The Signal Extraction Problem Revisited: A Note on Its Impact on a Model of Monetary Policy*

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Abstract

This paper develops a dynamic stochastic general equilibrium (DSGE) model with sticky prices and sticky wages, where agents have imperfect information on the stance and direction of monetary policy. Agents respond by using Kalman filtering to unravel persistent and temporary monetary policy changes in order to form optimal forecasts of future policy actions. Our results show that a New Keynesian model with imperfect information and real rigidities can account for several key effects of an expansionary monetary policy shock: the hump-shaped increase in output, the delayed and gradual rise in inflation, and the fall in the nominal interest rate.

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1 Introduction

Most empirical studies indicate that inflation responds gradually to an expansionary monetary policy shock with the peak occurring several quarters afterward.[1] A traditional New Keynesian model, however, is unable to produce the substantial lag in the peak inflation response. Its failure to account for that behavior has led to numerous efforts to solve the problem. Three of the most popular approaches are Christiano et al.'s [2005] "dynamic indexation," Gali and Gertler's [1999] "backward-looking rule of thumb," and Mankiw and Reis' [2002, 2007] "sticky information."[2] Using Calvo [1983] style price and wage setting, Christiano et al. [2005] assume that some firms and households can readjust optimally their price or wage, respectively, while the remaining agents must index their price and wage to the previous period's inflation rate. Dellas [2006], however, argues that a shortcoming of dynamic indexation is that it involves the "full or partial abandonment of rational expectations." In fact, Figure 1 illustrates that when dynamic indexation is replaced with static indexation (nonadjusting prices and wages are indexed to the steady state inflation rate) inflation peaks immediately after an expansionary monetary policy shock.[3] Similarly, Gali and Gertler's [1999] assumption that some firms adjust their prices based on last period's inflation rate is also inconsistent with rational expectations. Mankiw and Reis [2002, 2007], on the other hand, assume that information about monetary policy shocks disseminates gradually throughout the economy. When Mankiw and Reis [2007] incorporate sticky information into a general equilibrium model with a nominal interest rate policy rule, inflation peaks only two periods after a temporary monetary policy disturbance. Furthermore, the modest inflation lag is generated by a model in which informational stickiness needs to be applied to price setting, wage setting, and consumption decisions.

Our paper seeks to replicate observed inflation behavior in a New Keynesian model by assuming that agents have imperfect information on the stance of monetary policy.[4] This modern adaptation of Lucas' [1975] "islands" model has been of interest recently to some monetary theorists. Amano et al. [1999], for example, analyze the impact of a temporary

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[1] See Leeper et al. [1996], Christiano et al. [1999] and Altig et al. [2005].

[2] Other studies that seek to account for the gradual response of inflation to a monetary policy shock include Erceg [1997], Woodford [2003], Adam [2005], Burstein [2006], and Tsuruga [2007].

[3] The static indexation model is simply the perfect information specification of the model presented in Section 2. The dynamic indexation model is that same model modified by the assumption that prices and wages, which are not optimally reset, rise by last period's inflation rate.

[4] Orphanides [2001] discusses the informational problems experienced by central banks that utilize real-time data in setting their policy instrument.

but persistent monetary policy shock in a limited participation model when agents observe imperfectly the monetary authority's actions. Despite the presence of imperfect information, inflation peaks either immediately after a monetary policy shock or within one period. Using a New Keynesian model, Dellas [2006] shows that a temporary monetary policy shock generates a modest lag in the peak inflation response when variables like output, inflation, investment, and employment are mismeasured. Both Amano et al. [1999] and Dellas [2006] designate the money supply as the monetary authority's policy instrument. Evidence, however, suggests that most central banks target the nominal interest rate and not the money supply. The distinction between a money growth target and a nominal interest rate target is important because endogenous feedback of variables like inflation and output into a nominal interest rate rule causes inflation to behave differently.

In a recent imperfect information specification with a nominal interest rate policy rule, Erceg and Levin [2003] focus on modeling the Volcker disinflation by presupposing that agents learned slowly about the permanent reduction of the target inflation rate in the Federal Reserve's nominal interest rate rule.[5] The permanent monetary policy shift analyzed by Erceg and Levin [2003] differs from efforts to theoretically generate inflation's empirically observed behavior after a temporary monetary policy shock. Specifically, inflation monotonically adjusts to its new and permanently lower inflation target in Erceg and Levin [2003] while, after a temporary monetary policy shock, inflation slowly rises to its peak and then gradually returns to its preshock level. The monetary policy specification utilized by Erceg and Levin [2003] can be modified to address the impact of a temporary monetary policy shock by assuming that shifts to the target inflation rate revert to the mean instead of following a random walk.

This paper develops a New Keynesian model where agents have imperfect information on the target inflation rate in the monetary authority's nominal interest rate policy rule. Specifically, agents can observe temporary shifts in the target inflation rate but cannot observe the process underlying those changes. Agents respond by using the Kalman filter to form optimal forecasts of future changes in the target inflation rate. When we make that assumption, our results show that a dynamic stochastic general equilibrium (DSGE) model with nominal and real rigidities can generate a hump-shaped change in output, a gradual and delayed response in inflation, and a countercyclical movement in the nominal interest

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[5] Gurkaynak et al. [2005] and Kozicki and Tinsley [2005] also analyze the impact of permanent inflation rate target shifts in an imperfect information specification, while Schorfheide [2005] analyzes shifts between high and low inflation regimes in an imperfect information model.

rate after a temporary monetary policy shock. A sensitivity analysis then is conducted to determine the impact of nominal and real rigidities on our results. We also discuss any key differences with Christiano et al.'s [2005] perfect information model. That analysis reveals that the output and nominal interest rate responses require investment adjustment costs and either sticky prices or sticky wages, while the inflation result is robust over all nominal and real rigidities considered. Our findings suggest that integrating imperfect information into monetary models is a promising development that needs further study.

The remainder of the paper is structured as follows. Section 2 outlines our DSGE model with sticky prices and sticky wages. Section 3 parameterizes the model. Section 4 presents impulse responses of key economic variables to a temporary expansionary monetary policy shock for both the perfect and imperfect information models. Section 5 examines the sensitivity of our results from the imperfect information specification to various nominal and real rigidities. Section 6 concludes.

2 The Model

Our New Keynesian model with Calvo [1983] style wage and price contracts is similar to Christiano et al.'s [2005] model with two key exceptions. One, nonadjusting households and firms raise their wage and price, respectively, by the steady state inflation rate rather than by last period's inflation rate.[6] Two, market participants in our model have imperfect information, as opposed to perfect information, on the stance of monetary policy.

2.1 The Monetary Authority

The monetary authority targets the nominal interest rate, R_t, according to a Taylor [1993] style rule. That is, the nominal interest rate target rises when inflation, π_t, increases above either its steady state or its target, π_t^*, or output, y_t, rises above its steady state:

$$\ln(R_t/R) = \ln(\pi_t/\pi) + \theta_\pi \ln(\pi_t/\pi_t^*) + \theta_y \ln(y_t/y), \tag{1}$$

where $\theta_\pi \geq 0$ and $\theta_y \geq 0$. Like Erceg and Levin [2003], both the policy, π_t^p, and nonpolicy, π_t^{np}, shocks shift the inflation rate target:

$$\ln(\pi_t^*/\pi) = \ln(\pi_t^p/\pi) + \ln(\pi_t^{np}). \tag{2}$$

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[6] Christiano et al. [2005] assume that firms must borrow to finance labor costs. Since that assumption causes labor demand to depend on the nominal interest rate, our model does not include that feature.

Unlike Erceg and Levin [2003], policy shocks represent intentional changes to the monetary policy rule that are extremely persistent but ultimately temporary.[7] Persistent policy shocks are especially plausible in the U.S., where although the Federal Reserve does not have an explicit inflation target, its members have suggested informal target ranges from less than 1 percent to 2.5 percent or more annually (Bernanke [2004]). Specifically, the policy component of (2) is a first-order autoregressive [AR(1)] process:

$$\ln(\pi_t^p/\pi) = \rho_\pi \ln(\pi_{t-1}^p/\pi) + \varepsilon_t^p, \tag{3}$$

where $0 < \rho_\pi < 1$ and $\varepsilon_t^p \sim N(0, \sigma_p^2)$. Nonpolicy shocks, on the other hand, symbolize unintentional monetary policy changes that are short-lived. For example, the Federal Reserve's practice of making discrete changes in the nominal interest rate target (in increments of 25 basis points) creates a stochastic difference between the actual target and the implied target from the Taylor [1993] rule.[8] To capture those errors, the nonpolicy component of (2) is a one-period stochastic shock:

$$\ln(\pi_t^{np}) = \varepsilon_t^{np}, \tag{4}$$

where $\varepsilon_t^{np} \sim N(0, \sigma_{np}^2)$ and ε_t^p and ε_t^{np} are orthogonal.[9]

Most business cycle models assume that private agents have perfect information about monetary policy. Although circumstances exist where the monetary authority clearly signals its intentions, there are also many situations in which the objectives of the monetary authority are less clear. This paper seeks to analyze the effects of the latter situation. To conduct our analysis, we use techniques similar to those used by Kydland and Prescott [1982] and Cooley and Hansen [1995]. When private agents have imperfect information about the intentions of the monetary authority, they are unable to observe π_t^p and π_t^{np}. Instead, private agents form expectations of the future inflation target based on their observations of the current inflation target (π_t^*) and their knowledge of the driving process of monetary policy shocks (ρ_π, π, σ_p^2, and σ_{np}^2). Given that information, we can use the Kalman filter easily to

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[7] Erceg and Levin [2003] assume that the policy shock component to the inflation rate target follows a random walk. That specification implies that any policy shock will generate a permanent change in monetary policy. Since our objective is to analyze the impact of a temporary monetary policy change, we specify the policy shock as a stationary process.

[8] Using a dynamic ordered probit model, Dueker [2000] estimates that the standard deviation of the spread between the central bank's desired prime lending rate and the actual prime lending rate was 26 basis points from 1973 to 1999. Since the prime lending rate is closely linked to the federal funds rate, this finding indicates that discrete shifts in the nominal interest rate target introduce nonpolicy shocks into the economy.

[9] Ireland [2007] and Gavin et al. [2009] also assume that multiple shocks impact the monetary policy rule.

derive private agents' expectations of the future target inflation rate.[10]

To utilize the Kalman filter, the monetary policy rules in (2), (3), and (4) are converted into the following state space representation:

$$x_t = H' \cdot \xi_t \tag{5}$$

$$\xi_t = F \cdot \xi_{t-1} + \varepsilon_t, \tag{6}$$

where $x_t = [\ln(\pi_t^*)]$ is the observed variable, $\xi_t = [\ln(\pi_t^p), \ln(\pi_t^{np})]'$ is a vector of unobserved variables, H and F are matrices of parameters, and $\varepsilon_t = [\varepsilon_t^p, \varepsilon_t^{np}]'$ is a vector of error terms. The Kalman filter then is employed on the state space system in (5) and (6) to form expectations of the future target inflation rate (i.e., $E_t[\ln(\pi_{t+i}^*)]$ for $i = 1, 2, ..., \infty$).[11]

2.2 Households

Household h supplies differentiated labor services, $n_{h,t}$, to the firms in a monopolistically competitive market. Total labor hours, n_t, is a Dixit and Stiglitz [1977] aggregate of the differentiated labor services:

$$n_t = \left[\int_0^1 {n_{h,t}}^{(\epsilon_W - 1)/\epsilon_W} dh\right]^{\epsilon_W/(\epsilon_W - 1)},$$

where $-\epsilon_W$ is the wage elasticity of demand for $n_{h,t}$. Cost minimization by the firms indicates that the demand schedule for $n_{h,t}$ is

$$n_{h,t} = \left(\frac{W_{h,t}}{W_t}\right)^{-\epsilon_W} n_t,$$

where $W_{h,t}$ is household h's nominal wage and W_t is considered the aggregate nominal wage:

$$W_t = \left[\int_0^1 {W_{h,t}}^{1-\epsilon_W} dh\right]^{1/(1-\epsilon_W)}.$$

Each period, household h chooses consumption, c_t, and total labor hours to maximize its expected utility:

$$U = E_t\left[\sum_{j=0}^{\infty} \beta^j \left(\ln(c_{t+j} - bc_{t-1+j}) - \phi_n \frac{n_{h,t+j}^{1+\zeta} - 1}{1+\zeta}\right)\right],$$

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[10] Brunner et al. [1980] and Amano et al. [1999] construct similar imperfect information models of monetary policy, except that money is the policy instrument and not the nominal interest rate.

[11] See Chapter 13 of Hamilton [1994] for a detailed description of the Kalman filter.

subject to its budget constraint and the capital accumulation equation. E_t is the expectational operator at time t, $0 < \beta < 1$ is the discount factor, habit formation in consumption preferences exists when $b > 0$, $\phi_n > 0$, and $1/\zeta$ is the labor supply elasticity.

The budget constraint describes the flow of funds for household h:

$$P_t(c_t + i_t) + B_t = R_{t-1}B_{t-1} + W_{h,t}n_{h,t} + P_t q_t u_t k_t + D_t + A_{h,t},$$

where P_t is the price level, i_t is investment, B_t is nominal bond holdings, R_t is the gross nominal interest rate from period t to $t+1$, q_t is the capital rental rate, u_t is the capital utilization rate, k_t is the capital stock, D_t is nominal dividends, and $A_{h,t}$ is a payment from a state contingent securities market.

The capital accumulation equation is

$$k_{t+1} - k_t = i_t \left[1 - S\left(\frac{i_t}{i_{t-1}}\right)\right] - \delta(u_t)k_t.$$

The function for the investment adjustment costs, $S(\cdot)$, represents the resources lost in the conversion of investment to capital. Following Christiano et al. [2005], we assume that $S(1) = S'(1) = 0$ and $\kappa = S''(1) > 0$. The function, $\delta(\cdot)$, assumes that the depreciation rate depends on the capital utilization rate such that $\delta(\cdot)' > 0$ and $\delta(\cdot)'' > 0$.

Finally, wage setting follows a Calvo [1983] model of random adjustment. In each period, the probability that household h receives an opportunity to negotiate a new wage is η_W, while the probability that its wage rises only by the steady state inflation rate, π, is $(1 - \eta_W)$.

2.3 Firms

Firms produce differentiated products in a monopolistically competitive market. Specifically, firm f hires labor, $n_{f,t}$, and rents capital services, $u_t k_{f,t}$, to produce its product, $y_{f,t}$, according to a Cobb-Douglas production function:

$$y_{f,t} = (u_t k_{f,t})^{\alpha}(n_{f,t})^{1-\alpha},$$

where $0 < \alpha < 1$. Each firm's differentiated output is aggregated to get total output, y_t:

$$y_t = \left[\int_0^1 {y_{f,t}}^{(\epsilon_P - 1)/\epsilon_P} df\right]^{\epsilon_P/(\epsilon_P - 1)},$$

where $-\epsilon_P$ is the price elasticity of demand for $y_{f,t}$ and $y_t = c_t + i_t$. Cost minimization on the part of households implies the following demand schedule for $y_{f,t}$:

$$y_{f,t} = \left(\frac{P_{f,t}}{P_t}\right)^{-\epsilon_P} y_t,$$

where $P_{f,t}$ is the price for firm f's output and P_t is a nonlinear price aggregate index:

$$P_t = \left[\int_0^1 {P_{f,t}}^{1-\epsilon_P} df\right]^{1/(1-\epsilon_P)}.$$

Finally, firm f sets its price based on a Calvo [1983] style pricing rule. In each period, the probability that firm f receives an opportunity to set a new price is η_P, while the probability that it can increase its price only by the steady state inflation rate, π, is $(1 - \eta_P)$.

3 Parameterizing the Model

The parameter values are specified at a quarterly frequency. The discount factor, β, is 0.99, the habit persistence parameter, b, is 0.7, the labor supply elasticity, $1/\zeta$, is 3 (i.e., $\zeta = 1/3$), and preference parameter ϕ_n is selected so that steady state labor, $\overline{n}$, equals 1/3. Following Erceg et al. [2000], the price elasticity of demand, ϵ_P, and the wage elasticity of labor demand, ϵ_W, are both set to 6. The conditional probability of price adjustment, η_P, and wage adjustment, η_W, are set equal to 0.25. That calibration implies that the average time between price and wage adjustment opportunities is four periods (quarters). Capital's share of output, α, is set to 0.33 and the investment adjustment costs parameter, κ, is calibrated to 2.5, which is consistent with estimates of Christiano et al. [2005]. We also set the average depreciation rate, δ, to 0.025, the steady state capital utilization rate, u, to 1, and the elasticity of the marginal depreciation rate with respect to the capital utilization rate, $\chi = [u \cdot \delta''(\cdot)/\delta(\cdot)']$, to Basu and Kimball's [1997] estimated value of 1.

Parameterization of the monetary policy rule is consistent with the literature. The weight on inflation, θ_π, and output, θ_y, in the policy rule, (1), are set to Taylor's [1993] values of 0.5 and 0.125, respectively.[12] The steady state gross inflation rate, π, is set to 1.01. The autocorrelation coefficient on the policy shock, ρ_π, in (3) is set to 0.95, which implies the policy shock is extremely persistent, but ultimately temporary. Erceg and Levin [2003] calibrate ρ_π to 0.999. That value, however, is only relevant for examining the effects of a

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[12] The calibration of θ_y is equivalent to a coefficient of 0.5 on an annualized rate of change.

permanent monetary policy change, such as a disinflation, and not for analyzing the impact of a temporary monetary policy shock. Finally, we calibrate the ratio of the policy shock's variance, σ_p^2, to the nonpolicy shock's variance, σ_{np}^2, to Erceg and Levin's [2003] estimated value, which results in a Kalman filter gain value of 0.13.[13] That parameterization suggests that agents' expectations take into account about 50% of a shift in π_t^p within four quarters.

4 Results

This paper analyzes the ability of a New Keynesian model with imperfect information on the monetary authority's intentions to account for key business cycle facts. Many empirical studies of the impact of a monetary policy shock generate similar results. For example, Figure 2 presents the responses of key economic variables in Altig et al.'s [2005] vector autoregressive (VAR) model to an expansionary monetary policy shock.[14] Like many other empirical studies, Altig et al. [2005] find that an expansionary monetary policy shock produces a large hump-shaped increase in output that dies out within a few years, a gradual rise in the inflation rate over several periods, and a decline in the nominal interest rate.[15] Since those findings are robust across many empirical studies, many economists believe that a plausible model of the monetary transmission mechanism should account for that behavior.[16]

This section investigates the ability of our New Keynesian model to replicate those key U.S. business cycle facts. Initially, the impact of both a policy and nonpolicy shock when agents have perfect information is examined. We then present the effects of a policy shock when agents have imperfect information on the intentions of the monetary authority.

4.1 Perfect Information: Policy Shock

We begin by examining the effect of a 1% expansionary policy shock on our New Keynesian model when agents have perfect information on the intentions of the monetary authority. Figure 3 illustrates the responses of output, inflation, the nominal interest rate, the real interest rate, consumption, and investment to a policy shock. The monetary authority operationally increases its inflation target, which puts downward pressure on its nominal

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[13] In a model with perfect information, the Kalman filter gain value equals 1.

[14] Figure 2 uses the same model and data as described in Altig et al. [2005].

[15] The size of the monetary policy shock is set so that the initial response in the nominal interest rate is equivalent to its response in our baseline imperfect information model, which is presented in Figure 5.

[16] Leeper et al. [1996] and Christiano et al. [1999] are other studies that find similar results.

interest rate target, by transferring additional money to the households. Since prices are sticky, real household wealth rises, which households spend on additional consumption and investment. Growth in consumption and investment, however, is slowed by real rigidities in the model. Habit formation in consumption breaks the link between consumption growth and the real interest rate in the standard business cycle model. Households, who receive utility from habit formation, adjust slowly their consumption path with the peak response occurring five periods after the initial policy shock. Investment adjustment costs moderate investment's response by imposing a cost on households to increase their level of investment. As a result, investment continues to rise for seven periods after the policy shock. Investment adjustment costs also lower the elasticity of investment demand with respect to the real interest rate, which enhances the fall in the real interest rate. The magnitude of the real interest rate decline moderates in subsequent periods as the cost of adjusting investment diminishes.

The presence of nominal rigidities enables output to increase after a policy shock. The high persistence in the policy shock, however, prevents output from returning to its preshock level as fast as is empirically observed. Price stickiness prevents some firms from optimally resetting their prices, which causes those firms to charge a lower than optimal price. Wage stickiness, on the other hand, limits the increase in production costs, so that price-adjusting firms do not increase their prices as much. Nevertheless, price-adjusting firms raise their prices substantially because they have perfect knowledge that the policy shock will persist, but do not know the timing of their next price-adjusting opportunity. The aggressive response of price-adjusting firms generates a surge in inflation and expected inflation with the inflation peak occurring on impact and not with a lag as observed empirically. Escalation in expected inflation dominates the decline in the real interest rate, which causes the nominal interest rate to rise counterfactually.[17] The nominal interest rate continues to increase for the next three periods as expected inflation remains persistently high, while the decline in the real interest rate weakens. Only after that period does the nominal interest rate begin to fall toward its steady state. As a result, a New Keynesian model with perfect information generates hump-shaped increases in output, consumption, and investment after an expansionary policy shock, but cannot produce a lagged inflation peak or a decline in the nominal interest rate.

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[17] In terms of the monetary authority's policy rule, (1), the rise in output and inflation puts enough upward pressure on the nominal interest rate to dominate the downward pressure from the increase in the inflation rate target. As a result, the nominal interest rate rises counterfactually.

4.2 Perfect Information: Nonpolicy Shock

We now analyze the impact of a 1% positive, nonpolicy shock on our New Keynesian model when agents have perfect information that the shock will last for one period. Figure 4 shows the impulse responses of output, inflation, the nominal interest rate, the real interest rate, consumption, and investment to a nonpolicy shock. In the period that the shock occurs, price-adjusting firms increase their prices slightly because they realize that the inflation target will be elevated for just one period. Firms simply produce more output, but the size of that increase is limited by the behavior of consumption and investment. Households' preference for habit formation in consumption constrains the rise in consumption and instead, increases the supply of resources available for investment. Investment adjustment costs, however, dampen investment's rise, which causes the nominal and real interest rates to fall.

When the nonpolicy shock is reversed in the following period, inflation, the nominal interest rate, and the real interest rate return to their pre-shock values. Consumption, investment, and output also fall, but the drop is spread out over the next few periods. That protracted decline occurs because habit formation and investment adjustment costs slow the fall in consumption and investment, respectively, which is then transmitted to output. While the impact of a nonpolicy shock is short-lived, it has some important implications for the effects of a policy shock on our New Keynesian model with imperfect information.

4.3 Imperfect Information: Policy Shock

We next examine the impact of a 1% expansionary policy shock on our New Keynesian model when agents have imperfect information on monetary policy. Figure 5 reports the impulse responses for the imperfect information specification and compares them with the responses from the perfect information specification. Our results indicate that a New Keynesian model generates several important U.S. business cycle facts when agents have imperfect information about the persistence of the monetary policy shock. A key factor influencing those results is that agents initially believe there is a high probability that the policy shock will not persist beyond the current period. Therefore, immediately after the policy shock, the price-adjusting firms increase prices only modestly in comparison to their responses in the perfect information specification. As firms begin to learn that the monetary disturbance is persistent, price-adjusting firms increasingly set higher prices. Such behavior enables the inflation rate to peak in the eighth period following the policy shock ($t = 9$). One difference between our work and Christiano et al. [2005] is that inflation immediately increases in our model, while it initially decreases in their model.

Consumption, investment, and output respond sluggishly to the policy shock in the imperfect information model. Habit formation in consumption and investment adjustment costs discourage households from rapidly changing their consumption and investment paths. Since households believe the higher inflation target will likely last for one period, their consumption and investment does not drastically increase. As households learn that the shock is persistent, they increase their consumption and investment. Therefore, consumption, investment, and output all exhibit a hump-shaped response after a policy shock, but, as in the perfect information model, their responses persist longer than is observed in the data. The expectation of a short-lived rise in the inflation target also lowers the increase in expected inflation. The decline in the real interest rate then dominates that more modest jump in expected inflation, which enables the nominal interest rate to fall. As a result, a New Keynesian model with imperfect information successfully accounts for the following empirical effects of an expansionary policy shock: a gradual and lagged response in inflation, a large hump-shaped increase in output, and a decline in the nominal interest rate.

Our analysis of the impulse responses in Figures 3-5 contains several important results. One, a New Keynesian model with perfect information can produce a hump-shaped response in output, consumption, and investment after an expansionary monetary policy shock. Two, the model cannot generate the lagged peak in inflation or the decline in the nominal interest rate after such a shock. Three, results for a New Keynesian model change dramatically when we assume that agents have imperfect information on monetary policy. Specifically, the imperfect information specification produces a hump-shaped output response, a gradual rise in inflation, and a fall in the nominal interest rate after a monetary policy shock.

5 A Sensitivity Analysis

A policy shock's effect on a New Keynesian model with imperfect information depends, sometimes critically, on the model's structural features. This section examines the impact of key nominal and real rigidities on the response of output, inflation, and the nominal interest rate to a policy shock in the imperfect information specification. First, the importance of the price stickiness and wage stickiness assumptions are assessed both individually and jointly. Second, we examine the effects of eliminating variable capital utilization, habit formation in consumption, and investment adjustment costs from the model.

5.1 The Impact of Nominal Rigidities

Figure 6 illustrates the effect of nominal rigidities on the responses of output, inflation, and the nominal interest rate to a 1% expansionary policy shock in the imperfect information specification. The left column of Figure 6 displays the impulse responses when prices are flexible ($\eta_P = 1$). Our results indicate that price stickiness only slightly impacts the magnitude of the impulse responses, but its qualitative response to a policy shock does not change. When prices are perfectly flexible, every firm can immediately adjust its price to a policy shock. As a result, the increase in output is slightly smaller, while the inflation rate rises more rapidly. The higher response in inflation pushes up inflation expectations, which dampens the decline in the nominal interest rate. The humped-shaped responses of output and inflation, however, are unaffected by the elimination of price stickiness.

The sensitivity of the responses of output, inflation, and the nominal interest rate to the sticky wage friction is displayed in the center column of Figure 6. When wages are flexible ($\eta_W = 1$), the real marginal cost rises more after a positive policy shock than in the benchmark specification. Higher production costs cause price-adjusting firms to lower their output by boosting the size of their price increases. As a result, a specification with flexible wages produces a more moderate output increase and an enhanced inflation response. The higher inflation also raises expected inflation, which limits the magnitude of the nominal interest rate decline. Interestingly, the omission of wage stickiness, like in the flexible price case, does not affect the model's ability to generate the hump-shaped responses in output and inflation after a policy shock.

That result differs from Christiano et al.'s [2005] finding that wage stickiness is critical for a policy shock to generating a hump-shaped inflation response. The reason why wage stickiness is not as critical in our model is evident by examining the New Keynesian Phillips curve. That equation states that current inflation depends on the real marginal cost and expected inflation. In Christiano et al. [2005], the elimination of wage stickiness causes the real marginal cost to rise substantially after a policy shock, which pushes up inflation. In our model, imperfect information constrains the increase in expected inflation after a policy shock, so that wage stickiness is not essential to producing a hump-shaped inflation response.

The right column of Figure 6 shows the impulse responses for our imperfect information specification with both flexible prices and flexible wages ($\eta_P = 1$ and $\eta_W = 1$). Without any nominal rigidities, every price and wage immediately adjusts after a policy shock, which causes all of the real variables to remain unchanged. The persistent increase in the inflation target also raises expected inflation that in turn drives up the nominal interest rate. Inflation, nonetheless, still peaks several periods later. Thus, agents' imperfect information on monetary policy is key to generating a hump-shaped inflation response after a policy shock.

5.2 The Impact of Real Rigidities

Figure 7 shows the impact of variable capital utilization, habit formation in consumption, and investment adjustment costs on the responses of output, inflation, and the nominal interest rate to a 1% expansionary policy shock in a New Keynesian model with imperfect information. The first column of Figure 7 illustrates the impact of variable capital utilization on our model's impulse response functions. When the capital utilization rate is fixed ($\chi = 1000$), firms cannot to employ additional capital services immediately after a policy shock. Consequently, the rental rate of capital rises more, which causes a greater increase in the real marginal cost. A higher real marginal cost encourages price-adjusting firms to enhance their price increases and further moderate production. The result is that inflation is slightly higher and output is slightly lower when the capital utilization rate cannot vary. Faster price adjustment also raises expected inflation, which dampens the decline in the nominal interest rate. Our results indicate that variable capital utilization is not essential to producing a lagged inflation peak, while Christiano et al. [2005] find that it is critical to generating that result. We reach different conclusions because in our model, imperfect information constrains inflation by dampening expected inflation, while in Christiano et al.'s [2005] model, variable capital utilization moderates inflation by restraining the real marginal cost.[18]

In the second column of Figure 7, we examine the effect of habit formation in consumption on the impulse responses of output, inflation, and the nominal interest rate. Output's initial response to an expansionary policy shock is much larger without habit formation ($b = 0$) than in the baseline model, while responses of inflation and the nominal interest rate are essentially unchanged. The larger reaction in output is caused by greater household consumption. Without habit formation, the growth rate of consumption depends on the real interest rate.

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[18] Christiano et al. [2005] assume that the capital utilization rate is much more variable than in our baseline model. When we apply their specification and calibration of the capital utilization rate to our imperfect information model, our qualitative results are unchanged.

The real interest rate falls immediately to its lowest level after the policy shock. As a result, consumption peaks on impact and not with a lag as is observed empirically. Habit formation breaks the link between consumption growth and the real interest rate by assuming that households prefer to adjust their consumption gradually. That feature enables consumption to rise slowly for several periods after an expansionary policy shock.

The third column of Figure 7 displays the effect of investment adjustment costs on our imperfect information model. Our results indicate that investment adjustment costs are key to generating a hump-shaped output response and a countercyclical nominal interest rate response. When there are no investment adjustment costs ($\kappa = 0$), households can increase the level of investment without sacrificing any resources. Households, who prefer to spread out consumption over their lifetime, then decide to save most of the temporary increase in wealth generated by the policy shock. The result is a small rise in consumption but a dramatic increase in investment. That large jump in investment forces output to surge to its peak immediately and initially puts additional upward pressure on inflation. Future inflation, however, is slightly lower because the higher capital stock in subsequent periods increases the economy's future productive capacity that in turn helps limit expected price increases. Absence of investment adjustment costs also raises the elasticity of investment demand with respect to the real interest rate, which prevents the real interest rate from declining in any meaningful way. As a result, the positive expected inflation effect dominates the real interest rate effect, so that the nominal interest rate rises counterfactually following a positive policy shock. Christiano et al. [2005], in comparison, find that output surges with a two-period lag in its peak response when the degree of investment adjustment costs are small ($\kappa = 0.5$). Their model, however, is unsolvable without investment adjustment costs.

In the fourth column of Figure 7, we examine the impact of having no real rigidities on the behavior of output, inflation, and the nominal interest rate after an expansionary policy shock. Specifically, the capital utilization rate is unable to vary ($\chi = 1000$), the utility function does not exhibit habit formation in consumption ($b = 0$), and investment adjustment costs are dropped from the model ($\kappa = 0$). Impulse responses for the no real rigidities model look similar to the model with no investment adjustment costs; yet, they differ substantially from the benchmark model, especially for output and the nominal interest rate. That result suggests that investment adjustment costs are the key real rigidity needed to generate a hump-shaped output increase and a nominal interest rate decline after an expansionary monetary policy. Inflation, on the other hand, still peaks several periods after a policy shock when the model includes no real rigidities. One crucial finding from our sensitivity analysis then is that the lagged inflation peak is generated by agents' imperfect information on the stance of monetary policy and not by any nominal or real rigidity.

6 Conclusion

Most DSGE models of the monetary transmission mechanism assume that agents fully understand the nature of the monetary policy process. Analysis of post-war U.S. business cycle data, however, suggests that instances exist where the public has not clearly understood the intentions of the monetary authority. This paper examines the impact of a monetary policy shock when agents do not observe the persistence of an inflation rate target shock. Instead, agents learn about shifts in the target inflation rate slowly over time by monitoring changes in the nominal interest rate target. To obtain their optimal forecasts of the future inflation rate target, agents use the Kalman filter to disentangle persistent and transitory changes in monetary policy. Those forecasts are instrumental in determining the responses of both real and nominal variables in our model.

Incorporating imperfect information on monetary policy into the New Keynesian model moderates the rise in inflation expectations after an expansionary monetary policy shock. Those reduced expectations assist in producing a hump-shaped response in output, a gradual increase in the inflation rate that peaks several periods later, and a fall in the nominal interest rate. One drawback, however, is that output is more persistent than is observed in the data. A sensitivity analysis indicates that a mix of nominal and real rigidities is key to producing a hump-shaped output response and a countercyclical movement in the nominal interest rate, while imperfect information is critical to generating a lagged peak in the inflation rate.

Our assumption is that agents have imperfect information on the stance of monetary policy, but that the central bank is fully informed when setting its policies. A natural extension of this research is to assume that the central bank also sets its policies with imperfect information on the aggregate economy. Along those lines, Aoki and Kimura [2008] examine an endowment economy where informational imperfections are two-way between agents and the central bank. When uncertainty faces both agents and the central bank, Aoki and Kimura [2008] find that inflation persistence and volatility increase even when the central bank is responding to only transitory shocks. Incorporating that form of imperfect information into a DSGE model is one of many possible topics for future research.

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Figure 1: Inflation`s Response to a 1% Expansionary Monetary Policy Shock

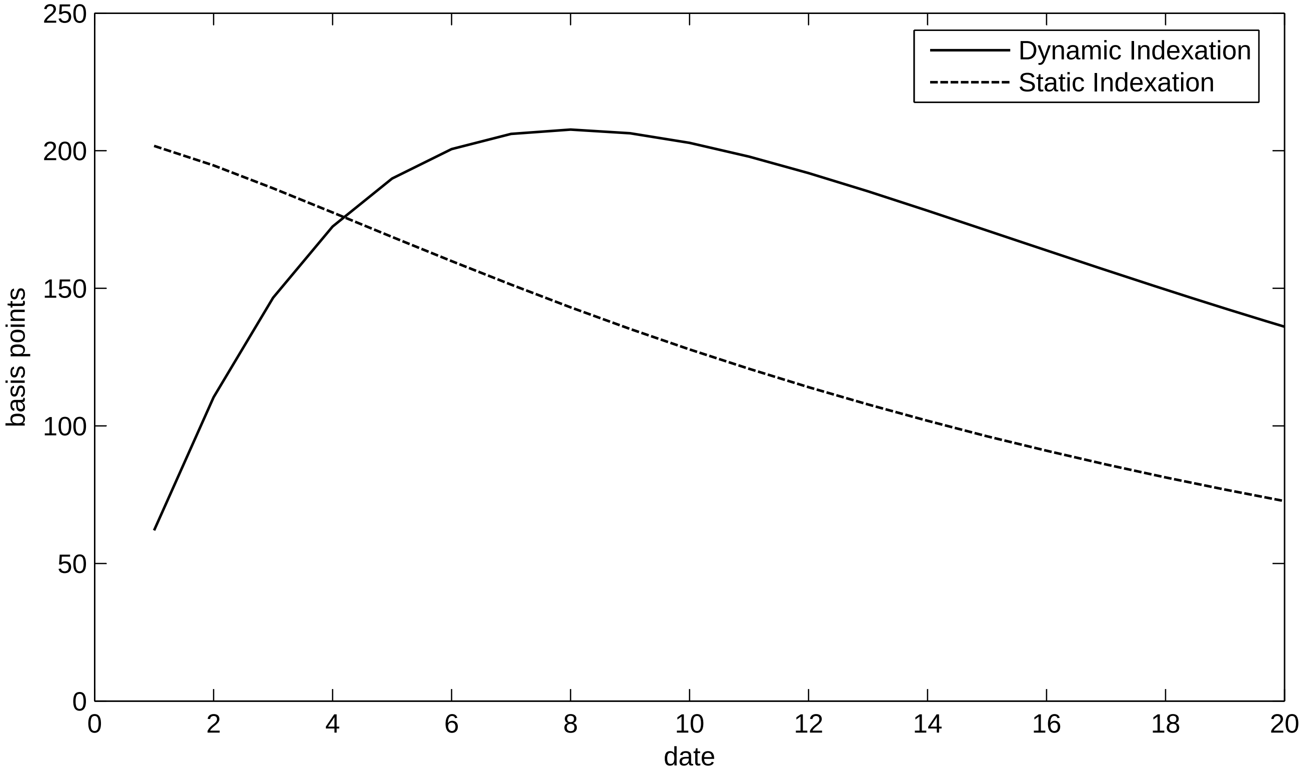

Figure 2: The Impact of a Policy Shock in a VAR Model (see Altig et al. (2005))

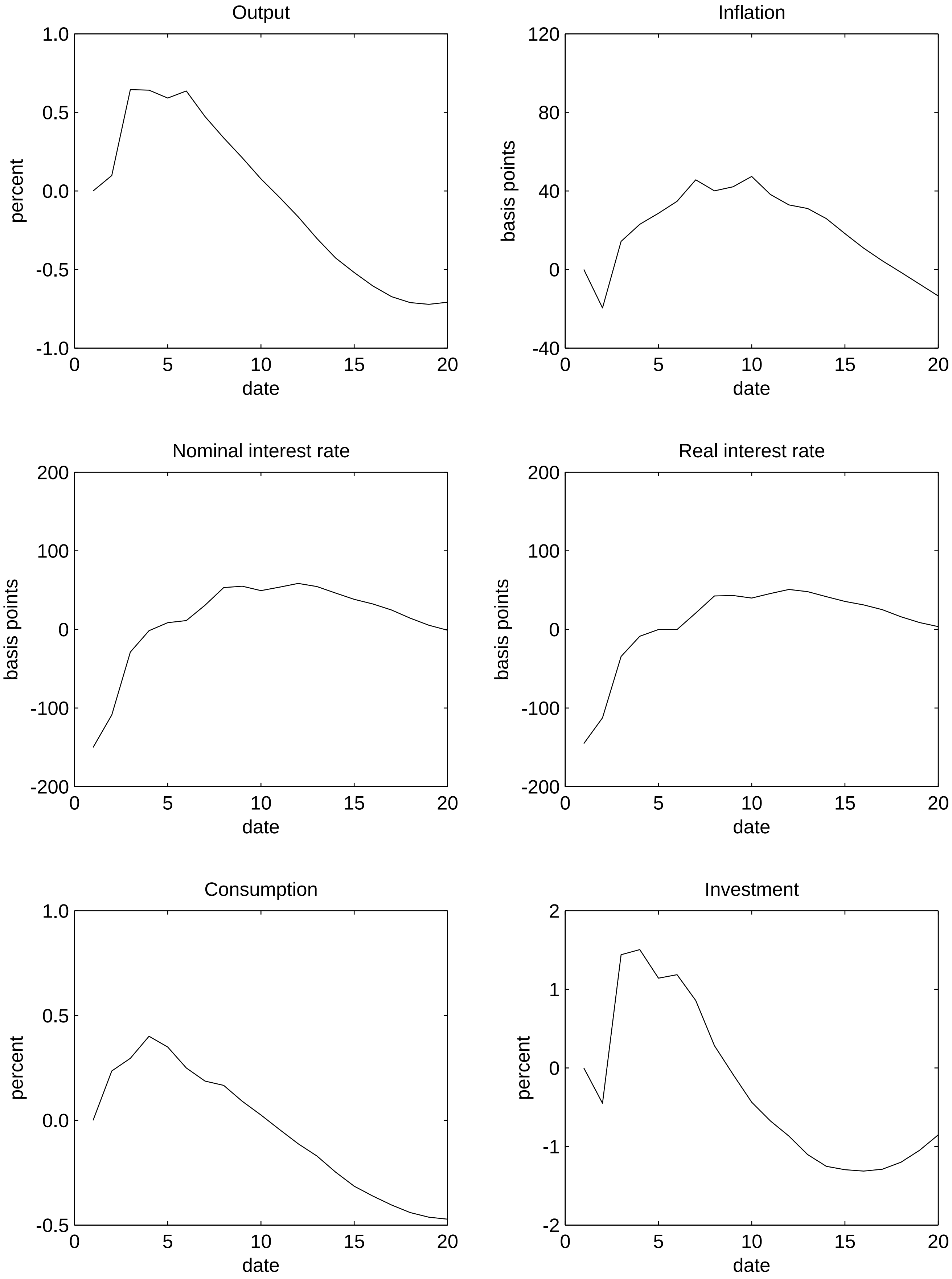

Figure 3: The Impact of a Policy Shock with Perfect Information

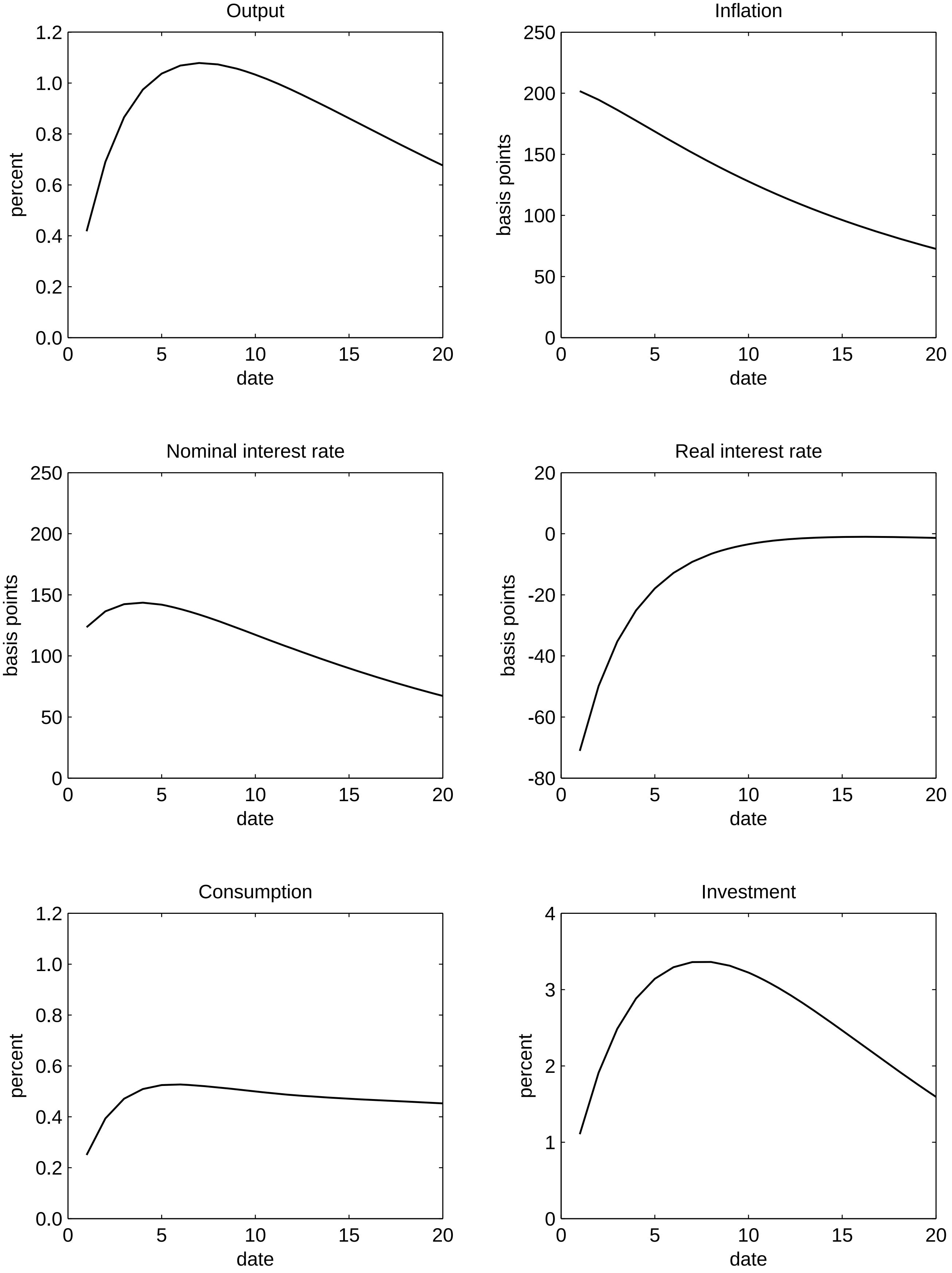

Figure 4: The Impact of a Nonpolicy Shock with Perfect Information

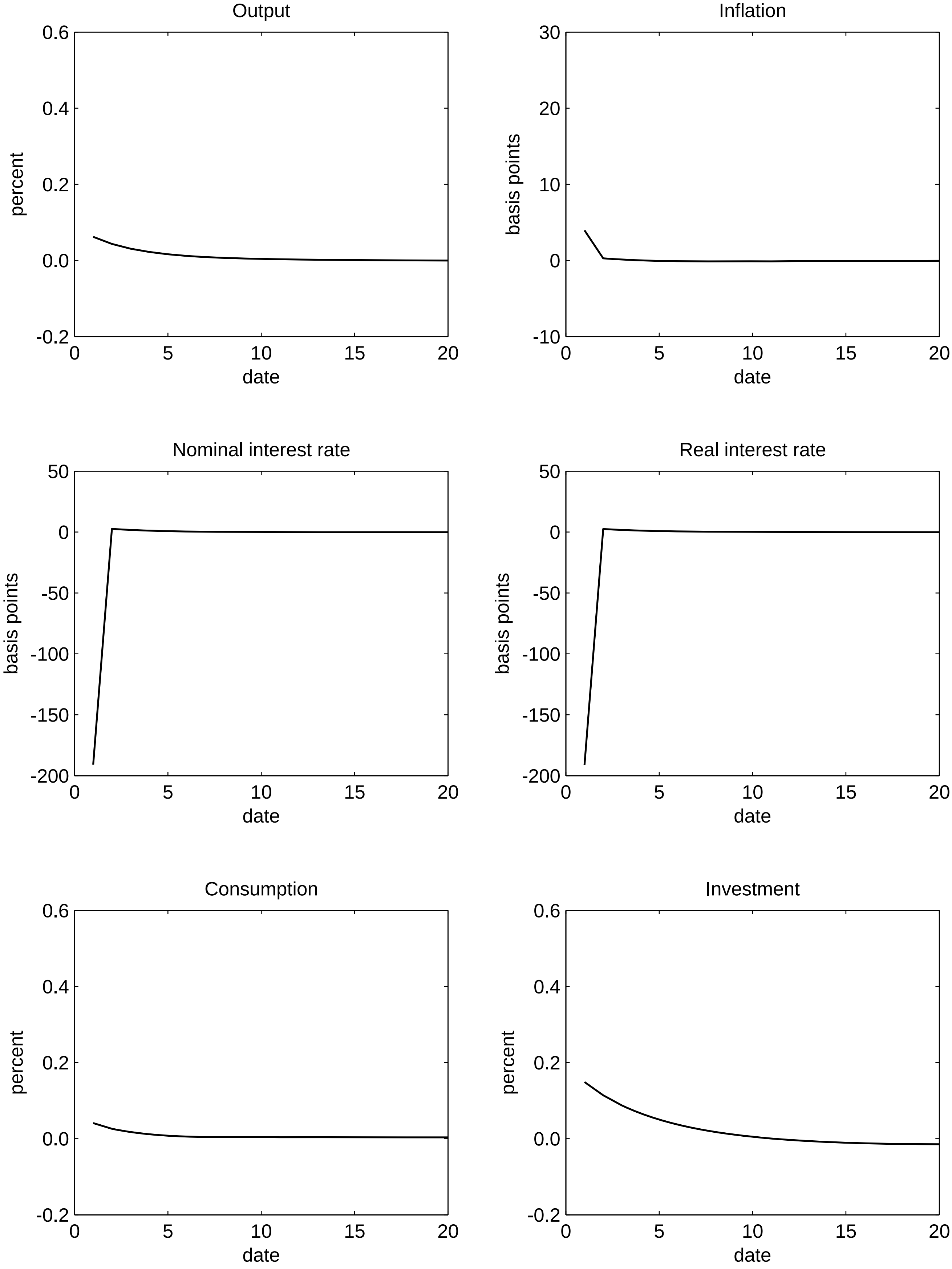

Figure 5: The Impact of a Policy Shock with Imperfect Information

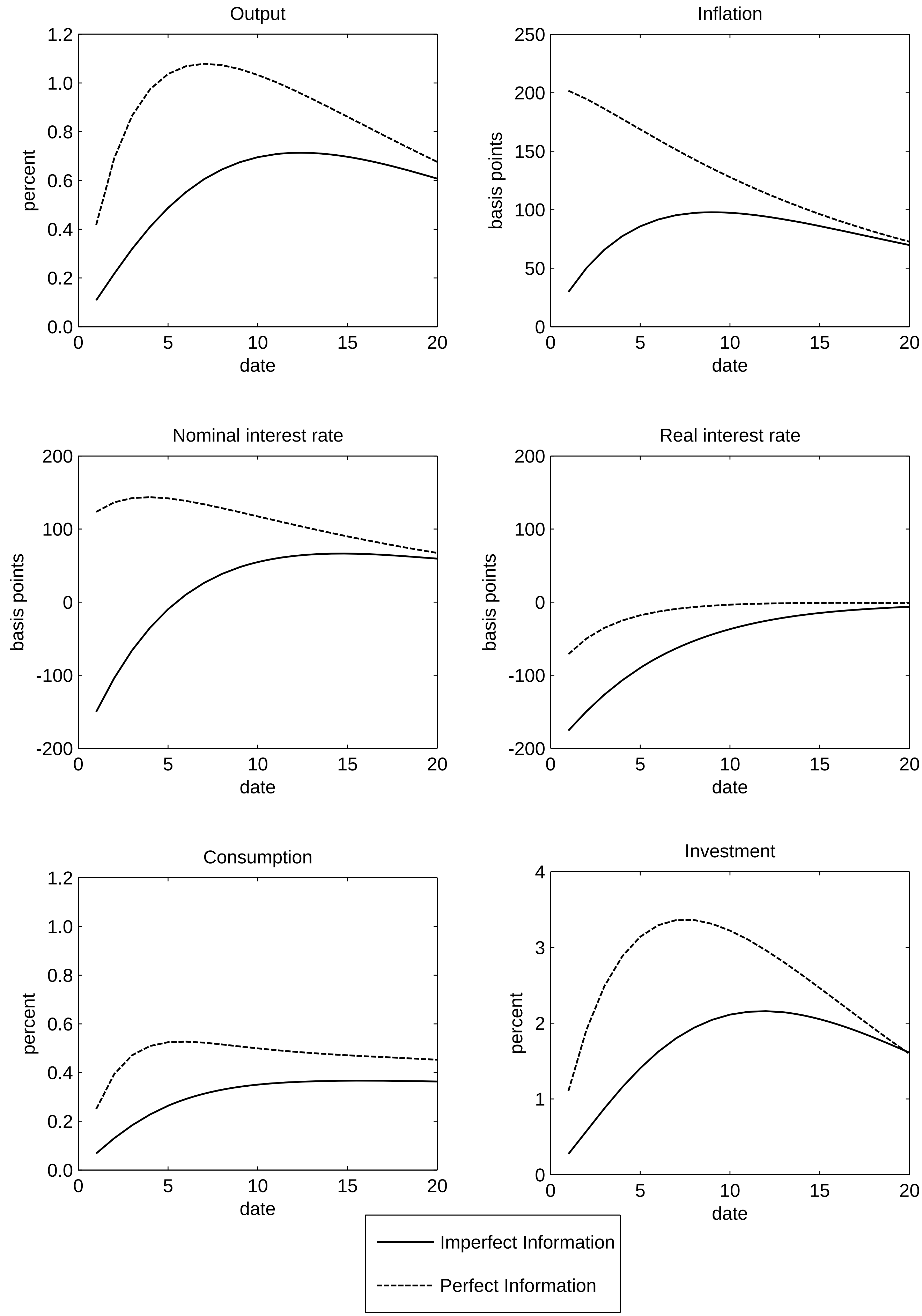

Figure 6: The Impact of Nominal Rigidities

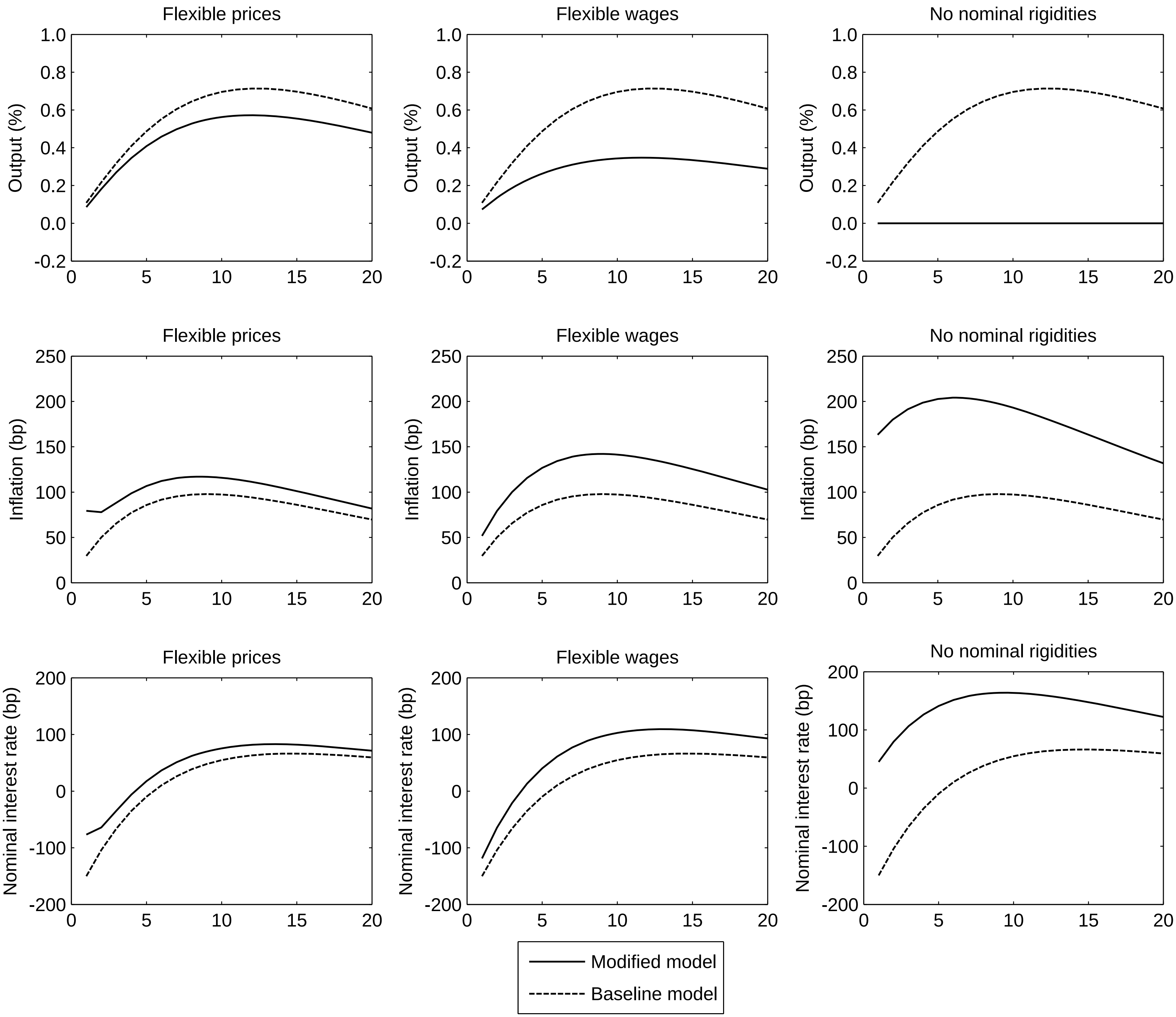

Figure 7: The Impact of Real Rigidities