

Problem Set 3

ECON 5153

For full credit you must show all your work.

1. Prove that, in a finite dimensional space, the intersection of compact sets is compact. Show that the infinite union of compact sets need not be compact.

2. Let A and B be the following sets in \mathbf{R}^2 . Graph their intersection $A \cap B$ and their union $A \cup B$, and indicate which of these sets are convex.

(i) $A = \{(x_1, x_2) : x_1^2 + x_2^2 \leq 4\}$, $B = \{(x_1, x_2) : (x_1 - 1)^2 + x_2^2 \leq 4\}$;

(ii) $A = \{(x_1, x_2) : x_1 \in [0, 1], x_2 \in [0, 1]\}$, $B = \{(x_1, x_2) : (x_1 - 1)^2 + x_2^2 \leq 1\}$.

3. For the Cobb-Douglas production function $f(x_1, x_2) = x_1^{1/3} x_2^{2/3}$. Use second order Taylor expansion to estimate the value of f at $(x_1, x_2) = (1.1, 0.9)$.

4. Consider the function $f(x, y) = xy^2 + x^3y$. In what direction should one move from the point $(4, -2)$ to increase this function most rapidly? In what direction would the function remain the same (locally)? Express your answers as vectors of length 1.

5. Exercise 15.6 on Simon and Blume page 342.

6. Let $f(z_1, z_2)$ be a strictly concave, CD2 production function. Moreover, p, w_1 and w_2 denote the output and input prices respectively. A perfectly competitive firm chooses z_1 and z_2 to maximize profits,

$$\pi = pf(z_1, z_2) - w_1z_1 - w_2z_2.$$

(1) Use the Implicit Function Theorem to find the comparative statics of the input demand functions, $z_1(p, w_1, w_2)$ and $z_2(p, w_1, w_2)$, with respect to the output and input prices.

(2) Under what conditions the following two inequalities, $\frac{\partial z_1}{\partial w_2} \geq 0$ and $\frac{\partial z_2}{\partial w_1} \geq 0$, hold?