

## Problem Set 6

ECON 5153

For full credit you must show all your work.

1. (1) Prove that if  $f$  and  $g$  are functions on  $\mathbf{R}^n$  that are homogeneous of different degrees, then  $f + g$  is not homogeneous;

(2) Which of the following functions are homothetic? Give a reason for each answer.

$$a) 2 \ln x + 3 \ln y, \quad b) xy^2 + xy, \quad c) x^2 y^2 / (xy + 1).$$

2. (1) Check concavity/convexity for the following functions on  $\mathbf{R}^n$ :

$$a) f(x) = 4x^4 + 3e^x - \ln x, \quad b) f(x, y, z) = 3e^x + 5y^4 - \ln z, \quad \text{and}$$

$$c) f(x, y, z) = Ax^a y^b z^c, \quad A, a, b, c > 0.$$

(2) For each of the following functions on  $\mathbf{R}^1$ , determine whether it is quasiconcave, quasiconvex, both or neither:

$$a) \ln x, \quad b) 2x^2 + x - 4, \quad c) 3x^3 - 5x^2 + 7x, \quad d) \sin x.$$

3. (1) Prove that summation of concave functions is still concave. Give an example to show that concavity might not be preserved under monotonic transformations.

(2) Prove that quasiconcavity is preserved under monotonic transformations. Give an example to show that summation of quasiconcave functions may not be quasiconcave.