has made significant progress in addressing this question in markets served by middlemen and a monopoly market maker. We observe however that many markets are served by multiple market makers and these market makers have significant market power even in homogeneous product markets. An extension of the models of Spulber and Rust and Hall is required to accommodate this market feature. We fill this gap in the literature by developing a market microstructure model in which oligopolistic competition among market makers coexists with an active search market populated by middlemen. Market makers' intermediation behavior is decomposed into a two-stage process: capacity setting in the first stage and bid and ask price setting in the second stage. It is shown that the two-stage competition among market makers is

[11] Per our assumptions, a consumer of type v consumes one unit of the good if the price she pays is at most v, and a producer of type c produces one unit of the good at the cost of c. Therefore the trade volume is equal to the number of consumers and producers served.

[12] We do not present an analysis of endogeneous entry and exit decisions here to conserve space. A monopoly market maker's entry and preemptive decisions are studied in Rust and Hall (2003). Our result that price dispersion is determined by Q^m still holds when market makers' entry, exit, and preemptive decisions are considered.

equivalent to Cournot competition when the capacity cost is sufficiently large.

Our theory has much to say about homogeneous product markets where capacity choice plays an important role. Our model predicts the total volume handled by all market makers will decreased and that price dispersion will increase following a market maker's exit. Under certain conditions, if the total trade volume handled by market makers recovers to the pre-exit level and then price dispersion will also return to its pre-exit level. Our model provides one explanation for the behavior of the dispersion of natural gas spot prices around the time that a major intermediary, Enron, exited from the North American natural gas market. Price dispersion jumped 4-fold immediately following Enron's exit at the end of 2001 but returned to its pre-exit level within roughly 2 months following the exit date.

Our focus is on how online market structure influences offline price dispersion. The structure of our model matches closely the North American natural gas market. Specifically our model describes markets in which online price setting results in a common set of prices across market makers so price dispersion arises from the offline search market. An alternative view on the source of price dispersion has been proposed by Baye and Morgan (2001, 2004). In their model firms which list their prices on a monopoly market maker's screen choose mixed strategies. A distinguishing feature of our model is that market makers in our setting are capacity constrained. If capacity cost is small, market makers in our model may indeed choose a mixed price quoting strategy. Therefore, both online dispersion and offline dispersion may contribute to overall price dispersion. We leave an investigation of this extension to future research.

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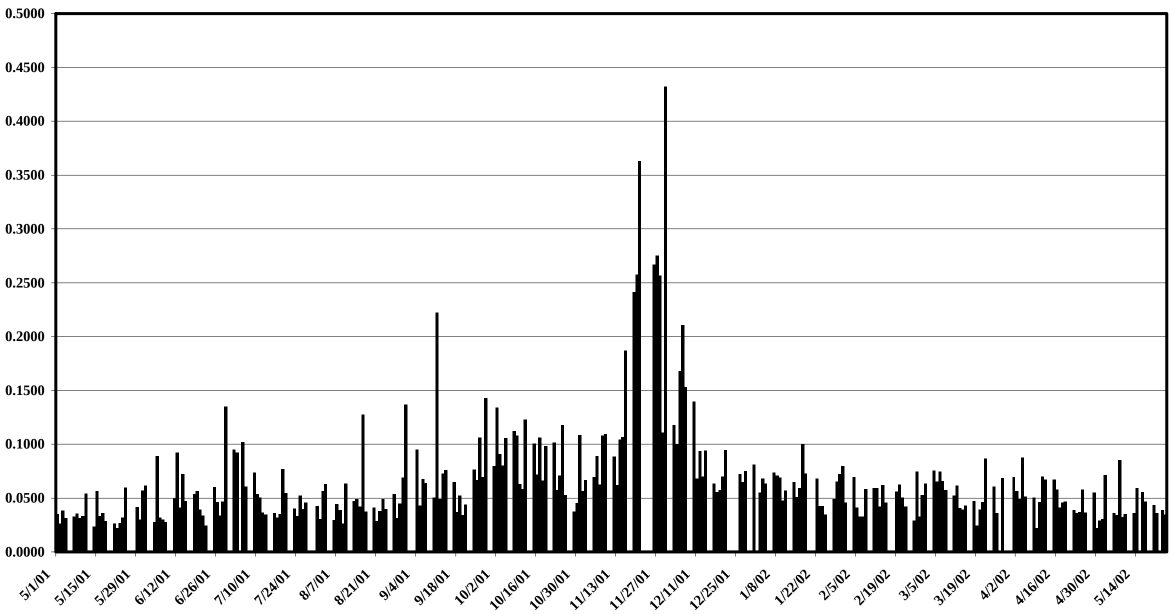

FIGURE 1: **Spot Natural Gas Price Dispersion.** Plot of the relative range of daily natural gas spot prices recorded at the Henry Hub for the period 05/01/2001 through 05/24/2002. The Relative Price Range is defined as $Rg_t =$ (Daily High Price - Daily Low Price)/Daily Midpoint Price. We approximate the Daily Midpoint Price with the average of the High and Low prices for the day. The spot natural gas price data were obtained from Platts (http://www.platts.com).

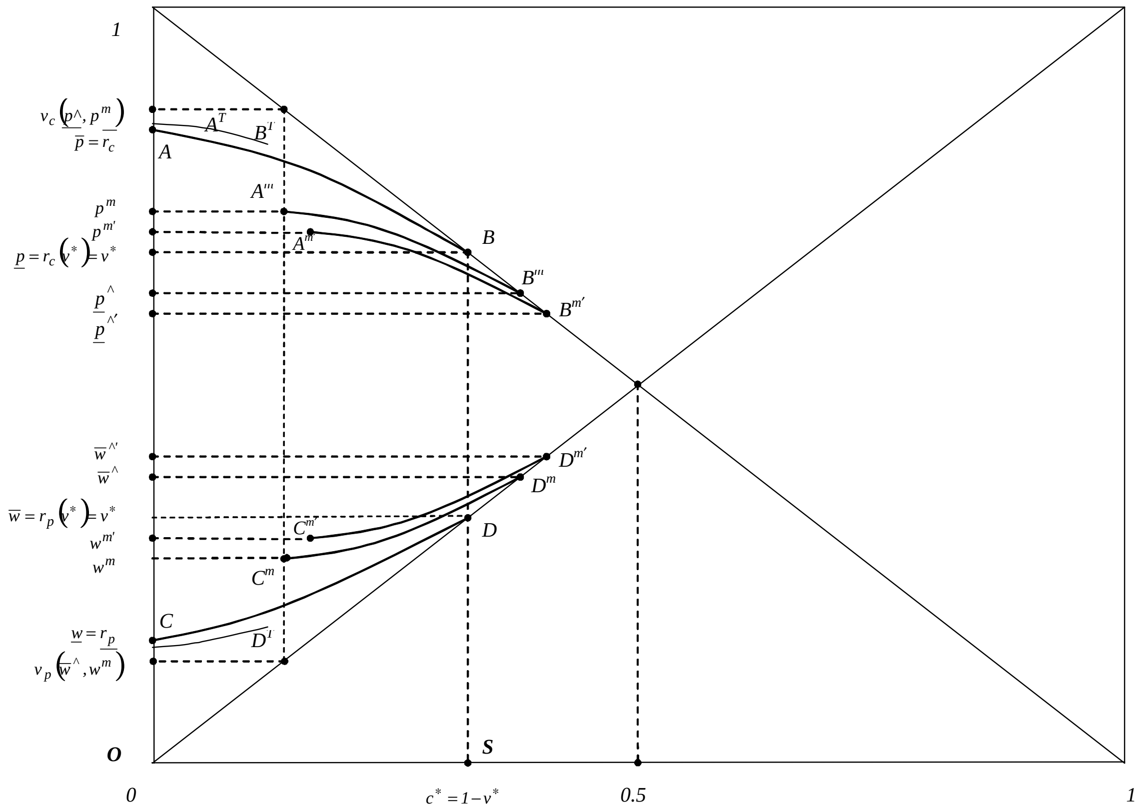

FIGURE 2: The Behavior of Bid and Ask Prices. The figure illustrates the behavior of the reservation price curves for the ask price p and the bid price w in the presence of middlemen only (the benchmark case) and in the presence of middlemen and oligopolistic market makers. Please see Table A1 for descriptions of the notation used in the figure.