

## Homework #7: Computationally Solving Rational Expectations Models

ECON 5163

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**(Include copies of all your Matlab code)** Let us consider the neoclassical growth model from Homework #6. At any date  $t$ , households in the economy gain utility from consumption,  $C_t$ , and disutility from labor,  $N_t$ . Those preferences are represented by the following utility function:

$$U = \sum_{t=0}^{\infty} \beta^t E_0 [\ln(C_t) + \theta \ln(1 - N_t)],$$

where  $\beta$  is a discount factor. Households' demands are restricted by their budget constraint:

$$Z_t(K_t)^\alpha(N_t)^{1-\alpha} = C_t + K_{t+1} - (1 - \delta)K_t,$$

where  $K_t$  is capital,  $\alpha$  is capital's share of output, and  $\delta$  is the capital depreciation rate. The parameter  $Z_t$  is an exogenous parameter that represents the level of productivity in the economy. That productivity factor is a stochastic process that follows an AR(1) representation:

$$\ln(Z_t/Z^*) = \rho_Z \ln(Z_{t-1}/Z^*) + \eta_t,$$

where  $Z^*$  is the steady state value of  $Z_t$  and  $\eta_t \sim N(0, \sigma^2)$  [Hint:  $\ln(Z_t/Z^*) \approx \widehat{Z}_t$ ]. The parameters are set to the following values:  $\alpha = 0.33$ ,  $\beta = 0.99$ ,  $\delta = 0.025$ ,  $\rho_Z = 0.95$ ,  $\sigma^2 = 1$ , and  $Z^* = 1$ . Finally, the time constraint forces the steady state value of labor,  $N^*$ , to be between 0 and 1. Since the preference parameter  $\theta$  is key in determining  $N^*$ , a value for  $\theta$  is selected so that  $N^* = 0.2$ . That is,  $N^*$  is set equal to 0.2 and  $\theta$  is determined from the first-order condition for labor.

1. Using the first-order conditions for  $C_t$ ,  $K_{t+1}$ ,  $N_t$ , and  $\lambda_t$  from Question 2 part *c* from Homework #6, find the steady state values for  $C_t$ ,  $K_t$ , and  $\lambda_t$  and the value for  $\theta$  [Hint: Use the first-order condition for  $N_t$  to find  $\theta$ ].
2. Linearize the first-order conditions for  $C_t$ ,  $K_{t+1}$ ,  $N_t$ , and  $\lambda_t$  and place them into matrix form:  $AE_t\widehat{Y}_{t+1} = B\widehat{Y}_t + C_0\widehat{X}_t + C_1E_t\widehat{X}_{t+1}$ , where  $\widehat{Y}_t$  is the percent deviation of the endogenous variables from their steady state, and  $\widehat{X}_t$  is the percent deviation of the exogenous variable,  $Z_t$ , from its steady state [Hint: This is the same answer as in Question 5 from Homework #6]. Use that matrix form and the King and Watson [1998, 2002] Matlab programs to find the rational expectations solution in the state space form:  $Z_t = \Pi S_t$  and  $S_t = MS_{t-1} + \varepsilon_t$ . In your answer, you just need to specify the vectors  $Z_t$  and  $S_t$  and the resulting matrix  $\Pi$  from the state space solution.
3. Suppose there is a positive 1% shock to the technology factor in period 1 (i.e.,  $\varepsilon_1 = 1$ ). Plot the impulse response functions for  $C_t$ ,  $K_t$ , and  $N_t$  in periods 1 – 30. Be sure to properly label your graphs. Why is the period 1 response of capital equal to zero?
4. Calculate the 3-step ahead forecast error variance for  $C_t$ ,  $K_t$ , and  $N_t$ .
5. What are the unconditional variances for  $C_t$ ,  $K_t$ , and  $N_t$ ?
6. Find the correlation between  $K_t$  and  $N_t$ ,  $\text{corr}(K_t, N_t)$ , and the correlation between  $K_t$  and  $N_{t-1}$ ,  $\text{corr}(K_t, N_{t-1})$ .