

## Exam 1

ECON5153

For full credit you must show all your work.

1. (a) Find the following limit:  $\lim_{x \rightarrow -\infty} xe^{1-x} + (1-x)e^x$ .

(b) Consider the function  $f(x) = x^3 - x^2 - 5x$ . Find out the local/global minimum/maximum.

2. Use either elementary row operations OR adjoint matrix to solve for the inverse of the following matrix,

$$A = \begin{pmatrix} 1 & -3 & -2 \\ 2 & -5 & 7 \\ -1 & 2 & -3 \end{pmatrix}.$$

3. Check whether the following set of vectors are linearly independent or not.

(a) (3,4,5), (-3,-1,8) and (0,0,0).

(b) (9, -3, 1), (2,0,2) and (-4,0, 8).

4. Given  $f(x_1, x_2) = x_1^{1/4} x_2^{3/4}$ .

(1) Derive the gradient vector and the Hessian matrix.

(2) Use second order Taylor approximation to estimate  $f(x_1, x_2)$  at  $(x_1, x_2) = (1.1, 0.9)$ .

(3) In what direction should one move from the point (16, 1) to increase this function most rapidly?

5. Consider the following nonlinear equations:

$$y_1^5 + y_2 - x_1 x_2 + 2x_1^2 + 29 = 0$$

$$y_1^2 + y_2^3 + x_1^3 - 3x_2^2 - 2 = 0.$$

Calculate  $\frac{\partial \mathbf{y}}{\partial \mathbf{x}}$ , where  $\mathbf{y} = (y_1, y_2)^T$ ,  $\mathbf{x} = (x_1, x_2)^T$ .