Price Discrimination in Two-Sided Markets*

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Abstract

We examine the profitability and welfare implications of targeted price discrimination in two-sided markets. The conventional wisdom from one-sided horizontally differentiated markets is that price discrimination hurts the firms and benefits consumers, *prisoners' dilemma*. Moreover, it is well-known that the presence of indirect externalities in two-sided markets can intensify the competition. Despite all these, we show that price discrimination, in a two-sided market, may actually *soften* the competition. Therefore, the implications of price discrimination from one-sided markets may not carry over to two-sided markets. Our analysis also sheds light on the welfare properties of price discrimination in intermediate goods markets, such as Business-to-Business (B2B) markets.

Keywords: Price discrimination; Two-sided markets; Indirect network externalities.
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1 Introduction

The aim of this paper is to study the implications of price discrimination in two-sided markets. Two-sided (or multiple-sided) markets are markets that are organized around intermediaries or "platforms" with two (or multiple) sides who should join a platform in order for successful exchanges (trade) to take place.[1] For example, videogame platforms (e.g., Nintendo, Sony, Microsoft) need to attract both gamers and game developers. Newspapers need to attract advertisers and readers. Credit cards need merchants and users.[2] More formally, a two-sided market is defined as one where the volume of transactions between end-users depends on the structure of the fees and not only on the overall level of fees charged by platforms [Rochet and Tirole (2006)].

The development of the Internet and the rapid growth of sophisticated software tools have enabled firms to collect large amounts of information about consumer preferences, characteristics and purchasing history. Firms can use such information to segment consumers into distinct groups and target each group with different prices and products of different qualities and attributes.

Consider, for example, two rival newspapers who compete to attract readers and advertisers. Advertisers are willing to pay more to place an ad in a given newspaper if that newspaper has more readers and readers are willing to pay more if the newspaper has attracted more ads (e.g., job listings, coupons). Each newspaper has some information about each reader's and each advertiser's willingness to pay relative to the willingness to pay for the rival newspaper. Newspapers can engage in third-degree price discrimination by offering, for instance, low introductory rates to new subscribers and charging low advertising fees to new advertisers. Asplund et. al. (2008), using data on regional Swedish newspapers, find evidence that newspapers target subscribers (readers) of rival newspapers with low introductory offers.

There exists a relatively large literature on oligopolistic third-degree price discrimination in "one-sided" markets, but this paper is among the first ones that examine this problem in the context of a two-sided market.[3] We assume that there are two platforms that are horizontally differentiated. The literature on price discrimination in one-sided markets where products are horizontally differentiated suggests that price discrimination leads to lower prices (and profits) for the firms (prisoners' dilemma), relative to uniform prices.[4] A necessary condition for this is best

[1] See Armstrong (2006a), Caillaud and Jullien (2003) and Rochet and Tirole (2003, 2006).

[2] Most media and advertising markets are two-sided markets, e.g., Anderson and Coate (2005) and Anderson and Gabszewicz (2006). Other examples of two-sided markets include, newspapers, scholarly journals, magazines, shopping malls, dating services and Business-to-Business (B2B) markets. See also the papers cited above for more detailed discussions and examples.

[3] By "one-sided" markets we simply mean markets with no externalities. For a survey of the literature on oligopolistic price discrimination in one-sided markets we refer the reader to Armstrong (2006b) and Stole (2007). Jullien (2008) investigates how price discrimination helps a platform to coordinate the choices of consumers.

[4] See, for example, Thisse and Vives (1988), Shaffer and Zhang (1995), Bester and Petrakis (1996), Chen (1997), Fudenberg and Tirole (2000) and Liu and Serfes (2004).

response asymmetry [Corts (1998)]. Under horizontal differentiation, one firm's strong market is the other firm's weak market and vice versa. When price discrimination is feasible, a firm can charge a low price to the loyal customers of the rival firm, while at the same time it can keep its price high to its own loyal customers. The problem is that the other firm can follow the same strategy, resulting in a very intense competition among the firms for consumers.

Therefore, the conventional wisdom is that in horizontally differentiated markets with no indirect externalities, price discrimination is beneficial for the consumers (at least on average).[5] The advice then given to policymakers and antitrust authorities is that they should not worry much about firms acquiring and using consumer information with the intention to customize prices, because after all firm competition for consumers dissipates profits and transfers most of the surplus to consumers.

Furthermore, it is well-known that the presence of indirect externalities in two-sided markets can intensify the competition, e.g., Armstrong (2006a). Platforms have strong incentives to lower prices in order to sign-up more agents. Therefore, putting together the results from one-sided models with price discrimination and from two-sided models with no price discrimination, one would expect that price discrimination in a two-sided market will generate a very competitive environment with low prices and profits. This is true, but not always. We show that price discrimination may actually *soften* the competition, even in a market with symmetric and horizontally differentiated platforms. The game need not be a prisoners' dilemma. In particular, price discrimination in two-sided markets is possible to increase prices for (almost) all consumers (agents) relative to uniform prices.

Our result has important theoretical and policy implications because it demonstrates that price discrimination is more likely to be anti-competitive in two-sided markets than it is in one-sided markets. More fundamentally, it suggests that two-sided markets can be very different from one-sided markets (see Economides and Tåg (2007), where a similar conclusion, regarding the difference between one-sided and two-sided markets, is reached).

The model we develop consists of two platforms that are horizontally differentiated. There are two groups of agents and each agent is assumed to join only one platform (single-homing).[6] Agents from one group that contemplate joining a given platform care about the number of agents from the other group that will join the same platform. This (indirect) externality is captured by the cross-group externality parameters. Each platform charges lump-sum prices. Under a uniform pricing rule each member of a group that joins a platform pays the same price (across groups the prices of a platform are allowed to differ). Under price discrimination each agent pays a different price (perfect price discrimination). We show that if the cross-group network externalities are strong and/or the

[5] When some kind of firm asymmetry is introduced the game may no longer be a prisoners' dilemma, e.g., Shaffer and Zhang (2002) and Liu and Serfes (2005). Price discrimination benefits the firm with the larger market share.

[6] We relax this assumption in section 4 by allowing agents to make a choice about whether to join only one platform or both (multi-homing).

marginal cost is low (e.g., digital products), then price discrimination increases platform profits and hurts consumer welfare.

The intuition for this result is as follows. Uniform equilibrium prices do depend on the cross-group externality. A stronger externality increases each platform's incentives to cut prices and as a result equilibrium prices fall. Discriminatory prices, on the other hand, are, under certain conditions, *independent* of the cross-group externality. The presence of the indirect externality intensifies competition and discriminatory prices fall. Under the reasonable assumption that prices cannot become negative, each platform, in the symmetric equilibrium, will charge zero prices to the agents that are located closer to the rival platform and to its own agents will charge a premium which *only* depends on the transportation cost. Due to the "limit price" nature of the problem under perfect price discrimination and the assumption of non-negative prices, the feedback effect disappears in equilibrium. Hence, strong externalities imply that uniform prices will fall while discriminatory prices do not change, which further implies that price discrimination in such a case is more profitable. Price flexibility is a curse in one-sided markets, but it can be a blessing in a two-sided market. The result and intuition are similar even when we allow for imperfect price discrimination or when we allow agents on both sides to multi-home.

Moreover, this is the first paper that is concerned with perfect price discrimination in a two-sided market. As we show, the features of the perfect price discrimination equilibrium are qualitatively very different from those in a one-sided market.

Caillaud and Jullien (2003) and Armstrong (2006a) also allow for price discrimination. In Caillaud and Jullien agents in each group are homogeneous and therefore price discrimination means different prices charged to each group of agents, while within each group the price is constant. This is also the meaning of price discrimination in Armstrong (2006a), although he allows for heterogeneous populations of agents. In contrast, we allow the prices within each group to vary. Price discrimination in Armstrong's model can lead to higher or lower prices and profits, but the condition that determines the profitability of price discrimination is qualitatively different from the condition (and intuition) we derive in this paper. In Armstrong the *differences* between: i) the degrees of platform differentiation and ii) the cross-group externalities across groups play an important role. In contrast, in our paper the *levels* matter. As a consequence, if we assume complete symmetry (i.e., same degrees of platform differentiation and same cross-group externalities across groups) then price discrimination always yields the same prices and profits with uniform prices in Armstrong's paper. In our paper, however, this need not be the case.

Our analysis can also apply to intermediate goods markets where price discrimination is more likely to raise antitrust concerns than in final goods markets. Indeed, in the Unites States price discrimination is illegal in intermediate goods markets under the Robinson-Patman act. Each platform in our model can be viewed as a Business-to-Business (B2B) website which matches input

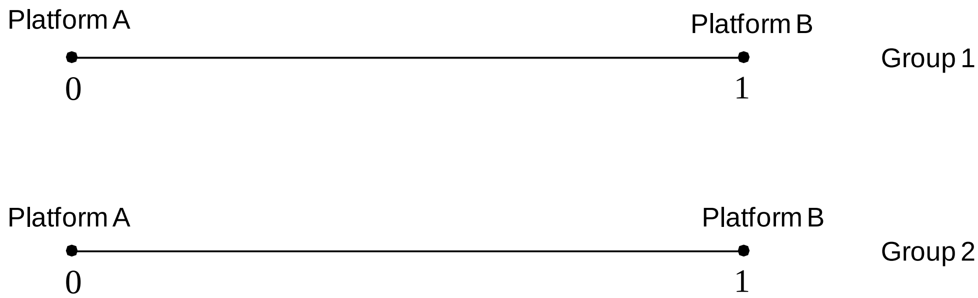

Figure 1: Graphical represenation of the benchmark model

suppliers with producers [e.g., Caillaud and Jullien (2003)]. The Internet facilitates the collection and application of information about the users' preferences and characteristics, see FTC (2000). An interesting question which arises then is whether platforms should be restricted to charge uniform prices. We will return to this interpretation of our model in section 4.

The rest of the paper is organized as follows. In section 2, we present the benchmark model. In section 3, we perform the analysis. In section 4, we extend the benchmark model to allow agents to multi-home. We conclude in section 5.

2 The description of the benchmark model

There are two groups of agents $\ell = 1, 2$ and two horizontally differentiated platforms $k = A, B$.[7] We will denote the "other" group of agents by m. We capture platform differentiation as follows. There is a continuum of agents of group ℓ that is distributed on the $[0, 1]$ interval according to the distribution function $F_\ell(\cdot)$ with density f_ℓ. The distributions are independent across the two groups of agents and symmetric about $\frac{1}{2}$, i.e., $F_\ell\left(\frac{1}{2}\right) = \frac{1}{2}$ and $f_\ell(x) = f_\ell(1-x)$. The two platforms are located at the two end points of each interval, with platform A located at 0 and platform B located at 1, see figure 1.

The common per-unit transportation cost of both groups is denoted by $t > 0$. We assume that each agent joins only one platform (single-homing).[8] Each member of a group who joins a given platform cares about the number of members from the other group who join the same platform. Denote by $n_{\ell k}$ the number of participants from group ℓ that platform k attracts. The maximum willingness to pay for a member of group ℓ if he joins platform k is given by $V + \alpha_\ell n_{mk}$, where V is a stand-alone benefit each agent receives independent of the number of participants from the other group on platform k. The parameter $\alpha_\ell > 0$ measures the cross-group externality for group

[7] Our benchmark model follows closely the model in Armstrong (2006a).

[8] We relax this assumption by allowing agents to multi-home in Section 4.

ℓ participants. The indirect utility of an agent from group ℓ who is located at point $x \in [0,1]$ is given by,

$$U_\ell = \begin{cases} V + \alpha_\ell n_{mA}^e - tx - p_{\ell A}(x), & \text{if he joins platform } A \\ V + \alpha_\ell n_{mB}^e - t(1-x) - p_{\ell B}(x), & \text{if he joins platform } B \end{cases} \quad (1)$$

where $p_{\ell k}(x)$ is platform k's lump-sum charge to a group ℓ participant who is located at point x and n_{mk}^e denotes the expectations agents from group ℓ have about how many agents from group m will join platform k. Under a uniform pricing rule prices are constant across all agents in the same group (prices are allowed to vary across groups), while under discriminatory pricing the price each agent pays depends on his preferences (location). We assume that V is high enough which ensures that the market is covered. Platforms have constant marginal cost $c \geq 0$. We assume that prices cannot be negative.[9]

The timing of the game is as follows. In stage 1, the two platforms make, simultaneously, their pricing decisions. In stage 2, the agents decide which platform to join.

3 Analysis

We study two different price regimes. In the first regime each platform charges uniform prices to the agents of each group. In the second regime each platform can price discriminate perfectly the agents of each group. Then, we compare prices and profits between these two price regimes. We assume that each agent has rational expectations about how many agents from the other group will join each platform. Each agent observes all prices before he decides which platform to join (public prices), [e.g., Caillaud and Jullien (2003) and Armstrong (2006a)].[10]

3.1 No price discrimination (uniform prices within each group of agents)

The next Proposition summarizes the main result when platforms cannot price discriminate within each group of agents with a general distribution of preferences.[11]

Proposition 1 ***(Uniform prices)*** *If a symmetric equilibrium exists, then it is given by:*

[9] In most cases negative prices are unrealistic and create perverse incentives [see also Armstrong (2006a) for a discussion on this issue]. If negative prices were allowed, viewers could, for example, subscribe to a TV channel, never watched it and got paid for that. More generally, agents will have incentives, when they get paid, to make multiple purchases and it will be difficult for the platforms to prevent this from happening.

[10] We consider private prices in section 3.2.2.

[11] We were not able to come up with clean conditions on the distribution functions that would ensure the strict concavity (or quasi-concavity) of the objective functions. For instance, the monotone hazard rate property is not enough. When the distribution is uniform (see below), then the profit functions are strictly concave provided that $2t > (\alpha_1 + \alpha_2)$. When this condition holds, then a symmetric sharing equilibrium exists. Otherwise, one platform may corner the entire market.

$$p_{1A}^* = p_{1B}^* = \frac{t - \alpha_2 f_1\left(\frac{1}{2}\right)}{f_1\left(\frac{1}{2}\right)} + c \textit{ and } p_{2A}^* = p_{2B}^* = \frac{t - \alpha_1 f_2\left(\frac{1}{2}\right)}{f_2\left(\frac{1}{2}\right)} + c. \tag{2}$$

The equilibrium profits are,

$$\pi_A = \pi_B = \frac{t - \alpha_2 f_1\left(\frac{1}{2}\right)}{2f_1\left(\frac{1}{2}\right)} + \frac{t - \alpha_1 f_2\left(\frac{1}{2}\right)}{2f_2\left(\frac{1}{2}\right)}. \tag{3}$$

Proof. See Appendix. ■

Each platform serves one half of the members of each group. The equilibrium prices depend positively on the differentiation parameter t, negatively on the strength of the cross-group externality α_ℓ and negatively on the number of marginal agents $f_\ell\left(\frac{1}{2}\right)$. When the externality for group ℓ is stronger platforms offer lower prices to the members of group m, all else equal. Potentially, prices can be negative, but we do not allow for this possibility.

3.1.1 Uniform distribution

If we assume that the distribution is uniform ($f_1(x) = f_2(x) = 1$), then the equilibrium prices and profits are.[12]

$$p_{1A}^* = p_{1B}^* = t - \alpha_2 + c,\ p_{2A}^* = p_{2B}^* = t - \alpha_1 + c \tag{4}$$

and

$$\pi_A = \pi_B = t - \frac{(\alpha_1 + \alpha_2)}{2}. \tag{5}$$

3.2 Perfect price discrimination

Now we assume that platforms can price discriminate perfectly and prices cannot be negative. Agent utility is given by (1) and platforms compete on an agent-by-agent basis. Each agent receives a targeted offer. Platform A's own territory is the $[0, 1/2]$ interval and platform B's own territory is the $[1/2, 1]$ interval. The next Proposition summarizes the equilibrium. We focus on symmetric equilibria.

Proposition 2 ***(Perfect price discrimination)*** *There are two distinct cases:*[13]

[12] The existence of a symmetric equilibrium is guaranteed if $2t > \alpha_1 + \alpha_2$, see Armstrong (2006).

[13] There is also a third case, case (iii), that falls in between the two cases presented in this Proposition, i.e., $\min\{\alpha_1, \alpha_2\} < c < \max\{2\alpha_1, 2\alpha_2\}$. Relative to case (i) some prices in the rival platform's territory become negative. In equilibrium, these prices are replaced by zero and the prices charged by a platform in its own territory are equal to the transportation cost premium, as in (9). For the prices that are not negative the equilibrium is the same as in (7). We do not pursue this case further, as it does not add anything to our understanding of the problem.

(i) High marginal cost and/or low cross group externality, $c \geq \max\{2\alpha_1, 2\alpha_2\}$. *Suppose that*

$$t > \max\left\{(\alpha_1 + 2\alpha_2) f_2(x), (2\alpha_1 + \alpha_2) f_1(x)\right\}. \tag{6}$$

The equilibrium prices are

$$\begin{aligned} p^*_{\ell A} &= t(1-2x) + c - 2\alpha_m(1 - F_m(x)) \text{ and } p^*_{\ell B} = c - 2\alpha_m F_m(1-x), \text{ for } x \leq \frac{1}{2} \text{ and} \\ p^*_{\ell A} &= c - 2\alpha_m F_m(x) \text{ and } p^*_{\ell B} = t(2x-1) + c - 2\alpha_m F_m(x), \text{ for } x \geq \frac{1}{2}. \end{aligned} \tag{7}$$

(ii) Low marginal cost and/or high cross-group externality, $c \leq \min\{\alpha_1, \alpha_2\}$. *All prices in the rival platform's own territory are negative. Since negative prices are not allowed, they are replaced by zero. Suppose that*

$$t > c + \max\{\alpha_1, \alpha_2\} \text{ and } t < (\alpha_1 + \alpha_2)\min\{f_1(x), f_2(x)\}. \tag{8}$$

The equilibrium prices are

$$\begin{aligned} p^*_{\ell A}(x) &= t(1-2x) \text{ and } p^*_{\ell B}(x) = 0, \text{ for } x \leq \frac{1}{2} \text{ and} \\ p^*_{\ell A}(x) &= 0 \text{ and } p^*_{\ell B}(x) = t(2x-1), \text{ for } x \geq \frac{1}{2}. \end{aligned} \tag{9}$$

Proof. See Appendix. ■

There are two main differences between the perfect price discrimination equilibrium in a one-sided market and in a two-sided market. These differences manifest themselves in case (i) in the above Proposition. In a one-sided market a firm charges prices equal to marginal cost in the rival firm's own territory and in its own territory prices exceed marginal cost. This is not the case in a two-sided market. First, in a two-sided market, the prices a platform charges in the rival's market (and some of the prices in its own market) are below marginal cost. Second, they are not constant. Notice, for example, from case (i) in Proposition 2, that $p^*_{\ell A} < c$ and decreases in x when $x \geq 1/2$. In addition, $p^*_{\ell A} < c$ for $x \leq 1/2$ but close to 1/2. The intuition behind these two differences is as follows. A platform may be willing to charge a price below marginal cost to an agent, because an extra agent from one group is valuable to *all* the agents in the other group. This is a direct consequence of the network externalities and a platform's ability to customize prices. The second difference we mentioned above is more subtle. For an equilibrium to exist, a platform should be losing increasingly more money as it tries to poach the rival platform's agents. The reason for this is that the benefit from signing up one more agent increases with the market share of a platform, i.e., the more agents a platform has already signed up the higher the benefit of an additional agent. Thus, the price in the rival platform's territory should be decreasing.

Similar to perfect price discrimination in one-sided markets, each agent is indifferent between the two platforms. The platform that is located closer to an agent charges a transportation cost premium over the rival's price for that agent.

When the marginal cost is high, as in case (i) in Proposition 2, the equilibrium prices depend on the cross-group externalities. Actually, the presence of cross-group network externalities intensifies the competition further under perfect price discrimination relative to uniform pricing. Under uniform pricing, prices are reduced by α_m, see (2), whereas under perfect price discrimination the price decline, according to (7), ranges from $2\alpha_m$ for the agents located at the extremes to α_m for the agents located at $x = 1/2$.

Nevertheless, the above comparison fails when the marginal cost is low and/or the network externalities are strong as in case (ii) in Proposition 2. Precisely because perfect price discrimination in a two-sided market generates a very competitive environment, prices fall so low that they reach the natural floor of zero. In this case, the network externality is priced-out of the equilibrium prices, see (9). Case (iii) falls in between the other two cases.

Therefore, when the marginal cost is high and/or the externalities are low, perfect price discrimination unambiguously yields lower profits than uniform pricing. This result is in line with the comparison in one-sided markets, e.g., Thisse and Vives (1988). The difference arises when the marginal cost c is low (e.g., digital products) and/or α is high, $c \le \min\{\alpha_1, \alpha_2\}$, case (ii) in Proposition 2. (More on this comparison in the next section). After the next subsection, in the remaining of the paper we assume that we are in case (ii).

3.2.1 Uniform distribution

Suppose that the distribution is uniform, i.e., $f_1(x) = f_2(x) = 1$. The equilibrium profits when all prices are positive, as in case (i) in Proposition 2, are

$$\begin{aligned} \pi_A &= \int_0^{1/2} [t(1-2x) - 2\alpha_2(1-x)]\, dx + \int_0^{1/2} [t(1-2x) - 2\alpha_1(1-x)]\, dx \\ &= \frac{2t - 3(\alpha_1 + \alpha_2)}{4} = \pi_B. \end{aligned} \tag{10}$$

Condition (6) guarantees that the above profits are positive.

On the other hand, when all unconstrained prices in the rival platform's territory are negative (in which case they are replaced by zero), as in case (ii) in Proposition 2, the equilibrium profits are

$$\pi_A = \int_0^{1/2} t(1-2x)dx + \int_0^{1/2} t(1-2x)dx = \frac{t}{2} - c = \pi_B. \tag{11}$$

Condition (8) guarantees that the above profits are positive. Notice that the network externalities affect the equilibrium profits given by (10), while they do not affect the equilibrium profits given by (11).

3.2.2 Prices are private

So far we have assumed that each agent observes all prices before he chooses which platform to join. This assumption is reasonable in the case platforms compete via uniform prices, but it may not be realistic in the price discrimination case. This is because each agent receives a customized price and also there are too many prices. It may then seem more reasonable to assume that prices are private, in the sense that each agent only observes his own price. Given the cross-group externalities, beliefs are important in this case. What is an agent's belief about the offers made to other agents if he receives an out-of equilibrium offer? If beliefs are passive [e.g., McAfee and Schwartz (1994)] and price offers are secret, then equilibrium prices do not depend on the cross-group externalities. In particular, the equilibrium prices are,

$$\begin{aligned} p_{\ell A} &= t\,(1-2x)+c \text{ and } p_{\ell B}=c, \text{ for } x\leq \frac{1}{2} \text{ and} \\ p_{\ell A} &= c \text{ and } p_{\ell B}=t\,(2x-1)+c, \text{ for } x\geq \frac{1}{2}. \end{aligned}$$

To see this, suppose that a platform raises unilaterally its prices to a group of agents in its territory. Each agent, however, continues to believe that market shares will not change and *given* that agents are indifferent, in equilibrium, between the two platforms they will *all* switch to the rival platform. Hence, such a deviation is unprofitable. Price cuts would also be unprofitable because a reduction in price to an agent (or a group of agents) will not lead to higher market share. Thus, when prices are private equilibrium discriminatory prices do not depend on the externality parameter, as in the case of public prices and low marginal cost, i.e., case (ii) of Proposition 2. Given that no new insights are derived under the assumption of private prices, in the rest of the paper we go back to assuming that prices are public.

3.3 Price and profit comparison

We compare the equilibrium uniform prices given by (4) with the discriminatory prices given by (9). So, we assume that $c\leq \min\{\alpha_1,\,\alpha_2\}$.[14] We will exploit the fact that uniform (non-discriminatory) prices depend on the cross-group externality parameters, while discriminatory prices do not. Discriminatory prices, as it is the case in one-sided markets that are characterized by horizontal differentiation, are decreasing in the degree of agent loyalty to a platform. Agents located very close to one or the other platform pay higher prices than those located in the middle. The highest price is t and the lowest is 0. If we compare these prices with the no discriminatory prices, $\frac{t-\alpha_\ell f_m\left(\frac{1}{2}\right)}{2f_m\left(\frac{1}{2}\right)}+c$, we will see that it is possible that nearly all agents pay higher prices under price discrimination if c is small and α_ℓ is high enough. It then becomes obvious that there exists a threshold for the

[14] As we pointed out before, when the marginal cost is high and/or the network externalities are weak, perfect price discrimination yields lower prices and profits relative to uniform pricing, a result that is consistent with the prediction from one-sided models.

cross-group externality parameters above which perfect price discrimination benefits the platforms (relative to uniform pricing). The next Proposition summarizes the profit comparison.

Proposition 3 ***(Profit comparison)*** *If the marginal cost is low enough and/or the cross-group externalities are strong enough, then perfect price discrimination is more profitable than uniform pricing.*

When the distribution is uniform, equilibrium profits increase with price discrimination if and only if $t/2 - c > t - (\alpha_1 + \alpha_2)/2$, (we compare (11) with (5)). This is the case if and only if,

$$t < (\alpha_1 + \alpha_2 - 2c). \tag{12}$$

Furthermore, the necessary and sufficient condition for a market sharing equilibrium under uniform pricing to exist is $2t > (\alpha_1 + \alpha_2)$. Under perfect discrimination the condition we need, from Proposition 2 condition (8), is

$$t > c + \max\{\alpha_1, \alpha_2\} \text{ and } t < (\alpha_1 + \alpha_2).$$

Therefore, if c is low and/or the externalities are strong, i.e., $c \leq \min\{\alpha_1, \alpha_2\}$, there exists a range of parameters such that uniform equilibrium prices are given by (4), discriminatory prices are given by (9) and price discrimination leads to higher profits. For example, if $\alpha_1 = \alpha_2 = \alpha$, for the above assertion to be true we need t to be in the nonempty interval $[c + \alpha, 2(\alpha - c)]$.

The main idea behind the price and profit comparison is that the externalities are priced in the equilibrium uniform prices, but not in the discriminatory prices. When platforms lower their prices below marginal cost due to the externality, the natural price floor of zero is reached before the price goes down all the way to $c - 2\alpha_m(1 - F_m(x))$, for $x \leq 1/2$, (which is negative). So, strong enough externalities combined with low marginal cost (i.e., $c \leq \min\{\alpha_1, \alpha_2\}$) make perfect price discrimination more profitable (relative to uniform prices). This intuition does not rely on specific modeling assumptions and it is likely to hold in more general models.

Armstrong (2006a) compares price discrimination with uniform prices. In his model price discrimination is defined as the uniform pricing rule in our model, i.e., when platforms charge each group a different price. A uniform pricing rule in Armstrong's model is when a platform charges both groups the same price. Armstrong shows that price discrimination is profitable if and only if,

$$(t_1 - t_2)^2 > (\alpha_1 - \alpha_2)^2. \tag{13}$$

Our condition (12) for a profitable price discrimination is qualitatively different from Armstrong's condition (13). In our case the levels matter, whereas in Armstrong's case the differences matter more. If the transportation parameters are the same across groups ($t_1 = t_2$), as it is the case in our model, then price discrimination is never profitable in Armstrong's model, while it may be in our model.

Finally, as it is well-known [e.g., Thisse and Vives (1988)], price discrimination, in one-sided markets when preferences are uniformly distributed and platforms are symmetric, *always* leads to a prisoners' dilemma. The profits under perfect price discrimination are $t/4$, while under a uniform pricing rule they are $t/2$ (with marginal cost c equal to zero). In contrast, in two-sided markets, when (12) is satisfied perfect price discrimination yields higher profits than uniform prices.

3.4 Intermediate goods markets

When the market is an intermediate goods markets, e.g., B2B market, then our result implies that price discrimination will lead to higher input prices if and only if platforms have detailed information about the preferences of the participants and the marginal cost is low (and/or externalities are strong). To arrive at this result, we can assume that each firm is seeking to buy only one unit of the input and each input seller sells only one unit. The platforms facilitate the matching process between the two sides [as in Caillaud and Jullien (2003)]. Let's assume that platforms have very good (perfect) information about the agents. If platforms are allowed to customize their prices then firms end up paying higher prices for the right to trade a unit of the input. Now if we assume that the prices the participants pay to join a platform do not affect the bargaining process between an input supplier and a firm that will ensue once a matching takes place, then a higher price charged by a platform will lead to a higher overall price a firm will have to pay in order to acquire its input. If firms can pass part of this extra cost on to consumers, then price discrimination is anti-competitive. However, the reverse is true if platforms do not possess very detailed information about the participants (as in the uniform price case). In this case the cost of acquiring the input is reduced due to price discrimination.

4 Agents are allowed to multi-home

We would like to investigate the robustness of our results to an extension to the benchmark model. We allow agents to multi-home. Prices are public. Our main results do not change qualitatively. One difference is that equilibrium discriminatory prices, when agents multi-home, depend positively (on average) on the cross-group externality. In order to cut down on the number of different cases that we will have to examine, we assume that the marginal cost c is zero. This allows us to maintain some consistency with the case of single-homing, where we mainly focused on the case of low marginal cost. We maintain the assumption that the distribution is uniform and we set $\alpha_1 = \alpha_2 = \alpha$. The indirect utility of an agent from group ℓ who is located at point $x \in [0, 1]$ is given by,[15]

$$U_\ell = \begin{cases} V + \alpha n^e_{mA} - tx - p_{\ell A}(x), & \text{if he joins platform } A \\ V + \alpha n^e_{mB} - t(1-x) - p_{\ell B}(x), & \text{if he joins platform } B \\ V + \theta + \alpha - t - p_{\ell A}(x) - p_{\ell B}(x) & \text{if he joins both platforms.} \end{cases} \quad (14)$$

The incremental maximum willingness to pay of an agent from group ℓ who chooses to multi-home by joining platform k is given by $\theta + \alpha(1 - n^e_{mk})$. The first effect (*product variety* effect) is captured by the parameter θ, where $\theta \in [0, V]$, and the second effect (*indirect externality* effect) is given by the term $\alpha(1 - n^e_{mk})$. For example, the utility of an agent who chooses to read a second newspaper increases because he gets to see more classified advertisements (indirect externality effect), but also because the second newspaper covers different issues than the first one (product variety effect). Or, a second credit card allows the holder to have transactions with more merchants (indirect externality effect), but also increases his credit limit (product variety effect). More generally, agent utility can increase, when he joins a second platform, independent of the indirect externality effect, because platforms are differentiated and agents value "variety."

The disutility of the agent who chooses to multi-home also increases and this is captured by the parameter t ($t = tx + t(1-x)$). We assume that the total transportation cost is additive. Agents choose the option that gives them the highest indirect utility. We maintain the assumption that $t > \alpha$.

In general, there are three possible type of equilibria: i) single-homing, ii) partial multi-homing and iii) complete multi-homing. In the first equilibrium, no agent multi-homes, in the second one a fraction of the agents multi-homes and in the third one all agents multi-home. Due to symmetry the outcomes are the same across the two groups of agents. We will focus on the second type of equilibrium. Figure 2 is consistent with the partial multi-homing equilibrium and depicts the indirect utilities when prices within each group are uniform.

There are two marginal agents in group ℓ, where $\ell = 1, 2$, located at $x_{\ell L}$ and $x_{\ell R}$ respectively (the subscript L stands for left and the subscript R stands for right). $x_{\ell L}$ is indifferent between joining platform A only and joining both platforms, whereas $x_{\ell R}$ is indifferent between joining both platforms and joining B only. Agents in $[0, x_{\ell L}]$ and in $[x_{\ell R}, 1]$ single-home and in $[x_{\ell L}, x_{\ell R}]$ they multi-home.

4.1 Uniform prices (UP)

We assume that prices are constant within each group of agents. Let $p_{\ell A}$ and $p_{\ell B}$ denote platform A and B's price in group $\ell = 1, 2$ respectively. The marginal agents in group $\ell = 1, 2$ are defined

[15] This utility specification has also been used in Kim and Serfes (2006) in a one-sided framework.

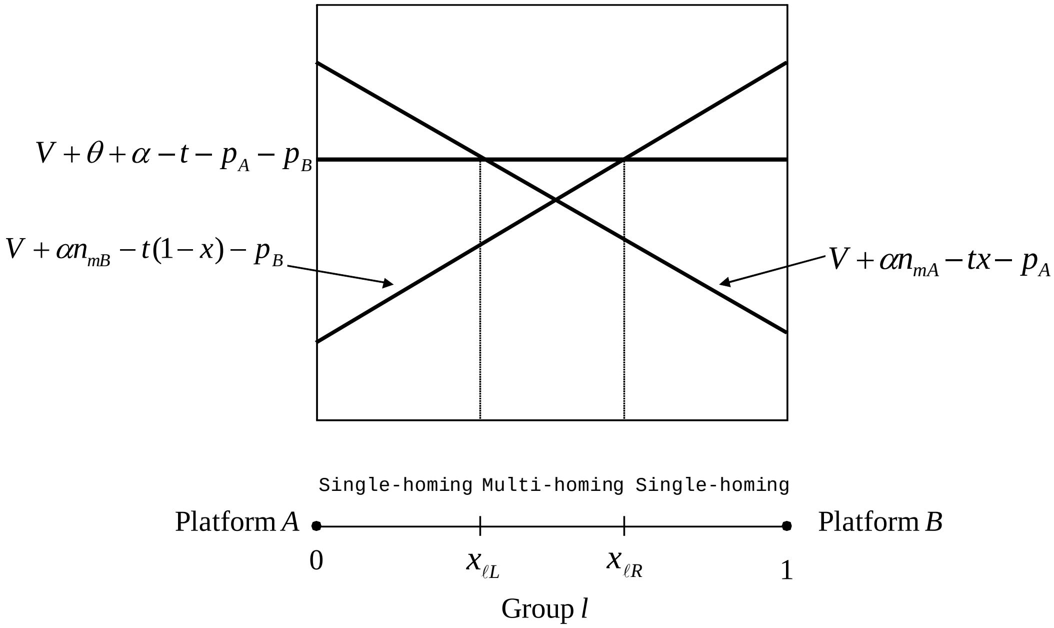

Figure 2: Indirect utilities: Partial multi-homing

by,

$$x_{\ell L}^{UP}: \quad V+\alpha n_{mA}^{e}-tx-p_{\ell A}=V+\theta+\alpha-t-p_{\ell A}-p_{\ell B},$$

$$x_{\ell R}^{UP}: \quad V+\alpha n_{mB}^{e}-t\,(1-x)-p_{\ell B}=V+\theta+\alpha-t-p_{\ell A}-p_{\ell B},$$

with $n_{mA}^{e}=x_{mR}^{UP}$, and $n_{mB}^{e}=1-x_{mL}^{UP}$, $m=1,2$. From these equations, we can obtain the marginal agents as follows,

$$x_{1L}^{UP}=\frac{(-t^2+\alpha p_{2A}-\alpha\theta+t\alpha-tp_{1B}+t\theta)}{(\alpha-t)(t+\alpha)}$$

$$x_{1R}^{UP}=\frac{(\alpha\theta-t\alpha-t\theta+tp_{1A}+\alpha^2-\alpha p_{2B})}{(\alpha-t)(t+\alpha)}$$

$$x_{2L}^{UP}=\frac{(t\theta+t\alpha-\alpha\theta+\alpha p_{1A}-tp_{2B}-t^2)}{(\alpha-t)(t+\alpha)}$$

$$x_{2R}^{UP}=\frac{(-t\theta-t\alpha+\alpha\theta+tp_{2A}-\alpha p_{1B}+\alpha^2)}{(\alpha-t)(t+\alpha)}.$$

Then, the platforms' problems are,

$$\max_{p_{1A},\,p_{2A}} \quad \pi_A=p_{1A}x_{1R}^{UP}+p_{2A}x_{2R}^{UP},$$

$$\max_{p_{1B},\,p_{2B}} \quad \pi_B=p_{1B}(1-x_{1L}^{UP})+p_{2B}(1-x_{2L}^{UP}).$$

Solving the first order conditions, we can obtain the candidate equilibrium prices and profits as,

$$p_{1A}^{*}=p_{1B}^{*}=p_{2A}^{*}=p_{2B}^{*}=\frac{(t-\alpha)(\alpha+\theta)}{2t-\alpha},$$
$$\pi_A=\pi_B=\frac{2t(t-\alpha)(\alpha+\theta)^2}{(2t-\alpha)^2(\alpha+t)}. \tag{15}$$

In this candidate equilibrium,

$$0<x_{\ell L}^{UP}\equiv\frac{2t^2-\alpha^2-t\theta}{(t+\alpha)(2t-\alpha)}<x_{\ell R}^{UP}\equiv\frac{t(\alpha+\theta)}{(t+\alpha)(2t-\alpha)}<1,$$

if $\theta\in\left[\theta_1\equiv\frac{6t(t-\alpha)}{(5t-\alpha)},\theta_2\equiv\frac{2t^2-\alpha^2}{t}\right]$.[16] That is, only the agents strictly in the middle multi-home (see figure 2), and we have partial multi-homing.

What is worth observing is that the equilibrium profits (15) are decreasing in α when α exceeds a given threshold (its specific value is omitted) and approach zero as α tends to t. For low values of α equilibrium profits can be increasing in the externality parameter. There are two opposing effects present as the indirect externality α increases. First, as in the single-homing case, incentives for unilateral price cuts increase. Second, agents are willing to pay more to join a second platform, which gives platforms incentives to raise their prices. This second effect arises because of the multi-homing assumption.[17] When α is high, the first effect is more dominant, while for low α the second effect may be more dominant.

4.2 Perfect price discrimination (PD)

Now, we assume that platforms can identify the exact location of each agent. We first need to identify the locations of the marginal agents. The agent who is located at $x_{\ell L}^{PD}$ is indifferent between joining from platform A only and joining from both platforms. This implies that this agent obtains zero utility from joining B, when platform B is charging zero price. Then, x_{1L}^{PD} is defined by,

$$x_{1L}^{PD}:\quad V+\alpha n_{2A}^{e}-tx-p_{1A}=V+\theta+\alpha-t-(p_{1A}+0)\Rightarrow\theta+\alpha(1-n_{2A}^{e})=t(1-x).$$

Similarly the other marginal agents are defined by,

$$x_{1R}^{PD}:\quad V+\alpha n_{2B}^{e}-t(1-x)-p_{1B}=V+\theta+\alpha-t-(0+p_{1B})\Rightarrow\theta+\alpha(1-n_{2B}^{e})=tx.$$

[16] The restrictions on the parameter θ can be understood as follows: i) $\theta<\theta_2$ guarantees that $x_{\ell L}^{UP}>0$ and $x_{\ell R}^{UP}<1$ and ii) $\theta>\frac{2t^2-\alpha^2-\alpha t}{2t}$ guarantees that $x_{\ell L}^{UP}<x_{\ell R}^{UP}$. Moreover, $\theta\geq\theta_1\geq\frac{2t^2-\alpha^2-\alpha t}{2t}$, ensures that no global unilateral deviation is profitable. Profit functions are strictly concave when partial multi-homing is assumed, but a platform can deviate in a way that partial multi-homing vanishes. We want our solutions to the first order conditions to be immune from such a deviation. So, if θ falls in the above interval, then the candidate equilibrium becomes an equilibrium.

[17] Multi-homing changes the platforms' incentives to unilaterally change prices and therefore it generates new insights. Choi (2006), for example, shows that, when multi-homing is allowed in two-sided markets, tying can be welfare-enhancing because it induces more consumers to multi-home.

$$x_{2L}^{PD}: \quad V+\alpha n_{1A}^e - tx - p_{2A} = V+\theta+\alpha-t-(p_{2A}+0) \Rightarrow \theta+\alpha(1-n_{1A}^e) = t(1-x).$$

$$x_{2R}^{PD}: \quad V+\alpha n_{1B}^e - t(1-x) - p_{2B} = V+\theta+\alpha-t-(0+p_{2B}) \Rightarrow \theta+\alpha(1-n_{1B}^e) = tx.$$

Also, $n_{mA}^e = x_{mR}^{PD}$, and $n_{mB}^e = 1 - x_{mL}^{PD}$, $m = 1,2$. From these equations, we can obtain the locations of the marginal agents, $\ell = 1,2$,

$$x_{\ell L}^{PD} = \frac{t-\theta}{t+\alpha} \text{ and } x_{\ell R}^{PD} = \frac{\theta+\alpha}{t+\alpha}.$$

Then, $n_{mA} = n_{mB} = \frac{\theta+\alpha}{t+\alpha}$. When $x \in (x_{\ell L}^{PD}, x_{\ell R}^{PD})$, both platforms set prices so that all agents are indifferent between joining both platforms or that platform alone,

$$V+\alpha n_{mA}^e - tx - p_{\ell A}(x) = V+\alpha n_{mB}^e - t(1-x) - p_{\ell B}(x) = V+\theta+\alpha-t-p_{\ell A}(x) - p_{\ell B}(x).$$

From this equation, we can solve for $p_{\ell A}(x)$ and $p_{\ell B}(x)$. When $\theta \in \left[\frac{t-\alpha}{2}, t\right]$ we have $0 \le x_{\ell L}^{PD} \le x_{\ell R}^{PD} \le 1$, and partial multi-homing is an equilibrium. The equilibrium prices and profits are given by,

$$p_{\ell A}^*(x) = \begin{cases} \frac{t(\theta+\alpha-x(t+\alpha))}{(t+\alpha)}, & \text{for } x \in \left[x_{\ell L}^{PD} \equiv \frac{t-\theta}{t+\alpha}, x_{\ell R}^{PD} \equiv \frac{\theta+\alpha}{t+\alpha}\right] \\ t(1-2x), & \text{for } x \le x_{\ell L}^{PD} \equiv \frac{t-\theta}{t+\alpha} \\ 0, & \text{for } x \ge x_{\ell R}^{PD} \equiv \frac{\theta+\alpha}{t+\alpha} \end{cases} \tag{16}$$

and

$$p_{\ell B}^*(x) = \begin{cases} \frac{t(\theta-t+x(t+\alpha))}{(t+\alpha)}, & \text{for } x \in \left[x_{\ell L}^{PD} \equiv \frac{t-\theta}{t+\alpha}, x_{\ell R}^{PD} \equiv \frac{\theta+\alpha}{t+\alpha}\right] \\ t(2x-1), & \text{for } x \ge x_{\ell R}^{PD} \equiv \frac{\theta+\alpha}{t+\alpha} \\ 0, & \text{for } x \le x_{\ell L}^{PD} \equiv \frac{t-\theta}{t+\alpha}. \end{cases}$$

$$\pi_A = \pi_B = \frac{t\left(t^2 - 2t\theta + 2\theta^2 + 2\alpha\theta + \alpha^2\right)}{(t+\alpha)^2}. \tag{17}$$

The solid lines in figure 3 depict the equilibrium prices as given by (16). (The dashed lines on the same figure depict the equilibrium prices under perfect discrimination when multi-homing is not allowed, as given by (9)). The price functions under multi-homing exhibit two kinks, one at $x_{\ell L}^{PD}$ and the other at $x_{\ell R}^{PD}$. The agents that multi-home are located in the interval $\left[x_{\ell L}^{PD}, x_{\ell R}^{PD}\right]$. The agents in $\left[0, x_{\ell L}^{PD}\right]$ join platform A exclusively and the agents in $\left[x_{\ell R}^{PD}, 1\right]$ join platform B exclusively. The differences between when multi-homing is not allowed and when it is are the following: i) when multi-homing is allowed platforms make more sales (i.e., $x_{\ell L}^{PD} < \frac{1}{2}$ and $x_{\ell R}^{PD} > \frac{1}{2}$) and ii) equilibrium prices are (weakly) higher when agents are allowed to multi-home. In particular, the prices are the same between the two regimes for the agents who join one platform exclusively, but higher when multi-homing is allowed for the agents who join both platforms. This is because

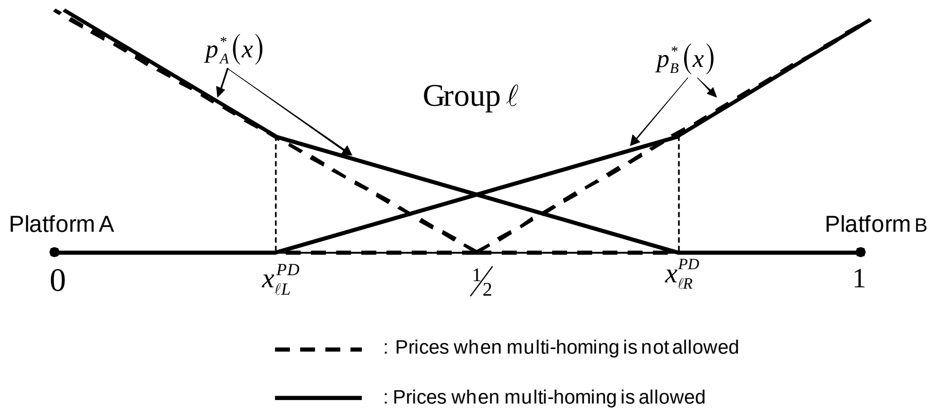

Figure 3: Price comparison under perfect discrimination between when multi-homing is allowed and when it is not

under multi-homing each agent is indifferent between joining one platform exclusively and joining both platforms, which softens price competition.[18]

It can be readily verified that equilibrium prices (for the agents who multi-home) and profits increase with α. When some agents multi-home equilibrium prices are affected by the cross-group externalities. The reason is that equilibrium prices keep the agents who multi-home indifferent between joining one and two platforms. In other words, platforms in equilibrium extract all the incremental surplus from the agents who choose to multi-home. Hence, the externalities are priced in the discriminatory equilibrium prices. (Recall that when multi-homing is not allowed, perfect discriminatory prices are free of the externality parameters, when $c < \min\{\alpha_1, \alpha_2\}$ (case (ii) of Proposition 2), which holds here because we have assumed that $c = 0$). As the indirect externality increases the incremental benefit from joining a second platform also increases. This allows platforms to sustain higher equilibrium discriminatory prices as a function of α. On the other hand, as expected, the prices for the agents who single-home are not affected by α.

4.3 Comparing uniform pricing and perfect PD

4.3.1 Price and profit comparison

In this comparison, for brevity, we focus implicitly on the parameter range that is common between the two parameter ranges for which (15) and (17) constitute an equilibrium. By comparing (15) with (17), it can be shown that price discrimination is always more profitable.

[18] This can be better seen by observing that in the multi-homing region the prices are falling slower (slope is equal to $-t$), as we move in the middle of the intervals, than when multi-homing is not allowed (slope is equal to $-2t$). In the latter case a unilateral price cut induces agents to switch platforms (*business stealing*), whereas in the former case a similar price cut results in more agents joining both platforms (*demand creation*).

This sharp prediction is very likely to be model specific. However, we believe that the effects we have identified at the end of each of the previous two subsections are likely to hold in more general models. These effects yield the following predictions as the indirect externality α increases: i) uniform prices decrease (after the externality exceeds a given threshold) and ii) discriminatory prices increase. Hence, price discrimination should yield higher profits than uniform pricing at least when these externalities are strong enough. This result echoes the prediction from our benchmark model where multi-homing is not allowed.

4.3.2 Social welfare

Due to multi-homing, aggregate demand is elastic, so social welfare comparisons are meaningful. The equilibrium under perfect price discrimination is efficient. This can be seen as follows. Given symmetry and the fact that each agent who single-homes joins the closest platform, what matters for efficiency is total output, i.e., the number of agents who multi-home. First, note that, in the partial multi-homing case, each platform is a local monopoly, because platforms do not compete head-on for agents. Second, each platform extracts each agent's entire incremental surplus from multi-homing (i.e., from joining a second platform). Therefore, the private benefit is aligned with the social benefit, which implies that social surplus is maximized under perfect price discrimination. If the equilibrium under uniform prices differs, then we can conclude that the uniform price equilibrium is inefficient. This is indeed the case. Comparing $x_{\ell R}^{UP}$ and $x_{\ell R}^{PD}$, we can easily find that,

$$x_{\ell R}^{UP} < x_{\ell R}^{PD},$$

since $t > \alpha$.

By symmetry, we can show that,

$$x_{\ell L}^{UP} < x_{\ell L}^{PD}.$$

This implies that more agents multi-home under perfect price discrimination than under uniform pricing. We can conclude by stating that perfect price discrimination is efficient, while uniform prices result in an inefficient equilibrium (less output than the first-best).

5 Conclusion

We examine the issue of price discrimination in two-sided markets. We assume that there are two symmetric horizontally differentiated platforms and two groups of agents. Agents from both groups must join a platform for successful trades to take place. Platforms possess information about the agents' brand preferences which can be used to customize prices. Our main result indicates that when the indirect externality effect is strong perfect price discrimination yields higher profits to the platforms relative to the profits under uniform prices. This result is in sharp contrast with the prisoners' dilemma prediction in oligopolistic one-sided price discrimination models.

Our main result, that price discrimination can yield higher profits than uniform pricing, is robust to an extension that allows agents to multi-home.

A Proof of Proposition 1

The location of the marginal agent from group ℓ, who is indifferent between A and B, is given by,

$$\begin{aligned} V + \alpha_\ell n^e_{mA} - tx - p_{\ell A} &= V + \alpha_\ell n^e_{mB} - t(1-x) - p_{\ell B} \Rightarrow \\ x_\ell &= \frac{\alpha_\ell \left(n^e_{mA} - n^e_{mB}\right) - p_{\ell A} + p_{\ell B} + t}{2t}. \end{aligned} \tag{18}$$

where $n^e_{mA} = F_m\left(x^e_\ell\right)$ and $n^e_{mB} = 1 - F_m\left(x^e_\ell\right)$. Therefore, the implicit functions for the market shares are given by,

$$x_1 = \frac{\alpha_1\left[2F_2\left(x^e_2\right) - 1\right] - \left(p_{1A} - p_{1B}\right) + t}{2t} \text{ and } x_2 = \frac{\alpha_2\left[2F_1\left(x^e_1\right) - 1\right] - \left(p_{2A} - p_{2B}\right) + t}{2t}.$$

Since expectations are rational we must have $x_\ell = x^e_\ell$, or $n^e_{mk} = n_{mk}$. By invoking the implicit function theorem we can derive the effect of prices on the market shares,

$$\begin{aligned} \frac{\partial x_1}{\partial p_{1A}} &= \frac{\partial x_2}{\partial p_{2A}} = -\frac{t}{2\left[t^2 - \alpha_1\alpha_2 f_1\left(x_1\right) f_2\left(x_2\right)\right]}, \frac{\partial x_1}{\partial p_{2A}} = -\frac{\alpha_1 f_2\left(x_2\right)}{2\left[t^2 - \alpha_1\alpha_2 f_1\left(x_1\right) f_2\left(x_2\right)\right]} \\ \text{and } \frac{\partial x_2}{\partial p_{1A}} &= -\frac{\alpha_2 f_1\left(x_1\right)}{2\left[t^2 - \alpha_1\alpha_2 f_1\left(x_1\right) f_2\left(x_2\right)\right]}. \end{aligned} \tag{19}$$

For the Jacobian of the system of the implicit functions to have a non-zero determinant it must be that $t^2 - \alpha_1\alpha_2 f_1\left(x_1\right) f_2\left(x_2\right) \neq 0$, for all x_1 and x_2. We further assume that $t^2 - \alpha_1\alpha_2 f_1\left(x_1\right) f_2\left(x_2\right) > 0$, for all x_1 and x_2.

The platforms' profit functions are given by,

$$\begin{aligned} \pi_A &= (p_{1A} - c)n_{1A} + (p_{2A} - c)n_{2A} = (p_{1A} - c)F_1\left(x_1\right) + (p_{2A} - c)F_2\left(x_2\right) \text{ and} \\ \pi_B &= (p_{1B} - c)n_{1B} + (p_{2B} - c)n_{2B} = (p_{1B} - c)\left[1 - F_1\left(x_1\right)\right] + (p_{2B} - c)\left[1 - F_2\left(x_2\right)\right]. \end{aligned}$$

The first order conditions of platform A are given by,

$$\begin{aligned} \frac{\partial \pi_A}{\partial p_{1A}} &= F_1\left(x_1\right) + (p_{1A} - c)f_1\left(x_1\right)\frac{\partial x_1}{\partial p_{1A}} + (p_{2A} - c)f_2\left(x_2\right)\frac{\partial x_2}{\partial p_{1A}} = 0, \\ \frac{\partial \pi_A}{\partial p_{2A}} &= F_2\left(x_2\right) + (p_{2A} - c)f_2\left(x_2\right)\frac{\partial x_2}{\partial p_{2A}} + (p_{1A} - c)f_1\left(x_1\right)\frac{\partial x_1}{\partial p_{2A}} = 0. \end{aligned}$$

Each first order condition has three terms. Suppose platform A lowers its price to group ℓ agents. The first two terms in each first order condition capture the reduction in inframarginal rents and the increase in marginal agents respectively. Since more agents from group ℓ join platform k, platform k becomes more attractive to the members of group m. The third term represents the additional revenue from the increase in the number of agents from group m that join platform k.

We look for a symmetric equilibrium where platforms charge the same prices to each group. We assume that regularity conditions hold so that a symmetric sharing equilibrium exists.[19] Using (19), the symmetric solution to the system of first order conditions is given by,

$$p_{1A}^* = p_{1B}^* = \frac{t - \alpha_2 f_1\left(\frac{1}{2}\right)}{f_1\left(\frac{1}{2}\right)} + c \text{ and } p_{2A}^* = p_{2B}^* = \frac{t - \alpha_1 f_2\left(\frac{1}{2}\right)}{f_2\left(\frac{1}{2}\right)} + c.$$

The equilibrium profits are,

$$\pi_A = \pi_B = \frac{t - \alpha_2 f_1\left(\frac{1}{2}\right)}{2f_1\left(\frac{1}{2}\right)} + \frac{t - \alpha_1 f_2\left(\frac{1}{2}\right)}{2f_2\left(\frac{1}{2}\right)}.$$

B Proof of Proposition 2

When we say "disadvantaged platform" we refer to a platform that prices for the agents in the rival platform's territory, e.g., in $[0, 1/2]$ for platform B. We first look at case (i) and then at case (ii) of Proposition 2. Case (iii), see footnote 13, falls somewhere in between and we do not examine it thoroughly since it does not add anything to our understanding of the problem.

Case (i): $c \geq \max\{2\alpha_1, 2\alpha_2\}$.

Due to symmetry, we only look at the case of platform A deviating. We divide the proof in two steps. In step 1, we show that in any deviation, platform A must be serving agents who are connected in each group. Then in step 2, we show that under this deviation structure, platform A has no incentive to deviate.

Step 1: Show that the agents served by either platform in each group are connected, i.e., there is no gap.

Equilibrium price $p_{\ell A}(x)$ decreases with x

When $x > 1/2$, it's easy to see that $p_{\ell A}(x)$ decreases with x. This comes from the fact that the value of an extra agent increases with the market shares of the platform, and consequently platform A is willing to lose more and more to sign up extra agents.

When $x \leq 1/2$, there are two effects when x increases. First, platform B's price goes up when x increases. All else the same, platform A's price can also go up. Second, platform A's advantage

[19] We were not able to come up with clean conditions on the distribution functions that would ensure the strict concavity (or quasi-concavity) of the objective functions. For instance, the monotone hazard rate property is not enough. When the distribution is uniform, then the profit functions are strictly concave provided that $2t > (\alpha_1 + \alpha_2)$. When this condition holds, then a symmetric sharing equilibrium exists. Otherwise, one platform may corner the entire market.

in terms of transportation cost goes down, and it has to lower its price. The overall effect of an increase in x is

$$\frac{dp_{\ell A}(x)}{dx} = -2t + 2\alpha_m f_m(x) < 0, \quad \text{since } t > \max\left\{(\alpha_1 + 2\alpha_2) f_2(x), (2\alpha_1 + \alpha_2) f_1(x)\right\}.$$

Therefore, when $x < 1/2$, the second effect dominates the first one and $p_{\ell A}(x)$ decreases with x as well.

Agents served by platform A are connected.

Suppose that platform A deviates by selling to a fraction of $x_\ell \leq 1$ agents in group $\ell = 1, 2$. Then it must be selling to agents on $[0, x_\ell]$ in group ℓ and no one else. This is because, while the premium of an extra agent is independent of where the agent is located, it is more costly for platform A to serve an agent located at higher x since $p_{\ell A}$ decreases with x. For each agent in $[0, x_\ell]$, platform A should choose a price to make the agent indifferent between buying from either platform. Of course, when platform A deviates and serves $x_m \neq 1/2$ of the agents in group m, its price in group ℓ ($p'_{\ell A}(x)$) will be adjusted from the equilibrium $p_{\ell A}(x)$, by $2\alpha_\ell x_m - 1$. But this adjustment is independent of the fraction and locations of agents served in group ℓ. Thus $\frac{dp'_{\ell A}(x)}{dx} = \frac{dp_{\ell A}(x)}{dx} < 0$ continues to hold.

Step 2: Show that platform A has no incentive to deviate

There are three types of deviations: (1) signing up fewer agents in both groups; (2) signing up more agents in both groups and (3) signing up more agents in one group, but fewer agents in the other group. Next we show that neither type of deviation can be profitable for platform A.

Type 1 deviation: $\max\{x_1, x_2\} \leq 1/2$.

In this case, platform A signs up fewer agents in both groups. We will show that this is dominated by $x_1 = x_2 = 1/2$, and thus platform A has no incentive for such a deviation.

Platform B still chooses the same prices, but platform A cannot choose the prices as in the equilibrium anymore, since its market shares in both groups are less now. Let's start with group 1. An agent located at x is indifferent between the two platforms if and only if

$$V - p'_{1A}(x) - tx + \alpha_1 F_2(x_2) = V - [c - 2\alpha_2(1 - F_2(x))] - t(1 - x) + \alpha_1(1 - F_2(x_2))$$

$$\Rightarrow p'_{1A}(x) = t(1 - 2x) + c + \alpha_1(2F_2(x_2) - 1) - 2\alpha_2(1 - F_2(x)).\text{[20]}$$

If platform A signs an extra agent in group 1, change of its profit in group 1 (due to an extra sale at price $p'_{1A}(x_1)$) is

$$p'_{1A}(x_1) - c = t(1 - 2x_1) + \alpha_1(2F_2(x_2) - 1) - 2\alpha_2(1 - F_2(x_1)).$$

[20] It's easy to see that $\frac{dp'_{\ell A}}{dx} = \frac{dp_{\ell A}}{dx} < 0$.

The change of its profit in group 2 (due to network externality) is $2\alpha_2 F_2(x_2)$. Thus the overall change in profit is[21]

$$\begin{aligned}\Delta\pi_{1A}(x_1, x_2) &= t(1-2x_1) + \alpha_1(2F_2(x_2) - 1) - 2\alpha_2(1 - F_2(x_1)) + 2\alpha_2 F_2(x_2) \\ &= t(1-2x_1) + 2(\alpha_1+\alpha_2)F_2(x_2) - \alpha_1 - 2\alpha_2(1 - F_2(x_1))\end{aligned}$$

Similarly, if platform A signs an extra group 2 agent, its change of profit is

$$\Delta\pi_{2A}(x_1, x_2) = t(1-2x_2) + 2(\alpha_1+\alpha_2)F_1(x_1) - \alpha_2 - 2\alpha_1(1 - F_1(x_2)).$$

It's easy to see that $\Delta\pi_{1A}(x_1, x_2)$ increases with x_2 and $\Delta\pi_{2A}(x_1, x_2)$ increases with x_1.

Assume that $x_1 \leq x_2$. Denote $\Delta\pi_{1A}(x) = \Delta\pi_{1A}(x_1 = x_2 = x)$, then $\Delta\pi_{1A}(x_1, x_2) \geq \Delta\pi_{1A}(x)$, since $\Delta\pi_{1A}(x_1, x_2)$ increases with x_2.

$$\Delta\pi_{1A}(x) = t(1-2x) + (2\alpha_1 + 4\alpha_2)F_2(x) - \alpha_1 - 2\alpha_2.$$

$$\frac{d\Delta\pi_{1A}(x)}{dx} = -2t + (2\alpha_1 + 4\alpha_2)f_2(x) < 0,$$

since $t > \max\{(\alpha_1 + 2\alpha_2)f_2(x), (2\alpha_1 + \alpha_2)f_1(x)\}$.

Combining it with the fact that $\Delta\pi_{1A}(x = 1/2) = 0$, we have

$$\Delta\pi_{1A}(x < 1/2) > 0 \Rightarrow \Delta\pi_{1A}(x_1, x_2) > 0.$$

Similarly, it can be shown that when $x_1 \geq x_2$, (1) $\Delta\pi_{2A}(x_1, x_2) > 0$.

To summarize, when $x_\ell < x_m$ ($\ell \neq m = 1, 2$), $\Delta\pi_{\ell A}(x_1, x_2) > 0$, implying that platform A has incentive to raise x_ℓ until it equals x_m. Then since $\Delta\pi_{\ell A}(x) > 0$ and $\Delta\pi_{mA}(x) > 0$, it will further raise both to $x_1 = x_2 = 1/2$. Therefore, we conclude that platform A has no incentive for case 1 deviations.

Type 2 deviation: $\min\{x_1, x_2\} \geq 1/2$.

Without loss of generality, assume that $x_1 \geq x_2$.

Show that $\pi_A(x_1 = x_2 = 1/2) = \pi_A(x_1 = x_2 > 1/2)$

We will show that when $x_1 = x_2 = x \geq 1/2$, platform A is indifferent between the following: (i) no change: $x_1 = x_2 = x$, (ii) signing up an extra agent in group 1; (iii) signing up an extra agent in group 2; (iv) signing up an extra agent in group 1 and 2. Let's compare (i) and (ii) for example. At any $x_1 = x_2 = x$, if platform A signs an extra agent in group 1, then it loses on this agent since $p_{1A}(x) < c$. Its change of profit is,

$$p_{1A}(x) - c = -2\alpha_2 F_2(x).$$

[21] $\Delta\pi_{\ell A}$, $\ell = 1, 2$ denote platform A's profit change if it signs an extra agent in group ℓ.

In the mean time, an extra agent is valuable to platform A in group 2, at a premium of $2\alpha_2$ per agent (platform A's share in group 1 increases, while platform B's share decreases, thus multiply by 2). A total of $F_2(x)$ agents are served in group 2. Thus the benefit is

$$2\alpha_2 F_2(x).$$

The loss and the benefit exactly offset, thus platform is indifferent between (i) and (ii). Indifference between (i) and (iii) and (i) and (iv) can be shown similarly.

Indifference between (i) and (iv) implies that

$$\pi_A(x_1 = x_2 = 1/2) = \pi_A(x_1 = x_2 > 1/2).$$

This implies that if platform A deviates by choosing $x_1 = x_2 > 1/2$, it enjoys the same profit as that in the equilibrium. Thus it has no incentive for such deviation. Next we show that $x_1 = x_2$ is the optimal deviation.

Show that $\pi_A(x_1 = x_2) > \pi_A(x_1 \neq x_2)$

Let $\Delta\pi_{\ell A}(x_1, x_2)$ denote the change of platform A's profit by signing up an extra agent in group ℓ, let $\Delta\pi_{\ell A}(x)$ denote $\Delta\pi_{\ell A}(x_1, x_2)$ when $x_1 = x_2 = x$. Then indifference between (i) and (ii) and (i) and (iii) implies

$$\Delta\pi_{\ell A}(x) = 0, \quad \ell = 1, 2.$$

It's easy to see that $\Delta\pi_{\ell A}(x_1, x_2)$ increases with x_m, $m \neq \ell$. This is because, the more agents platform A signs in group m, the more valuable an extra agent in group ℓ is. Without loss of generality, assume that $x_1 > x_2 \geq 1/2$. Then

$$\Delta\pi_{2A}(x_1, x_2) > \Delta\pi_{2A}(x_2, x_2) = 0.$$

That is, platform A can increase its profit by signing up an extra group 2 agent, and it has incentive to raise x_2 until $x_2 = x_1$.

Combining the two results $\pi_A(x_1 = x_2 = 1/2) = \pi_A(x_1 = x_2 > 1/2)$ and $\pi_A(x_1 = x_2) > \pi_A(x_1 \neq x_2)$, we have

$$\pi_A(x_1 = x_2 = 1/2) = \pi_A(x_1 = x_2 \geq 1/2) > \pi_A(x_1 > x_2 \geq 1/2).$$

That is, platform has no incentive to deviate and sign more agents.

Proof for the other case of $x_2 > x_1 \geq 1/2$ is similar.

Type 3 deviation: $x_\ell \geq 1/2 \geq x_m$, $\ell \neq m = 1, 2$.

Let's look at the case of $x_1 \geq 1/2 > x_2$ for example. We have explained that $\Delta\pi_{2A}(x_2, x_2) > 0$ in case 1. That is, when $x_1 = x_2 < 1/2$, platform A has incentive to raise x_2. Since $\Delta\pi_{2A}(x_1, x_2)$

increases with x_1, we have

$$\Delta\pi_{2A}(x_1, x_2) > 0, \text{when } x_1 \geq 1/2 > 1/2$$

and platform A has incentive to raise x_2 all the way until $x_2 = 1/2$. But then it becomes type 2 deviation, and we have shown that platform has no incentive for type 2 deviations.

Case (ii): $c \leq \min\{\alpha_1, \alpha_2\}$.

The following conditions are needed

$$t > c + \max\{\alpha_1, \alpha_2\} \text{ and } t < (\alpha_1 + \alpha_2)\min\{f_1(x), f_2(x)\}.$$

When $c \leq \min\{\alpha_1, \alpha_2\}$, the disadvantaged platform always chooses zero price. Still assume the platform A deviates. Without loss of generality, assume that $x_1 < x_2 \leq 1/2$ (platform A can't raise x_ℓ to be above 1/2 now). Next we show that platform A has incentive to raise x_1, x_2 to 1/2.

Let $p''_{1A}(x)$ denote platform A's price for agent located at x in group 1. Then

$$V - p''_{1A}(x) - tx + \alpha_1 F_2(x_2) = V - p_{1B}(x) - t(1-x) + \alpha_1(1 - F_2(x_2))$$

$$\Rightarrow p''_{1A}(x) = t(1 - 2x) + \alpha_1(2F_2(x_2) - 1).$$

Note that $\frac{dp''_{\ell A}(x)}{dx} < 0$, so platform A's agents are still connected. If platform A signs an extra agent in group 1, it's change of profit is

$$\begin{aligned} \Delta\pi_{1A}(x_1, x_2) &= p''_{1A}(x_1) - c + 2\alpha_2 F_2(x_2) \\ &= t(1 - 2x_1) + 2(\alpha_1 + \alpha_2)F_2(x_2) - \alpha_1 - c, \end{aligned}$$

which increases with x_2. Since $x_1 < x_2$, the minimum is obtained when $x_1 = x_2 = x$, and

$$\Delta\pi_{1A}(x) = t(1 - 2x) + 2(\alpha_1 + \alpha_2)F_2(x) - \alpha_1 - c.$$

$$\Delta\pi_{1A}(x = 0) = t - \alpha_1 - c \qquad \text{positive since } t > c + \alpha_1.$$

$$\frac{d\left(\Delta\pi_{1A}(x)\right)}{dx} = -2t + 2(\alpha_1 + \alpha_2) > 0.$$

The last two equations guarantee that $\Delta\pi_{1A}(x) > 0$ for $x < 1/2$. We also know that

$$\Delta\pi_{1A}(x_1, x_2) > \Delta\pi_{1A}(x_1, x_1) = \Delta\pi_{1A}(x_1) > 0.$$

This implies that whenever $x_1 < x_2 \leq 1/2$, platform A has incentive to raise x_1 to equal to x_2 (because $\Delta\pi_{1A}(x_1, x_2) > 0$), then raise both to 1/2 (because $\Delta\pi_{1A}(x) > 0$).

Similar conditions can be derived for $\Delta\pi_{2A}$ when $1/2 \geq x_1 > x_2$.

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