Frictional Unemployment and Periodical Price Adjustment in Transition Periods

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Abstract

Using a general equilibrium model with many goods and many factors, this paper studies the adjustment process in a transition from one equilibrium to another. An adjustment mechanism that applies a search process is developed to link the product and factor markets. Then a non-steady state optimal control problem is solved to show that optimal price adjustment is periodical and takes more than one period. Therefore, optimal price adjustment is gradual.

Key words: General equilibrium; Gradualism; Price adjustment; Trade liberalization; Unemployment

JEL classification: F13; F16; P2

1 Introduction

Changes in economic environment require changes in steady state equilibrium prices. Price adjustments then take place in the transition from one equilibrium to another. There are many kinds of price adjustments in the real world. The worldwide tariff reduction in the wake of the globalization movement is one example. The transition from planned economies to market economies can be viewed as another, in that both prohibitive taxes on market sectors and subsidies of planned sectors are being removed.[1]

Both general equilibrium models and game theory models have been used to study the price adjustment. This paper studies the adjustment process using a general equilibrium model with many goods and many factors. Applying a search process, an adjustment mechanism is developed to link the product and factor markets. An optimal control problem is then solved without assuming a steady state condition. Optimal price adjustment is shown to be periodical and at least two adjustment periods are required. Therefore, optimal price adjustment is *gradual*.

There are two difficult problems in the study of price adjustment using a general equilibrium framework. The first problem is modeling the adjustment mechanism.

[1] Besides price liberalization and factor adjustment, the transition process from a planned economy to a market economy includes many other important phases, such as privatization, political stabilization, institutional and legal reform, none of which are studied in this paper.

In studies of trade liberalization, two adjustment techniques have been used. Neary (1982) imposes an ad hoc mechanism where the rate of capital movement is proportional to the difference between the values of the marginal product of capital in different sectors. Mussa (1982, 1984) treats capital movement as an issue in investment theory without considering unemployment. Labor is assumed to be freely mobile between sectors. To transfer a unit of capital from one sector to another, however, a capital owner must purchase the services of a capital-moving firm. Contrary to the widely-held belief that adjustment costs call for gradualism in policy reform, Mussa shows that an immediate jump to free trade is optimal unless there exist specific market distortions.

The second problem in the study of price adjustment is to solve for the optimal path in an optimal control problem. The usual way to solve an optimal control problem in economics is to assume a steady state condition. Davidson and Matusz (2000), for example, recently studied the rate at which labor is adjusted from the contracting sector into expanding sector in a steady state using empirically observable parameters. However, the nature of adjustment periods is that economies are not in steady states. Thus, the adjustment rates of prices and factors are not zero during the entire transition process. The non-steady state nature of the adjustment process usually makes the optimal control problem very hard to solve.

While general equilibrium models in the literature have not provided a "satisfactory theory that would yield a gradual process of trade liberalization as an endogenous outcome," (Staiger 1995, pp 250) game theory models have been used to study the widely observed feature of gradualism in optimal reform. In a dynamic context without commitment, Dewatripont and Roland (1992) show that optimal reform will be gradual when a government takes the information revealed in a partial reform and uses it for the design of later. In a game between a government without commitment power and a domestic industry over when to liberalize and when to invest, Matsuyama (1990) argues that equilibrium liberalization may be delayed. Staiger (1995) considers a trade liberalization game between two symmetric countries and argues that liberalization reduces the size of the protected import-competing sector, and hence reduces the rents received as a result of protection. The lower the rents, the less will be the anti-liberalization interest-group pressure on the government. Therefore, a one-step liberalization would be averted by the protected interest group, but gradual tariff cuts will be supported and eventually lead to free trade. Bond and Park (1998) analyze an efficient trade agreement between large and small countries and show that a trade-off between static inefficiencies (tariff distortion) and dynamic inefficiencies (lack of consumption smoothing) generates the gradualism. Assuming that each moving worker must pay a fixed adjustment cost, Furusawa and Lai (1999) derive a most-cooperative, self-enforcing trade liberalization path for two large countries and find that, in general, trade liberalization is gradual.

Game theory models have discovered strategic ingredients of gradualism in reciprocal price liberalization. However, these models have not satisfactorily explained why unilateral price adjustment is usually gradual. The perception of unilateral price adjustment as a gradual process is straightforward and pervasive, yet has not been formally proved. Because of the costs and time involved in moving factors to alternative uses, the adjustment to any sudden changes in economic conditions will spread out over time. The more rapidly adjustment takes place, the higher costs tend to be. Therefore, economic system should not adjust too rapidly to changes in economic conditions. Using unemployed factors and a search process to represent the costs and time spent in factor reallocation, this pape rigorously proves such an intuition. Furthermore, we are able to provide equations under which the optimal time and amount of price adjustment are determined.

We assume that *factor adjustments take time*. Price adjustment brings changes in factor demands among different sectors. The reductions (increases) in factors demanded result in unemployment (vacancy) immediately after the price adjustment. The unemployed factors then find new positions through a search process. Price adjustment alone increases social welfare, but the resulting unemployment reduces social welfare. Optimal price adjustment is determined by the *net dynamic gain* of the adjustment, which equals the dynamic gain minus the dynamic cost. It is shown that the *net dynamic gain* is the sum of the *net intertemporal gain* and its *dynamic* *residual*. The price adjustment takes place when the *net dynamic gain* equals zero. The amount of the price adjustment is what it takes to make the *net intertemporal gain* equal to zero. However, since the *dynamic residual* is shown to be negative, the *net dynamic gain* becomes negative immediately after the price adjustment takes place. Negative *net dynamic gain* prevents further price adjustment. As time goes on, unemployed factors find new positions and the *net dynamic gain* reaches zero; the next price adjustment then begins. Therefore, the *optimal price adjustment path is periodical*.

In Section 2, a static general equilibrium model is built. Section 3 develops an adjustment technology in the factor market. Stability conditions are considered in Section 4. We set up an optimal control problem where intertemporal social utility is maximized in Section 5. Periodical price adjustment is proved optimal in Section 6. Finally, Section 7 contains conclusion and discussion.

2 The Static Model

There are n goods and m factors in a open economy[2]. Let $P^h = (p_1^h, p_2^h, \cdots, p_n^h)$ denote the equilibrium price vectors in state h, where $h \in \{0, 1\}$. $W^h = (w_1^h, w_2^h, \cdots, w_m^h)$ is the equilibrium factor price vector. The economy is in state 0 at time $t = 0^-$,

[2] These could be final or intermediate goods. Intermediate goods enter the output vector as negative elements and pure intermediate goods enter the demand vector as zeros. We will treat all vectors as column vectors and denote transposes by " $'$ "s.

while it will be in state 1 when $t \geq 0$. Assuming there are L sectors[3], let $X^l(P,W)$ and $V^{dl}(P,W)$ denote the output of goods and demand for factors by sector l. Firms maximize profits and

$$\begin{aligned} \Pi^l(P,W) &= \max_{X^l, V^{dl}} \{P'X^l - W'V^{dl} \mid (X^l, V^{dl}) \text{ feasible}\} \\ &= P'X^l(P,W) - W'V^{dl}(P,W) \end{aligned} \tag{1}$$

defines the maximized value of profits. We assume that $\Pi^l(.)$ has all the standard properties. Partial derivatives are denoted by the subscripts. All of the usual envelope results apply so that $\Pi^l_p(P,W) = X^l(P,W)$ and $-\Pi^l_w(P,W) = V^{dl}(P,W)$. The vectors of total supply and derived demands are defined as $X = \sum_{l=1}^{L} X^l$ and $V^d = \sum_{l=1}^{L} V^{dl}$, respectively. Then,

$$\begin{aligned} X &= \sum_{l=1}^{L} X^l = \sum_{l=1}^{L} \Pi^l_p(P,W) = \Pi_p(P,W), \\ V^d &= \sum_{l=1}^{L} V^{dl} = -\sum_{l=1}^{L} \Pi^l_w(P,W) = -\Pi_w(P,W) \end{aligned}$$

where total profits $\Pi(P,W) = \sum_{l=1}^{L} \Pi^l(P,W)$. The market is assumed to be perfect competitive. The production functions are constant return to scale, and profits equal zero.

[3] Each sector may produce several goods.

Let $U(C, V^s)$ be the utility function of a representative consumer where C and V^s denote the consumption and factor supply. The consumer's problem is:

$$\max_{C,V^s} U(C, V^s)$$

subject to the budget constraint:[4]

$$\begin{aligned} P'C &= W'V^s + T_m'M + T_c'C + T_w'W \qquad (2) \\ &= W'V^s + E \end{aligned}$$

where net import vector $M = C - X(P, W)$. $T_\alpha(\alpha = m, c, w)$ are vectors of tax on import, consumption, and factor income. E represents the lump sum transfer to the consumer. Maximizing the utility gives demand and factor supply functions $C = C(P, W, T_\alpha)$ and $V^s = (P, W, T_\alpha)$. Let $S = S(P - T_m, \phi)$ denote the world supply in the product market, which depends on the world price $P - T_m$ and a vector of other exogenous variables ϕ. The equilibrium of the economy is given by:

[4] For analytical simiplicity, we assume that the consumer maximizes instantaneous utility subject to the current budget constraint in the static equilibrium. The intuition behind this is that the consumer faces liquidity constraints; then the Bellman principle of optimality will ensure that the assumption is dynamically optimal for the consumer.

$$M(P, W, T_\alpha) = S(P - T_m, \phi) \tag{3}$$

$$V^d(P, W) - K = V^s(P, W, T_\alpha) \tag{4}$$

Equation (3) requires that the goods market clear internationally. The corresponding factor prices are determined by Equation (4) where factor markets may not be clear and K is the vector of frictional unemployment. Assume that $K \geq \underline{k}$ where $\underline{k}$ is a vector of natural unemployment. The unemployment equals the natural one in the steady state equilibria of both states. Natural unemployment may depend on the state; we assume that they are all the same for simplicity. Hence, $K^0 = K^1 = \underline{k}$. At $t = 0^-$, $P^0 = P(T_\alpha^0, \phi^0)$ and $W^0 = W\ (P^0, T_\alpha^0, \phi^0)$. After that, taxes and other exogenous variables change to state 1 so that the market equilibrium would require $P^1 = P(T_\alpha^1, \phi^1)$ and correspondingly, $W^1 = W\ (P^1, T_\alpha^1, \phi^1)$. Assuming that $P^1 \leq P^0$, a social planner will adjust P^0 to P^1 to maximize the intertemporal social utility and adjustment will take time t^*. The optimal adjustment path is studied in following sections.

3 Adjustment Mechanism

In real world economies, productive resources cannot be moved instantaneously among alternative uses. It is widely believed that the process of adjustment subsequent to a sudden price change will involve not only a gradual movement of resources, but also substantial unemployment of some resources for a period of time.

Let $P(t)$ and $W(t)$ denote the price vectors at time t. The demand for j^{th} factor in producing good i is denoted as $v_{ij}(t) = v_{ij}(P(t), W(t))$. The total derived demand of factor j is $v_j(t) = \sum_{i=1}^{n} v_{ij}(t)$. v_j then becomes the j^{th} component of the factor demand vector $V^d(P, W)$. If the factor demand at state 1, $v_{ij}^1 = v_{ij}(P^1, W^1)$, is less than the factor demand at state 0, $v_{ij}^0 = v_{ij}(P^0, W^0)$, in the i^{th} sector when factor markets are clear, factor j will flow out the i^{th} sector during the transition period. A subset of all sectors, $O \in \overline{N} = [1, 2, \cdots, n]$, represents a set of sectors in which factor demands are reduced due to the adjustment, while $I = \overline{N} \backslash O$ is the set of sectors in which factor demands are increased.

The order of decisions is the following: at time t, a product price adjustment $\dot{P}(t)$ is made by the planner. Firms then adjust their derived demands corresponding to such product price adjustment and frictional unemployed factors occur. After that, factor prices $W(t)$ are determined by equation (4) so that factor supply equals factor demand minus frictional unemployment.[5] For simplicity, we assume that the factor adjustment generated from changes in *factor prices* can be realized immediately. Therefore, the instantaneous unemployment (vacancy), $\dot{v}_{ij}$, only comes from the factor adjustment

[5] This order of decisions allows us to choose product price vector P and unemployment vector K as state variables.

generated from changes in *product prices* at t. The is:

$$\dot{v}_{ij} = \frac{\partial v_{ij}(P, W)}{\partial P} \dot{P}(t) \tag{5}$$

For a small time period $\Delta t > 0$,

$$v_{ij}(t + \Delta t) = v_{ij}(t) + \dot{v}_{ij}(t)\Delta t \tag{6}$$

where $\dot{v}_{ij}(t) \leq 0$ if $i \in O$. $\frac{\partial v_{ij}(P,W)}{\partial P} \geq 0$ for $i \in O$ since $\dot{P}(t) \leq 0$ as $P^1 \leq P^0$.[6] $\dot{v}_{ij}(t) \geq 0$ if $i \in I$. The total instantaneous unemployment of factor j at t is

$$\dot{v}_j^O(t) = \sum_{i \in O} \dot{v_{ij}}(t). \tag{7}$$

The total instantaneous vacancies of factor j at t is

$$\dot{v}_j^I(t) = \sum_{i \in I} \dot{v_{ij}}(t). \tag{8}$$

Unemployment and vacancies are assumed to yield a flow of new employment with a search process. We assume that the probability distribution per unit of $\dot{v}_j^O(t)$ to find a new position at time s is $F_j(s-t)$ where $F_j(0) = 0$ and $F_j(+\infty) = 1$. Similarly, the probability distribution per unit of $\dot{v}_j^I(t)$ to hire a new employee at time s is $F_j^I(s-t)$. The matching process requires that $\dot{v}_j^O(t)F_j(s-t) = \dot{v}_j^I(t)F_j^I(s-t)$.

Assume a partition of time $0 = t_0 < t_1 < \cdots < t_g = t$. The amount of unemployed factor j at time t due to price adjustments within the time interval $[t_v, t_{v+1}]$ for

[6] Reversal adjustment is not considered in this paper.

$v = 0, 1, \cdots, g$ is approximately $-\dot{v}_j^O(t_v)\Delta t_v\,(1 - F_j(t - t_v))$ where $\Delta t_v = t_{v+1} - t_v$. Thus the total amount of unemployed factor j at time t, $k_j(t)$, is the limitation of the summation of $-\dot{v}_j^O(t_v)\Delta t_v\,(1 - F_j(t - t_v))$ as $\Delta t = \max\{t_{v+1} - t_v \mid v = 0, 1, \cdots, g\}$ converges to zero. Hence,

$$k_j(t) = \lim_{\Delta t \to 0} -\sum_{v=0}^{g} \dot{v}_j^O(t_v)\Delta t_v\,(1 - F_j(t - t_v)), \tag{9}$$

which simply says that

$$k_j(t) = -\int_0^t \dot{v}_j^O(s)\,(1 - F_j(t - s))\,ds.$$

The factor market equilibrium condition is $V^d(P, W) - K = V^s(P, W)$ where K is the vector of k_js.[7] Note that any price change will result in a factor outflow that contributes positively to K, which implies that the change of K subject to price change, $\frac{DK}{DP}$, is non-negative.

Let $\dot{V}^O(.)$ denote the vector of $\dot{v}_j^O(.)$s and $I - F$ denote the diagonal matrix with the j^{th} element equals $1 - F_j(t - s)$ along the diagonal. Then,

$$K(t) = -\int_0^t (I - F(t - s))\,\dot{V}^O(s)ds$$

F_j is assumed to be an exponential distribution function[8] for each j so that $F_j(\tau) = 1 - e^{-\theta_j \tau}$. Note that $1 - F_j(\tau) = e^{-\theta_j \tau}$ and $\frac{d(1-F_j(t-s))}{dt} = -\theta_j e^{-\theta_j (t-s)} = -\theta_j\,(1 - F_j(t - s))$.

[7]It is assumed that changes of $V^s(.)$ can be realized instantaneously. Fixed factor supply is a special case of this assumption.

[8]If we assume that the unemployment will last at most for time period T, then a distribution function is defined as:

Thus,

$$\begin{aligned}\dot{K} &= \frac{dK(t)}{dt} = -\dot{V}^{O}(t) + \Theta \int_0^t (I - F(t-s))\,\dot{V}^{O}(s)ds \\ &= -V_p^O(P,W)\dot{P}(\tau) - \Theta K(t) \qquad (10)\end{aligned}$$

where Θ is the diagonal matrix in which the j^{th} element equals θ_j. The equation (10) is the unemployment path resulting from the price adjustment, through which the product market and the factor market are linked. Existing literature usually uses an ad hoc adjustment mechanism [Neary (1982)]. The above derived linkage between the product market and the factor market gives us an analytic ground for the optimal control problem in later sections. We now turn to stability conditions for the economy.

4 Stability Conditions

Suppressing the exogenous variables, factor market condition (4) gives that $W = W(P,K)$. Let

$$\begin{aligned}\hat{U}(P,K) &= U[C(P,W(P,K)), V^s(P,W(P,K))] \\ \hat{V}(P,K) &= V^d(P,W(P,K)).\end{aligned}$$

$$F_j(\tau) = \begin{cases} \frac{1-e^{-\theta_j \tau}}{1-e^{-\theta_j T}} & \text{for } \tau \le T \\ 1 & \text{for } \tau > T \end{cases}$$

$U\hat{}(P,K)$ and $V\hat{}(P,K)$ are utility function and derived demands that depend on state variables P and K. Denote (P^*, K^*) as equilibrium price and unemployment. It must be the case that $\frac{DU\hat{}(P^*,K^*)}{DP} = 0$ and $\frac{DU\hat{}(P^*,K^*)}{DK} = 0$. We first consider the simplest tâtonnement type of adjustment which is given by the differential equation system:

$$\dot{P} = \frac{DU\hat{}(P,K)}{DP}, \ \dot{K} = \frac{DU\hat{}(P,K)}{DK} \tag{11}$$

The above equations may be expressed in a small neighborhood of (P^*, K^*) by linearization:

$$\dot{P} = \frac{D^2U\hat{}(P^*,K^*)}{DP^2}(P-P^*) + \frac{D^2U\hat{}(P^*,K^*)}{DPDK}(K-K^*) \tag{12}$$

$$\dot{K} = \frac{D^2U\hat{}(P^*,K^*)}{DKDP}(P-P^*) + \frac{D^2U\hat{}(P^*,K^*)}{DK^2}(K-K^*) \tag{13}$$

A usual global stability condition is that the coefficient matrix of (12) and (13),

$$A = \begin{bmatrix} \frac{D^2U\hat{}(P,K)}{DP^2} & \frac{D^2U\hat{}(P,K)}{DPDK} \\ \frac{D^2U\hat{}(P,K)}{DKDP} & \frac{D^2U\hat{}(P,K)}{DK^2} \end{bmatrix} \tag{14}$$

is everywhere quasi-negative definite.

Let r be the time discount rate. The intertemporal marginal social welfare of price adjustment at time t is $r^{-1}\frac{DU\hat{}(P(t),K(t))}{DP}$, which is negative so that it pays to reduce the price. The increase of unemployment due to the price adjustment at t

is $V_p^{\hat{}O}(P(t), K(t))$ so that the instantaneous marginal social cost of the adjustment is $\frac{DU^{\hat{}}(P(t),K(t))}{DK} V_p^{\hat{}O}(P(t), K(t))$, which is negative since $\frac{DU^{\hat{}}(.)}{DK} \leq 0$ and $V_p^{\hat{}O}(.) \geq 0$. Because $K(t)$ itself is declining in the rate of Θ, the intertemporal marginal social cost of the price adjustment becomes $(r+\Theta)^{-1}\frac{DU^{\hat{}}(P,K)}{DK} V_p^{\hat{}O}(P, K)$. Let

$$R^0(P,K) = r^{-1}\frac{DU^{\hat{}}(P,K)}{DP} - (r+\Theta)^{-1}\frac{DU^{\hat{}}(P,K)}{DK} V_p^{\hat{}O}(P,K),$$

which measures the *net intertemporal marginal gain* of the price adjustment at time t. Correspondingly, $R^0\dot{P}$ measures the *net intertemporal gain* of the price adjustment at time t. When $R^0 \leq 0$, the intertemporal marginal social gain exceeds the cost, which requires that the prices should be reduced. We now consider another type of adjustment which is given by the differential equation system:

$$\dot{P} = R^0(P,K), \;\; \dot{K} = \underline{k} - K. \tag{15}$$

Linearization gives:

$$\dot{P} = \frac{DR^0(P^*,K^*)}{DP}(P-P^*) + \frac{DR^0(P^*,K^*)}{DK}(K-K^*) \tag{16}$$

$$\dot{K} = -(K-K^*) \tag{17}$$

The coefficient matrix of (16) and (17) is:

$$B = \begin{bmatrix} \frac{DR^0(P,K)}{DP} & \frac{DR^0(P,K)}{DK} \\ 0 & -I \end{bmatrix} \quad (18)$$

Again, a usual global stability condition requires that B is everywhere quasi-negative definite. Therefore, we assume:

Both A and B are quasi-negative definite.

A leading case of this arises when the off-diagonal elements of the matrix are positive and the diagonal elements are negative. Hence, the coefficient matrix is a Metzler matrix, which is assumed in this paper. R^0 is an $n \times 1$ vector. Let $R_i^0(P, K)$ represents the i^{th} component of the vector. If changes of p_i and k_j are considered only, B being a Metzler matrix ensures that $R_i^0(p_i, k_j) = 0$ is upward sloping.

5 The Optimal Control Problem

The social planner will adjust the prices to maximize the intertemporal social utility. The intertemporal social utility associated with the price adjustment is:

$$J = \int_0^\infty U\hat{}\,(P, K)e^{-rt}dt \quad (19)$$

Denote $Z = \dot{P}(t)$ as control variables. Then the transition equations for P and K can be written as:

$$\dot{P}(t) = Z(t) \tag{20}$$

$$\dot{K}(t) = -V_p^{\hat{}O}(P,K)Z(t) - \Theta K(t) \tag{21}$$

The optimal control problem is set to maximize J subject to transition equations (20) and (21) and state variable constraint

$$K - \underline{k} \geq 0 \tag{22}$$

as well as the initial and terminal conditions[9]

$$P(0) = P^0,\ K(0) = \underline{k},\ \text{and } P(t^*) = P^1.$$

The current value Hamiltonian is:

$$H = U^{\hat{}}(P,K) + \lambda_I' Z + \lambda_J' \left[-V_p^{\hat{}O}(P,K)Z - \Theta K\right] \tag{23}$$

$$= U^{\hat{}}(.) + \left(\lambda_I' - \lambda_J' V_p^{\hat{}O}\right) Z - \lambda_J' \Theta K \tag{24}$$

and the Lagrangian associated with the constraint is:

$$L = H + q' \left[K - \underline{k}\right] \tag{25}$$

[9] $\underline{k}$ may be zero. $K(t^*)$ may not equal $\underline{k}$ since frictional unemployment could still exist after the last price adjustment.

where λ_I, λ_J, q are vectors of multipliers,

$$q \geq 0 \text{ and } q' \left[K - \underline{k} \right] = 0. \tag{26}$$

In an optimal path, Z must be chosen to maximize H subject to the constraint. Note that $Z \leq 0$ as $P^1 \leq P^0$ is assumed. The problem is reduced to:

$$\max_z \left(\lambda_I' - \lambda_J' V_p^{\hat{O}} \right) Z \text{ for } Z \leq 0 \tag{27}$$

where λ_I is the marginal social welfare due to the price adjustment. λ_J is the marginal social cost due to the change in unemployment, while $V_p^{\hat{O}}$ is the amount of unemployment due to the price adjustment. So $\lambda_J' V_p^O$ represents the marginal social cost due to the price adjustment. (27) states that the net social benefits should be maximized due to the price adjustment at any time in the optimal path. (27) requires:

$$Z^*(t) = 0 \text{ if } \lambda_I' - \lambda_J' V_p^{\hat{O}} > 0, \tag{28}$$

If $\lambda_I' - \lambda_J' V_p^{\hat{O}} = 0$, $Z^*(t)$ is not determined. When $\lambda_I' - \lambda_J' V_p^{\hat{O}} < 0$ at time t, a reduction (a jump in negative direction) in P may occur. λ_I and λ_J satisfy the differential equations:

$$\dot{\lambda}_I = r\lambda_I - \left(\frac{\partial L}{\partial P} \right)' \tag{29}$$

$$\dot{\lambda}_J = r\lambda_J - \left(\frac{\partial L}{\partial K} \right)' \tag{30}$$

where

$$
\begin{aligned}
\frac{\partial L}{\partial P} &= \frac{DU\hat{\ }(P,K)}{DP} - \lambda'_J \frac{DV_p^{\hat{\ }O}(P,K)}{DP} Z &(31)\\
\frac{\partial L}{\partial K} &= -\lambda'_J \Theta + \frac{DU\hat{\ }(P,K)}{DK} - \lambda'_J \frac{DV_p^{\hat{\ }O}(P,K)}{DK} Z + q'. &(32)
\end{aligned}
$$

We denote $R = \lambda'_I - \lambda'_J V_p^{\hat{\ }O}$ and call it *net dynamic marginal gain,* which will be the key equation to determine the optimal price adjustment path. Correspondingly, $RZ = R\dot{P}$ is the *net dynamic gain* of the price adjustment.

6 Periodical Price Adjustment

The above optimal control problem is a "bang-bang" control problem. We will propose a solution candidate which satisfies all necessary conditions and then prove the candidate also satisfies the sufficient conditions. $\lambda_I(0)$ is the marginal social welfare due to price reduction at time zero, while $\lambda_J(0)$ is the marginal social cost at time zero. We therefore propose:

$$
\begin{aligned}
\lambda_I(0) &= r^{-1} \frac{DU\hat{\ }(P_0, K_0)}{DP} &(33)\\
\lambda_J(0) &= (r+\Theta)^{-1} \frac{DU\hat{\ }(P_0, K_0)}{DK} &(34)
\end{aligned}
$$

where $P_0 = P^0$ and $K_0 = \underline{k}$. Starting from $t = 0^-$, it must be the case that $\frac{DU\hat{\ }(P_0,K_0)}{DP}$ $= 0$ at the state 0 since P^0 are equilibrium prices. However, $\frac{DU\hat{\ }(P_0,K_0)}{DP} < 0$ at $t_0 = 0$ when economy is in state 1, so that it pays to reduce the prices. We assume that welfare is maximized at the nature unemployment so that $\frac{D\hat{U}(P_0,K_0)}{DK} = 0$ at $t = 0$. Therefore, $\lambda_I(0) - \lambda_J(0)\hat{V_p^O}(0) < 0$, which implies that the prices should be reduced at $t = 0^+$.

Let the prices be reduced to P_1. Then, unemployment at $t = 0^+$ becomes

$$K_1 = -\left[V^O(P_1, W_0) - V^O(P_0, W_0)\right]$$

where $W_0 = W^0$. Prices are reduced until the *net intertemporal marginal gain* $R^0 = 0$. Thus, P_1 is solved by:

$$r^{-1}\frac{D\hat{U}(P_1, K_1)}{DP} - (r + \Theta)^{-1}\frac{D\hat{U}(P_1, K_1)}{DK}\hat{V_p^O}(P_1, K_1) = 0$$

After price adjustment at $t = 0^+$, we let the price adjustment $Z = 0$ for $0^+ < t < t_1$ where t_1 is the time that the next price reduction takes place and which will be determined later. During this period $\frac{\partial L}{\partial P} = \frac{D\hat{U}(t)}{DP}$ and[10] $\frac{\partial L}{\partial K} = -\lambda_J'\Theta + \frac{D\hat{U}(t)}{DK} + q'$ by equations (31) and (32). Differential equations (29) and (30) then become:

$$\dot{\lambda_I} = r\lambda_I - \left[\frac{D\hat{U}(t)}{DP}\right]' \quad (35)$$

$$\dot{\lambda_J} = (r + \Theta)\lambda_J - \left[\frac{D\hat{U}(t)}{DK} + q\right]' \quad (36)$$

[10] $\frac{D\hat{U}(t)}{DP}$ represents $\frac{D\hat{U}(P(t),K(t))}{DP}$ for simplification. Similar simplifications apply to other notations.

Note that $q = 0$ when $K > \underline{k}$ due to the condition (26). Solving for (35) and (36)[11], we have:

$$\lambda_I'(t) = r^{-1}\frac{DU\hat{}(0^+)}{DP}e^{rt} - \int_0^t \frac{DU\hat{}(\tau)}{DP}e^{r(t-\tau)}d\tau \tag{37}$$

$$\lambda_J'(t) = (r+\Theta)^{-1}\frac{DU\hat{}(0^+)}{DK}e^{(r+\Theta)t} - \int_0^t \frac{DU\hat{}(\tau)}{DK}e^{(r+\Theta)(t-\tau)}d\tau \tag{38}$$

Furthermore,

$$\begin{aligned} -\int_0^t \frac{DU\hat{}(\tau)}{DP}e^{r(t-\tau)}d\tau &= \int_0^t r^{-1}\frac{DU\hat{}(\tau)}{DP}d\left[e^{r(t-\tau)}\right] = \\ r^{-1}\frac{DU\hat{}(\tau)}{DP}e^{r(t-\tau)} &\mid {}_0^t - \int_0^t \frac{D^2U\hat{}(\tau)}{DPDK}\dot{K}r^{-1}e^{r(t-\tau)}d\tau \\ &= r^{-1}\frac{DU\hat{}(t)}{DP} - r^{-1}\frac{DU\hat{}(0^+)}{DP}e^{rt} \\ &\quad -\int_0^t \frac{D^2U\hat{}(\tau)}{DPDK}\dot{K}r^{-1}e^{r(t-\tau)}d\tau \end{aligned} \tag{39}$$

since P is constant within the period and only K depends on time τ. Substituting (39) into (37), we have:

$$\lambda_I'(t) = r^{-1}\frac{DU\hat{}(t)}{DP} - \int_0^t \frac{D^2U\hat{}(\tau)}{DPDK}\dot{K}r^{-1}e^{r(t-\tau)}d\tau \tag{40}$$

Similarly,

[11] $\frac{DU\hat{}(.)}{DP}$ is independent from λ_I and $\left[\frac{DU\hat{}(.)}{DK} + q\right]$ is independent from λ_J. Thus, the differential equations are linear. The solution to linear differential equation $\frac{dy}{dx} + p(x)y = q(x)$ is $y = e^{-\int p(x)dx}[\int q(x)e^{\int p(x)dx}dx + y_0]$.

$$\lambda'_J(t) = (r+\Theta)^{-1}\frac{DU\hat{\ }(t)}{DK} - \int_0^t \frac{D^2U\hat{\ }(\tau)}{DK^2}\dot{K}(r+\Theta)^{-1}e^{(r+\Theta)(t-\tau)}d\tau \tag{41}$$

Now,

$$R(t) = \lambda'_I(t) - \lambda'_J(t)V_p^{\hat{\ }O}(t) = R^0(t) + R^d(t) \tag{42}$$

where $R^0(t) = r^{-1}\frac{DU\hat{\ }(t)}{DP} - (r+\Theta)^{-1}\frac{DU\hat{\ }(t)}{DK}V_p^{\hat{\ }O}(t)$ and

$$R^d(t) = -\int_0^t \frac{D^2U\hat{\ }(\tau)}{DPDK}\dot{K}r^{-1}e^{r(t-\tau)}d\tau \tag{43}$$

$$+\left[\int_0^t \frac{D^2U\hat{\ }(\tau)}{DK^2}\dot{K}(r+\Theta)^{-1}e^{(r+\Theta)(t-\tau)}d\tau\right]V_p^{\hat{\ }O}(t). \tag{44}$$

Therefore, $R(t)$, the *net dynamic marginal gain* of the price adjustment is decomposed into $R^0(t)$, the *net intertemporal marginal gain* evaluated at time t, and $R^d(t)$, its *marginal dynamic residual.* $\dot{K} = -\Theta K \leq 0$ as $Z = 0$ and $V_p^{\hat{\ }O}(t) \geq 0$ by the definition. Using the stability conditions assumed in Section 4, we know that the *dynamic residual* $R^d(t) \geq 0$.

When $K \geq \underline{k}$ is binding for some factor k_j, then the corresponding j^{th} component of vector q, q_j, becomes positive. Thus,

$$\lambda'_J(t) = (r+\Theta)^{-1}\frac{DU\hat{\ }(t)}{DK} - \int_0^t \left[\frac{D^2U\hat{\ }(\tau)}{DK^2}\dot{K}(r+\Theta)^{-1}e^{(r+\Theta)(t-\tau)} + q'\right]d\tau \tag{45}$$

$R = (R_1, R_2, \cdots, R_n)$ where $R_i (i = 1, \cdots n)$ is the *net dynamic marginal gain* for the adjustment of p_i.

Figure 1 depicts the adjustment path of p_i when only one factor j is considered. Zero *net intertemporal marginal gain* line, $R_i^0(p_i, k_j) = 0$, is depicted by upward sloping line AA. $R_i^0 > 0$ below AA and $R_i^0 < 0$ above AA since increasing k_j increases the marginal cost. Zero *net dynamic marginal gain* line, $R_i(p_i, k_j) = 0$ when $q = 0$, is depicted by upward sloping line CFE_1, which is above AA and the difference between them equals $R_i^d(p_i, k_j)$. $R_i > 0$ below CE_1 and $R_i < 0$ above CE_1. $B_hB_h(h = 1, \cdots, N)$ depicts downward sloping line

$$k_j = -\left[v_j^O(P, W_{h-1}) - v_j^O(P_{h-1}, W_{h-1})\right],$$

which is the reduction in derived demand due to the price adjustment. $E_0 = E^0$ is the equilibrium at state 0, while $E_{N+1} = E^1$ is the equilibrium at state 1. $E_1 = (p_{i1}, k_{j1})$ is the first price reduction arising at time 0^+ and E_1 is determined by the intersection between B_1B_1 and AA. After first reduction, $z_i^*(t) = 0$ for $0 < t < t_1$ since $R_i(t) > 0$. $k_j(t)$ is declining at the rate θ_j until D_1 is reached where $R_i(p_{i1}, k_j(t_1^-)) = 0$. The second price reduction occurs at t_1^+, which is denoted by $E_2 = (p_{i2}, k_{j2})$, the intersection between B_2B_2 and AA. After the second reduction, $z_i^*(t)$ again equals zero since $R_i(t) > 0$ for $t_1 < t < t_2$. D_2 will then be reached and the third reduction occurs at $t = t_2^+$.

After the s^{th} reduction[12] at E_s, $k_j(t)$ will reach the natural unemployment $\underline{k}_j$ at t_s^- while $R_i(p_{is}, k_j(t_s^-)) \mid_{q=0} > 0$. Hence, D_s is in the right of the line CFE_1. Now the j^{th}

[12] $t_h(h = 1, \cdots, s)$ is determined by $R_i(p_{ih}, k_j(t_h^-)) = 0$.

component of constraint (22) is binding so that q_j becomes positive. q_j is determined by $R_i = 0$ where λ_J is given by expression (45). From now on, zero *net dynamic marginal gain* line, $R_i(p_i, k_j) = 0$ when $q_j > 0$ is represented by line FE_{N+1}. The $s+1^{th}$ reduction at t_s^+ brings the path to E_{s+1}. The following price reductions occur when all amounts of the unemployed factor resulting from the last price reduction find new positions and the natural unemployment rate is reached. Finally, the new equilibrium is reached after N *periodical price reductions.*

A similar solution is proposed when all factors are considered. The only difference is that now as long as one unemployed factor has reached the natural rate, the new round of price reduction begins. Therefore, k_j may be viewed as the factor that spends the shortest time to reach the natural rate at s^{th} round while $R_i(p_{is}, K)\mid_{q=0} > 0$. The proposed solution candidate for vector P is similar to the above. We only need to replace p_i by P, R_i by R, and R_i^0 by R^0 in the above expressions.[13] We summarize the above proposed solution candidate as the following proposition and then prove it is indeed optimal.

Proposition 1 *Assuming that $U\hat{}\,(P, K)$ is concave, the optimal price adjustment is periodical. The transition period is partitioned by N time period $[t_{h-1}, t_h]$ for $h = 1, \cdots, N$ where $t_0 = 0$ and $t_N = t^*$. For each period, the price reduction takes place at*

[13] The unemployment immediately after the price adjustment, $k_j(t_h^+)$, is declining in Figure 1. However, this may not hold in the case of many goods and many factors.

t_{h-1}^+. *The amount of the price reduction is determined such that the net intertemporal marginal gain of the price adjustment* $R^0(P,K) = 0$. *Within the period* (t_{h-1}, t_h) *prices are fixed and the unemployed factors decline at the rate* Θ *until the net dynamic marginal gain of the price adjustment* $R(P,K) = 0$; *then the next round of price reduction begins.*

Proof: Note that we build up the solution candidate by standard necessary conditions in each period (t_{h-1}, t_h). To prove the proposition, we need to show that the corresponding sufficient conditions are satisfied. Follow Arrow's (1970) suggestion, the utility associated with a given policy, including reductions,[14] is:

$$\int_0^\infty U\hat{}\,(P,K)e^{-rt}dt + \sum_{h=0}^{N} - \left[\bar{U}_h(t_h^+)P(t_h^+) - \bar{U}_h(t_h^-)P(t_h^-)\right]$$

$$= \sum_{h=0}^{N} \int_{t_h^+}^{t_{h+1}^-} U\hat{}\,(P,K)e^{-rt}dt + \int_{t_{N+1}}^{\infty} U\hat{}\,(P,K)e^{-rt}dt \tag{46}$$

$$+ \sum_{h=0}^{N} - \left[\bar{U}_h(t_h^+)P(t_h^+) - \bar{U}_h(t_h^-)P(t_h^-)\right] \tag{47}$$

The second item of the expression (46) is the intertemporal utility in steady state 1, which does not depend on the adjustment path. For the first item of expression (46), we want to show that $Z^*(t)$ is optimal in each interval (t_h, t_{h+1}). Using standard sufficient conditions, we need to show that the maximized Hamiltonian H^* and the constraint are concave in that interval. Substituting $Z^*(t) = 0$ into (23), we have:

[14] Marginal utility associated with a unit reduction is negative.

$$H^* = U^{\hat{}}(P, K) - \lambda'_J \Theta K$$

and the constraint equation

$$G = K - \underline{k}$$

which are indeed concave in (P, K).

The only item left is expression (47). $\bar{U}_h(.)$ is the marginal utility associated with a unit reduction in state variable P. Thus, $\bar{U}_h(.) = a\frac{DU(t)}{DP} + b\frac{DU(t)}{DK}\frac{DK}{DP}$ where a and b are the intertemporal weight for marginal utilities of P and K, respectively. $a = r^{-1}$ and $b = (r + \Theta)^{-1}$, while $\frac{DK}{DP} = -V_p^O(t)$ by (10). Therefore, $\bar{U}_h(t) = R^0(t)$. To prove $-\left[\bar{U}_h(t_h^+)P(t_h^+) - \bar{U}_h(t_h^-)P(t_h^-)\right]$ is maximized, we need to show that

$$-R^0(P, K)P + R^0(P(t_h^-), K(t_h^-))P(t_h^-)$$

is maximized at $P = P(t_h^+)$, which is certainly the case since $R^0(P, K) \leq 0$ when $P \geq P(t_h^+)$, but $R^0(P, K) > 0$ when $P < P(t_h^+)$. **Q.E.D.**

The only possibility that optimal path is continuous is that the adjustment path follows zero *net dynamic marginal gain* line $R(P, K) = 0$. However, that is impossible, because $K = -V_p^O(P, W)\dot{P}(\tau) - \Theta K(t)$, which do not always coincide with $R(P, K) = 0$ for any price adjustment.

The *net dynamic gain* of the price adjustment, $R\dot{P}$, is the sum of the *net intertemporal gain,* $R^0\dot{P}$, and its *dynamic residual,* $R^d\ \dot{P}$. For any price adjustment, the dynamic gain and the dynamic cost of the adjustment do not always cancel each other since equations $R(P,K)=0$ and $K=-V_p^O(P,W)\dot{P}(\tau)-\Theta K(t)$ do not always give the same solution for (P,K). The optimal amount of price adjustment is determined such that the *net intertemporal gain* $R^0\dot{P}$ equals zero. However, since the *dynamic residual* $R^d\ \dot{P}$ is negative, the *net dynamic gain* becomes negative immediately after the price adjustment taking place. Negative *net dynamic gain* prevents any further price adjustment. As time goes on, unemployed factors find new positions and the *net dynamic gain* reaches zero; the next price adjustment then begins.

We now consider whether the price adjustment is gradual, hence, whether the adjustment takes more than one period. If the adjustment takes only one period, E_1 and E_{N+1} must either coincide or be at the same flat line. Therefore, either the B_1B_1 curve should be vertical or $R^0(P,K)=0$ should be flat . A vertical B_1B_1 curve means no frictional unemployment, which is ruled out in this paper. If $R^0(P,K)$ is flat, it means that the change in unemployment does not affect the *net intertemporal marginal gain* of the price adjustment, which is also ruled out by the stability condition. Therefore, we have:

Proposition 2 *Under the stability conditions, the optimal price adjustment is gradual.*

Although Mussa (1984) emphasizes immediate reform, he does recognize that the unemployments may provide a rationale for gradualism. As long as the searching process takes time, the productive resource cannot be moved instantaneously among alternative uses. Not only do explicit adjustment costs exist, which according to Mussa (1984) do not provide the rationale for gradualism, the substantial unemployment of some resources exists for a period of time, which is proven here to provide the rationale for gradualism.

7 Conclusion and Discussion

This paper develops an adjustment technique that links the product and the factor markets. Based on this technique, we then solve an optimal control problem to find the optimal price adjustment. It is shown that the optimal price adjustment is periodical and takes more than one period. Therefore, the optimal adjustment is gradual.

Two questions are usually raised about price adjustments: *when* and *how much* are the prices adjusted? We show that the timeing of adjustment is determined by the point at which the *net dynamic gain* of the price adjustment equals zero. The optimal amount of price adjustment is what it makes the *net intertemporal gain* equal to zero.

The existence of unemployed factors in a period of adjustment is supported by empirical studies in labor and capital markets. Studies by Topel (1990), Ruhm (1991), and Jacobson, Lalonde and Sullivan (1993) estimate the effects of reallocation on individual workers. They find that displaced workers experience prolonged periods of unemployment as well as significant losses in permanent income. A study by Ramey and Shapiro (1998) shows that the process of winding down operations results in a significant period of underutilization before the capital is finally sold. The importance of job flow across different sectors is documented by Davis and Haltiwanger (1999). Over the 1972:2-1988:4 period, they find that job flow is large in every sector, averaging in a quarterly job reallocation rate of 10.7 percent of employment.

In studies toward "a general theory of wage and price rigidities and economic fluctuations," Stiglitz (1999) makes the hypothesis that large economic fluctuations are a consequence of problems of the adjustment of wages and prices. Our analysis has been limited to a micro general equilibrium analysis. However, the idea that prices will not be adjusted when the benefit of adjustment is less than the cost of frictional unemployment due to the adjustment may shed light on some aspects of the micro-foundations of price rigidities and unemployment fluctuations.

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