

Exam 2

ECON5153

For full credit you must show all your work.

1. Check whether the following matrix A is (i) N.D., (ii) P.D., (iii) N.S.D., (iv) P.S.D, (v) indefinite.

$$A = \begin{bmatrix} 1 & -2 & 1 \\ 3 & 1 & 2 \\ 0 & 1 & 0 \end{bmatrix}.$$

2. Use Kuhn-Tucker conditions to solve the following problem

$$\max_{x_1, x_2} f(x_1, x_2) = 3x_1x_2 - x_2^3$$

$$\text{subject to } 2x_1 + 5x_2 \geq 20, x_1 - 2x_2 = 5, x_1 \geq 0, x_2 \geq 0.$$

Derive the optimal choices of x_1 and x_2 . Also check NDCQ and second order condition for the optimal choice.

3. Consider the following

$$f(x, y) = ax + y$$

$$\text{subject to } x^2 + ay^2 = 1, x \geq 0, y \geq 0,$$

with $a > 0$. Let the optimal choice be $(x^*(a), y^*(a))$, and let $M(a) \equiv f(x^*(a), y^*(a), a)$. Using envelope theorem to estimate the value of $M(a = 1.01)$.

4. (i) Is the function $f(x, y) = x^2 - 2xy + y^3$ homogeneous? Homothetic? Why?

(ii) Consider the function $f(x, y) = \sqrt{x} + \sqrt{y}$, $x > 0, y > 0$. Check whether the function is (i) concave, (ii) convex, (iii) quasiconcave and (iv) quasiconvex.