Problem Set 1

ECON5153

For full credit you must show all your work.

1. For each of the following functions, plot graph and then answer these questions:
a) Where is the function increasing and where is it decreasing?
b) Find the local and global minimum and maximum.

$$\text{i) } y = x^3 + 2x^2 - 4x; \text{ ii) } y = |x|; \text{ iii) } y = \tfrac{1}{2x+1}.$$

2. What are the domain and range for each of the following functions:

$$\text{i) } y = \tfrac{1}{\sqrt{x^2-2x-3}}; \text{ ii) } \tfrac{\ln(x)}{x^3-1}.$$

3. Calculating derivatives for the following functions:

$$\text{i) } 4x^2 + 3x - 8x^{3/4} + 3x^{1/2}; \text{ ii) } (x^2+1)(x^2+3x+1);$$
$$\text{iii) } \tfrac{x}{x^2-1}.$$

4. For each of the following functions, is it *continuous*, *differentiable*, *continuously differentiable*? Why?

$$\text{i) } y = \begin{cases} x^2, & \text{if } x \geq 0, \\ -x^2 & \text{if } x < 0; \end{cases} \quad \text{ii) } y = \begin{cases} x^2 + 1, & \text{if } x \geq 0, \\ -x^2 - 1, & \text{if } x < 0; \end{cases} \quad \text{iii) } y = \begin{cases} x^3, & \text{if } x < 1, \\ 3x - 2, & \text{if } x \geq 1. \end{cases}$$

5. Calculate the inverse for the following functions, specifying the domains carefully. Take derivatives for these functions and their inverse functions, and show that the part (c) of the Inverse Function Theorem holds.

$$\text{a) } \tfrac{1}{x+1}; \text{ b) } x^{3/4}; \text{ c) } x^2 + x + 2; \text{ d) } a^{x^2-2}.$$

Extra questions

1. Show that the following function is differentiable but not CD1:

$$f(x) = \begin{cases} x^2 \sin \frac{1}{x} & \text{if } x \neq 0, \\ 0 & \text{if } x = 0. \end{cases}$$

2. Show that the following function is CD1 but not CD2 functions:

$$f(x) = \begin{cases} +\frac{1}{2}x^2 & \text{if } x \geq 0, \\ -\frac{1}{2}x^2 & \text{if } x < 0. \end{cases}$$