

How Does the Modeling Choice Affect Deviations from Purchasing Power Parity?*

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Abstract:

Empirically, elements of both long memory and non-linearity are present in the real exchange rates of the G-7 countries against the US, especially in the EU countries. If one ignores non-linearity and only allows for linear long memory, the half lives of PPP adjustments are surprisingly long, sometimes exceeding 1,000,000 months. However, with nonlinear methods the PPP adjustments are much shorter, with half lives of less than 2 years. The adjustments remain similarly quick, and sometimes even quicker, when one allows for both nonlinearity and long memory.

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I. Introduction

After the work of Rogoff (1996), the empirical failure of the theory of purchasing power parity (PPP), the equality between the nominal exchange rate and relative prices of two countries, has been established as a puzzle in international macroeconomics. In fact, as pointed out by Rogoff, deviations from PPP are more likely to be the rule rather than the exception for the US, since half lives of shocks to the real exchange rate were estimated in the neighborhood of 3-5 years. Further, economic theory has not been able to reconcile the demonstrably long half lives. Most notably, economic models with price stickiness generally cannot reproduce the persistence of the real exchange rate (c.f. Chari, Kehoe, and McGrattan, 2002). Relative to standard model predictions, real exchange rates are overly volatile and persistent.

Yet, the theory of PPP remains the cornerstone of modern international macroeconomics, and thus there have been a number of attempts to explain the large half lives in the theoretical and especially the empirical literature. In empirical research, the central question revolves around whether or not the long estimates of half lives years are specific to the models employed.¹ Along these lines, researchers have questioned the standard univariate modeling approach to the real exchange rate. At least two plausible alternatives have arisen. On one hand, researchers have questioned the assumption of an integer order of integration for the real exchange rate and have argued that a fractional order of integration is present (Diebold, Husted, and Rush, 1991, and Cheung and Lai, 2001). On the other hand, a number of researchers have questioned the assumption of linearity and have suggested that previous estimates of half lives based on linear models are biased upward (Taylor, Peel, and Sarno, 2001, and Paya, Venetis, and Peel, 2003).

¹ Another strand of literature attempts to resolve the paradox in terms of aggregation bias and heterogenous agents (see Imbs, 2005 for a review). The current paper is interested in the extent to which the time series model alone affects measured deviations.

These authors have suggested that the appropriate modeling choice is one in which the real exchange rate is subject to two regimes, one with a strongly persistent real exchange rate and one in which deviations from PPP are short lived.

The dichotomous modeling approach in the PPP literature highlights the emerging debate among researchers employing macroeconometric techniques. The two modeling procedures have been employed simultaneously to study the same phenomena. Given the research of Diebold and Inoue (2001), who show that regime switching processes can be mimicked by long memory fractional processes, it is of interest to determine how these two models can be used to address the PPP paradox. In conjunction with determining the extent to which long memory and non-linearity exist in the real exchange rate through the use of an encompassing technique, I wish to determine how the modeling choice affects measured deviations from PPP.

In this paper, I employ a technique that extends the analysis of van Dijk, Franses, and Paap (2002). The model I employ (a FI-ESTAR model) allows one to account for long memory and smooth transition autoregressive (ESTAR) non-linearity simultaneously, where the transition from one regime to another is smooth and governed by an exponential function. The results show that the long memory model in isolation produces a parameterization that is quite close to a unit root process, and thus produces half lives of deviations from PPP that can persist for over 1,000,000 months. There is significant evidence of non-linearity for the EU countries in my sample of G-7 nations, with little evidence of non-linearity for the real exchange rates of Canada and Japan. Further, I am typically able to reject the restriction that the order of integration is zero or unity when non-linearity is also considered. Thus, elements of both non-linearity and fractional integration are present in the real exchange rate data especially for the EU countries. This implies that the order of integration of the real exchange rate is generally not zero even in a

non-linear framework as assumed in previous analysis. In terms of resolving the PPP puzzle, the introduction of non-linearity dramatically reduces half lives relative to the use of long memory in isolation. Interestingly, a model with both fractional integration and non-linearity can produce short half lives of fewer than two years, and it is not necessarily the case that restricting the order of integration to be zero produces shorter half lives.

The rest of the paper is organized as follows. In section II, I review the literature on the PPP puzzle, highlighting the empirical literature related to long memory and non-linearity. In section III, I present the ARFIMA, ESTAR, and FI-ESTAR models and review the testing procedures for linearity. In section IV, I present estimated ARFIMA, ESTAR, and FI-ESTAR models for measures of the post Bretton Woods real exchange rates for the G-7 countries relative to the US. In section V, I present estimates of the impulse response functions and accompanying half lives for each model, while Section VI concludes.

II. Relevant Literature

The theory of PPP asserts that the exchange rate and prices of good and services between two countries should be equal. In particular, let e_t denote the log foreign currency price of the home currency, and let p_t and p_t^* denote the logs of the home and foreign price levels respectively. If PPP holds, $e_t = p_t^* - p_t$, in which case the real exchange rate, $r_t = e_t - p_t^* + p_t$, should equal zero. Thus, PPP can be tested by analyzing the behavior of the real exchange rate.

As discussed by Rogoff (1996), a consensus has emerged that suggests that PPP does indeed hold in the very long run, but not so well in the short run. In particular, Rogoff cites a number of studies using primarily linear models with an autoregressive structure that yield an estimated half life of 3-5 years for shocks to the real exchange rate. More recently, Murray and

Papell (2005) use a median-unbiased estimation procedure and suggest that these estimates may actually be biased downward.

Empirical studies have emerged that contest the standard modeling approach to the real exchange rate. As pointed out by Taylor (2001), the selected model can have important implications for the measured half lives. One modeling alternative is based on the idea that integer orders of integration may be unduly restrictive. Cheung and Lai (2001) assert that fractional integration is likely for the real exchange rate, since econometrically, long memory accommodates the long swings that are prototypical of exchange rates. Cheung and Lai also argue that the use of fractional integration can be important in testing PPP as it allows one to separate permanent or low frequency components from high frequency or transitory components easily. Further, long memory is plausible given the aggregation of price indices and exchange rates themselves. A number of researchers have employed fractional integration to model the dynamics associated with PPP, including Diebold, Husted, and Rush (1991), Cheung and Lai (1993), Cheung and Lai (2001), and Achy (2003). In particular, Cheung and Lai assert that half lives are reduced by considering fractional integration for yen based real exchange rates.

A second modeling strategy employed in the PPP literature takes specific umbrage with the linearity assumption. The non-linear modeling approach is based on sound theoretical underpinnings. For example, as in Sercu, Uppal, and Van Hulle (1995), PPP may be expected to hold only after deviations are large enough to make arbitrage profitable because of impediments to trade such as transactions costs, trade restrictions, and transportation costs. In this case, there are two regimes, one in which the exchange rate is allowed to deviate from its equilibrium value, and one in which arbitrage is activated. Existing non-linear models, therefore, extend readily to this important problem. Recently, Michael, Nobay, and Peel (1997), Baum, Barkoulas, and

Caglayan (2001), Taylor, Peel, and Sarno (2001), and Paya, Venetis, and Peel (2003) have presented evidence of non-linearity in the real exchange rate. In particular, Taylor, Peel, and Sarno (2001) show that half lives of deviations from PPP with dollar based real exchange rates can be reduced through the use of a restricted non-linear model that allows for a smooth transition between two regimes. Paya, Venetis, and Peel (2003) present results that show very short half lives for EMS based real exchange rates when exogenous variables such as a time trend and a proxy for a relative measure of productivity are included in a nonlinear framework.

A number of the studies cited above argue that the aggregation of price indices and exchange rates likely induces a transition between regimes is smooth. Aggregation arguments have also been used to motivate long memory. A natural extension of the dichotomous methods allows for both fractional integration and non-linearity or regime-switching. A limited number of recent studies have attempted to utilize such models. Baum, Barkoulas, and Caglayan (1999) consider a model that allows for a double mean shift in the context of long memory for the real exchange rate, while Kapetanios (2002) presents evidence that yen based real exchange rates are subject to long memory and a general form of non-linearity. Smallwood (2005) performs preliminary estimation of the FI-ESTAR model for a number of countries. These studies do not consider the implications of the modeling choice on the central question being investigated in this paper, as none of these papers employ impulse response analysis, nor do they consider how the modeling choice affects half lives of deviations from PPP.

III. The FI-ESTAR Model

The FI-ESTAR model nests both the ARFIMA($p,d,0$) and ESTAR(p) models. The ARFIMA model was introduced by Granger and Joyeux (1980) and Hosking (1981) to bridge the

gap between I(0) and unit root processes. Let $\phi(L)$ and $\theta(L)$ denote p and q order autoregressive and moving average polynomials. Then, the standard ARFIMA(p,d,q) process is defined as,

$$\phi(L)(1-L)^d(y_t - \mu) = \theta(L)\varepsilon_t, \quad (1)$$

where $\phi(z)=0$ and $\theta(z)=0$ have all roots outside the unit circle, d is any real number, and ε_t is a martingale difference sequence, with $E(\varepsilon_t^2)=\sigma^2$. The differencing operator, $(1-L)^d$, is defined as

$$(1-L)^d = \sum_{k=0}^{\infty} \frac{\Gamma(k-d)}{\Gamma(-d)\Gamma(k+1)} L^k, \quad (2)$$

where $\Gamma()$ is the gamma function. The ARFIMA model is stationary and invertible if $-1/2 < d < 1/2$ and for stationary representations has an autocorrelation function that decays at a hyperbolic rate.

The use of time series methods that deviate from a strict linearity assumption emerged almost simultaneously with the use of long memory as a means of explaining Nelson and Plosser's (1982) pervasive finding of unit roots in macroeconomic data. The STAR model, which was made operational by Granger and Teräsvirta (1993) and Teräsvirta (1994), has proven to be an especially useful non-linear model. The two-regime STAR(p) model with transition function $G(s_t; \gamma, c)$ is defined as

$$y_t = \phi_{1,0} + \phi_{1,1}y_{t-1} + \dots + \phi_{1,p}y_{t-p} + (\phi_{2,0} + \phi_{2,1}y_{t-1} + \dots + \phi_{2,p}y_{t-p})G(s_t; \gamma, c) + \varepsilon_t, \quad (3)$$

where ε_t is as above, c is the equilibrium threshold, γ controls the degree of curvature of G , and s_t is the transition variable governing the non-linear behavior of the process. Although a number of choices are plausible for $G(s_t; \gamma, c)$, I concentrate on the exponential function,²

$$G(s_t; \gamma, c) = 1 - \exp[-\gamma(s_t - c)^2]. \quad (4)$$

² Alternatively, one could employ a logistic or simple threshold function. Baum, Barkoulas, and Caglayan (2001) argue that the most appropriate choice in the current context is the exponential function, a choice I follow.

Clearly, as $\gamma \rightarrow 0$ the function collapses to 0, whereas if $\gamma \rightarrow \infty$ the function converges to unity. In either case, the model in (3) becomes a linear AR(p) model. The inner regime corresponds to the case where $\gamma=0$, whereas the outer regime corresponds to the case where $\gamma \rightarrow \infty$.

The FI-ESTAR model, which is a simple extension of the FI-LSTAR model developed by van Dijk, Franses, and Paap (2002), is nothing more than a simple combination of a fractional long memory model and an ESTAR(p) model. The FI-ESTAR model is defined as

$$x_t = \phi_{1,0} + \phi_{1,1}x_{t-1} + \dots + \phi_{1,p}x_{t-p} + (\phi_{2,0} + \phi_{2,1}x_{t-1} + \dots + \phi_{2,p}x_{t-p})(1 - \exp[-\gamma(s_t - c)^2]) + \varepsilon_t, \quad (5)$$

with ε_t defined as above, and where $x_t \equiv (1-L)^d y_t$.

A number of estimation methods are available for the models presented here. Following van Dijk, Franses, and Paap (2002), I chose to use the time domain estimator of Beran (1995). Under the assumption that the pre-sample observations are non-stochastic, the estimator is asymptotically equivalent to MLE. Following Beran, I set $y_0=y_{-1}=\dots=\varepsilon_0=\varepsilon_{-1}=\dots=0$. The estimates of the FI-ESTAR model parameters are the set of parameters that maximize the following function:

$$\begin{aligned} AMLE(c, \gamma, \phi', d, \sigma^2) &= -\frac{T}{2} \log(2\pi) - \frac{T}{2} \log \sigma^2 - \frac{1}{2\sigma^2} \sum_{t=2}^T \varepsilon_t^2 \\ x_t &= (1-L)^d y_t \\ \varepsilon_t &= x_t - \phi_{1,0} - \sum_{j=1}^p \phi_{1,j} x_{t-j} - \dots - (\phi_{2,0} + \sum_{j=1}^p \phi_{2,j} x_{t-j})(1 - \exp[-\gamma(s_t - c)^2]) \end{aligned} \quad (6)$$

If the user wishes to estimate an ARFIMA($p,d,0$) model, then $\phi_{2,j}$ is set equal to zero for $j=0,\dots,p$.

If the user wishes to estimate the standard ESTAR model, then d is constrained to be 0.

The FI-ESTAR model allows one to account for both long memory and non-linearity. The introduction of the long memory parameter does introduce some complications with respect to standard tests for non-linearity. As is well known, the null hypothesis of linearity for the

standard STAR model can be built in more than one way ($\gamma=0, \phi_{2,0}=\phi_{2,1}=\dots=\phi_{2,p}=0$). Further, linearity tests are complicated by the fact that the nuisance parameters are not restricted under the null. The original linearity tests (with $d=0$) were developed by Teräsvirta (1994) to avoid this issue. In particular, one can take an appropriate Taylor series expansion of the model about $\gamma=0$. Under the null hypothesis, the test can be carried out through a standard OLS regression with restrictions on a subset of parameters. The extension to the FI-ESTAR model is straightforward. As discussed by Smallwood (2005), a first order Taylor series expansion of (5) about $\gamma=0$ results in the following auxiliary regression,

$$x_t = \beta_{1,0} + \sum_{j=1}^p \beta_{1,j} x_{t-j} + \sum_{j=1}^p \beta_{2,j} x_{t-j} s_t + \sum_{j=1}^p \beta_{3,j} x_{t-j} s_t^2 + u_t. \quad (7)$$

The null hypothesis for linearity is then given by $\beta_{2,j}=\beta_{3,j}=0$. The test must account for the change in the likelihood function that results from the value of d . Let $\hat{\varepsilon}_t$ denote the residuals from an estimated ARFIMA($p,d,0$) model. Under the hypothesis of linearity, and with a Gaussian assumption, the gradient of the likelihood function for the t^{th} observation for the model above with respect to d is given by:

$$\frac{\partial \ell_t}{\partial d} = -\frac{\hat{\varepsilon}_t}{\sigma^2} \sum_{j=1}^{t-1} \frac{\hat{\varepsilon}_{t-j}}{j}. \quad (8)$$

Given this expression, it is straightforward to construct the test for linearity using the following steps (see Teräsvirta, 1994, van Dijk, Franses, and Paap, 2002, and Smallwood, 2005 for details).

1. Estimate an ARFIMA($p,d,0$) model, obtain the estimated differencing parameter, \hat{d} , the residuals, $\hat{\varepsilon}_t$, and form the sum of squared residuals denoted SSE_R .
2. Form $x_t = (1-L)^{\hat{d}} y_t$. Regress $\hat{\varepsilon}_t$ on $-\sum_{j=1}^{t-1} \frac{\hat{\varepsilon}_{t-j}}{j}, 1, x_{t-i}, x_{t-i} s_t, x_{t-i} s_t^2$, for $i=1, \dots, p$.

3. From the regression in step 2, form the sum of squared errors, SSE_{UR} . With T observations, the χ^2 and F -version of the tests are given by:

$$LM_{\chi^2} = \frac{T(SSE_R - SSE_{UR})}{SSE_R}, LM_F = \frac{(SSE_R - SSE_{UR})/2p}{(SSE_{UR})/(T - 3p - 1)} . \quad (9)$$

The χ^2 and F versions of the tests are distributed as $\chi^2(2p)$ and $F(2p, T-3p-1)$ statistics.

The FI-ESTAR model was applied to the US unemployment rate by van Dijk et al. (2002), and the authors find that the degree of persistence as measured by the differencing parameter decreases once non-linearity is considered. Similarly, Taylor, Peel, and Sarno (2001) suggest, without reporting estimates from various models, that the persistence of the real exchange rate is expected to fall once the researcher extends their analysis beyond linear models and allows for non-linearity. In other words, the possibility exists that the integration parameter, d , may differ under the null and alternative hypotheses as pointed out by Kapetanios (2002). Therefore, below, I estimate d both under the hypothesis of linearity as above, and under the alternative hypothesis. I report the χ^2 version of the test and the minimized p-value in each instance.³

IV. Estimation Results

In this section, I consider the estimation results for ARFIMA, ESTAR, and FI-ESTAR models for the monthly real exchange rates of the G-7 countries vis-à-vis the US.⁴ All data employed in this study were obtained from the IFS-CD ROM. The nominal exchange rate data are monthly averages of the foreign currency price of the US dollar. For price indices, I employ both the consumer price index (CPI) and a measure of the wholesale price index (WPI). For the

³ Under the alternative hypothesis, the true model is the FI-ESTAR model. To obtain the value of d under the alternative, I estimate the full model, and then use this estimated value of d in place of \hat{d} above.

⁴ Below, I wish to compare how the modeling choice affects half lives of deviations from PPP, and thus consider estimation of all the models even for the countries where linearity was not rejected in this section.

euro-zone countries, I employ data from January 1973-December 1998. For Japan, Canada, and the UK, the data extend from January 1973-June 2004. As in Taylor, Peel, and Sarno (2001), I normalize the real exchange rate in January 1973 to be 0.

The analysis conducted in this paper is primarily concerned with the real exchange rate. However, some researchers have asserted a relationship between exchange rates and price indices may exist even if PPP fails to hold in the presence of measurement errors in prices (see Cheung and Lai, 1993). These researchers contend that a very weak version of PPP holds if $e_t = \alpha_1 p_t^* + \alpha_2 p_t$, with $\alpha_1, \alpha_2 \neq 0$. The parameters, α_1 and α_2 can be estimated using OLS, and the resulting residuals are taken to be the *estimated* real exchange rate. I conducted analysis based on the estimated real exchange rate, along with the real exchange rate based on the wholesale price index (WPI). The estimates for the WPI based real exchange rates, which are available upon request, are consistent with the estimates reported here, and so to conserve space I concentrate on the CPI based real exchange rates. There are only a couple of minor differences for the analysis conducted below between the estimated real exchange rate and the real exchange rate based on conventional PPP, which I discuss whenever relevant.

For the non-linear models, I allow s_t to be a lag of the real exchange rate, with $s_t = r_{t-d^*}$, where d^* is the delay parameter. Following Baum, Barkoulas, and Caglayan (2001), I let d^* range from 1 to 12. Here, it is sensible to allow the degree of non-linearity to be controlled by the extent to which the real exchange rate deviates from the equilibrium threshold c .

The estimates for the ARFIMA($p, d, 0$) model for the real exchange rates are given in Table 1. The autoregressive order is selected to ensure that the residuals are serially uncorrelated. As is clear from the table, $p=1$ is generally appropriate, although two autoregressive coefficients were needed for the UK. Based on the estimates that appear in Table 1, there is little evidence

that yields support for PPP. Although the estimates of d are less than 1, using the estimated numerical standard errors, they are never more than two standard deviations away from unity except in the case of Canada. For Canada, the estimated value of d is 0.1445, although the estimated autoregressive parameter is very close to unity. The results of the table suggest that linear ARFIMA models tell a morbid tale for the theory of PPP. In regards to diagnostics, I calculate the Ljung-Box Q statistics for serial correlation and autoregressive conditional heteroskedasticity (ARCH), along with the skewness and kurtosis of the estimated residuals. The results of the table show that the residuals are generally free of serial correlation, but that ARCH effects may be present for the real exchange rates of Canada, the UK, and possibly Japan.⁵

Conditional on $d=0$, I employ the Teräsvirta rule (1994) to obtain an appropriate ESTAR model. In particular, the selection of p for the ESTAR model is chosen to ensure that the estimated residuals are free of serial correlation. The delay parameter, d^* , is obtained by minimizing the probability value for the linearity test for d^* between 1 and 12. The results of the linearity tests of Teräsvirta are reported in Table 2. The results show that when d is constrained to be zero, there is substantial evidence of non-linearity for Italy and the UK, since the hypothesis of linearity can be rejected at the 5% level. When a 10% test is employed, the linearity hypothesis is also rejected for Germany, while the p-value for France is only marginally outside the 10% threshold. These results suggest that there is evidence of non-linearity for the real exchange rates of the four EU countries in my sample. Based on the linearity tests, there is little evidence of non-linear behavior in the real exchange rates of Canada and Japan. The estimates of the unrestricted ESTAR models are recorded in Table 3. Standard errors appear in

⁵ The results below indicate that there is substantially less evidence of ARCH effects once non-linearity is introduced. Although outside the scope of this paper, one could consider heteroskedastic effects directly, by replacing σ^2 with σ_t^2 in (6) and allowing σ_t^2 to follow a GARCH specification.

brackets beneath the coefficient estimates.⁶ The estimates of γ range from 2.4893 (for Germany) to 573.0928 (for Japan). The estimates of γ are large for Canada and Japan, supporting the results of the linearity tests in Table 2. The diagnostic tests are generally supportive of the non-linear models, as the residuals are free of serial correlation, and there is substantially less evidence of heteroskedasticity relative to the estimated linear ARFIMA($p,d,0$) models. In addition, I apply the test for no remaining non-linearity as discussed by Eitrheim and Teräsvirta (1996), whose p-values exceed 5% for every country suggesting that the ESTAR (p) model adequately accounts for all non-linearity.

The FI-ESTAR model allows for joint estimation of the non-linear and long memory model parameters. Further, the testing procedure allows the user to distinguish between the relative importance of the two modeling procedures. Table 4 presents the test results for linearity based on the FI-ESTAR model associated with equation (5) for the CPI based real exchange rates. When d is estimated under the alternative hypothesis, linearity is rejected at the 5% level for each of the EU countries, and it is interesting to note that the probability values can be lower than when d is constrained to be zero as in the standard tests of Teräsvirta (1994). Further, the test results from the estimated real exchange rate using OLS indicate that non-linearity is present for each of the EU countries, but not for Canada and Japan.

Finally, Table 5 reports the results from estimation of the FI-ESTAR model for the CPI-based real exchange rates. The estimated value of d is significantly different from 0 and 1 at the 5% level for France and Germany and at the 10% level for Italy. This suggests that there is evidence of both long memory and non-linearity for at least three countries in my sample. The

⁶ Teräsvirta (1994) and van Dijk, Franses, and Paap (2002) suggest multiplying γ by the inverse of the variance of the transition variable, which I follow.

estimated models for Canada and Japan in Table 5 are virtually identical to the estimated linear models in Table 1. For example, the large value of γ for Canada implies that the transition function in (5) is about one for all observations. This yields an ARFIMA(1,0.1227,0) model with an AR(1) coefficient equal to, $\phi_{1,1} + \phi_{2,1} = 0.9872$. The estimated linear ARFIMA model for Canada produces a value of d equal to 0.1445 with an AR(1) coefficient equal to 0.9868. It is interesting to note, that for the four remaining countries where some evidence for non-linearity has been uncovered, the estimated value of d is substantially lower than the estimated value of d reported in Table 1. As for the UK's real exchange rate, the results suggest that the restriction $d=0$ is binding. The results of the diagnostic tests in Table 5 show that the residuals are serially uncorrelated and largely homoskedastic (with the UK as the lone exception). For every country, I am unable to reject the hypothesis that there is no remaining non-linearity when I employ the test of Eitrheim and Teräsvirta (1996) as modified by van Dijk, Franses, and Paap (2002).

To get a sense of the estimated model, Figure 1 depicts the transition function for the estimated FI-ESTAR model for France. Referring to the results in Table 5, we see that the inner regime (with $G=0$) is affiliated with more persistence, since the autoregressive parameter in this regime is 1.0356. During the appreciation of the US dollar in the beginning of the 1980's, the French real exchange rate appears to be in the more persistent regime. As the deviation from equilibrium becomes large, market participants and potentially active intervention after the Plaza Accord (1985) and Louvre Accord (1987) begin to bring the real exchange rate back toward equilibrium. In this case, the real exchange rate has deviated substantially from its equilibrium value and is in the less persistent regime.

The results for the FI-ESTAR model for the estimated real exchange rate are very consistent with those in Table 5. The only glaring exception is the existence of both long

memory and non-linearity for the estimated real exchange rate of the UK. In particular, the value of d for the UK's estimated real exchange rate is 0.3549 and given a standard error of 0.1249, is easily more than two standard deviations away from 0 and 1.

To summarize, the results in this section indicate that there is substantial evidence of both long memory (with d significantly different from 0 and 1) and non-linearity for either the real exchange rate or the estimated real exchange for every EU country in my sample. There is very little evidence of non-linearity for any measure of the real exchange rates of Canada and Japan. Next, I turn to computation of the impulse response functions based on the models estimated in Tables 1, 3, and 5.

V. Impulse Response Analysis

The previous section formally tested the degree of non-linearity and long memory in the real exchange rates of the G-7 countries. In this section, I am concerned with the implications of the various modeling choices on the PPP puzzle. In particular, I wish to analyze the extent to which the modeling choice affects the estimated half lives of deviations from PPP. A number of researchers (see Rogoff, 1996) have estimated the half-lives using linear AR models through least squares (LS). An AR(p) model for the real exchange rate is given by:

$$r_t = c + \phi_1 r_{t-1} + \dots + \phi_p r_{t-p} + \varepsilon_t. \quad (10)$$

The half-lives can be estimated using an augmented Dickey-Fuller type regression,

$$r_t = c + \alpha r_{t-1} + \zeta_1 \Delta r_{t-1} + \dots + \zeta_{p-1} \Delta r_{t-p+1} + \varepsilon_t, \quad (11)$$

where $\alpha = \phi_1 + \phi_2 + \dots + \phi_p$, and $\zeta_j = -(\phi_2 + \dots + \phi_p)$, $j=1, \dots, p-1$. The half life can be calculated from the impulse response function of the estimated model in (10) by determining the number of periods it takes for the initial shock to dissipate by half. However, as pointed out by Andrews (1993) and Andrews and Chen (1994), the estimates of autoregressive parameters using LS are biased

downward, and thus estimated half lives are also biased. The problem can be corrected using median unbiased estimators. Suppose $\hat{\alpha}_{LS}$ is the estimated LS parameter from equation (11). The median unbiased estimator of α is that estimator which yields $\hat{\alpha}_{LS}$ as it's median value.⁷ To put the results in this section in the context of analysis associated with the PPP paradox, I report the LS estimates of the parameters from equation (11) and the associated median unbiased estimates of the model parameters along with the estimated half lives in Table 6. When uncorrected, the estimated half lives based on LS range from 33 months to 118 months. The median unbiased estimates yield half lives ranging from 49 months to an infinite half life. These results are similar to Murray and Pappell (2005) who demonstrate that median unbiased corrections yield half lives that may be even longer than previously considered.

The impulse response function for the ARFIMA($p,d,0$) model is based on the calculation of the moving average (MA) representation using equations (1) and (2), which is given by:

$$(y_t - \mu) = \frac{\sum_{i=0}^{\infty} c_i \varepsilon_{t-i}}{1 - \phi_1 L - \dots - \phi_p L^p}, \quad (12)$$

with $c_0=1$, $c_i = \frac{\Gamma(i+d)}{\Gamma(d)\Gamma(i+1)}$. Then the MA coefficients are calculated through recursion. For

example, for an ARFIMA($p,d,0$) model, the j^{th} coefficient is, $\psi_j = \phi^j + \phi^{j-1}c_1 + \phi^{j-2}c_2 + \dots + c_j$.

The calculation of the impulse response weights for non-linear models is more complicated, since the impulse response function is not independent of the history of the process, size of the shock, or future innovations of the process (see Koop, Pesaran, and Potter, 1996). Typically, therefore, researchers have utilized the generalized impulse response function

⁷ I use the procedure outlined in Chen and Andrews (1994) to obtain the estimated median unbiased estimates of the model parameters and associated half lives.

(GIRF). The GIRF for a horizon h of a time series process Y_t with a shock of size δ and conditional on the history, ω_{t-1} , is given by:

$$GIRF_Y(h, \varepsilon_t, \omega_{t-1}) = E[Y_{t+h} | \varepsilon_t = \delta, \omega_{t-1}] - E[Y_{t+h} | \varepsilon_t = 0, \omega_{t-1}], \quad (13)$$

where ε_t is the disturbance sequence associated with the process. The computation of the impulse response function involves the calculation of the conditional expectation of the process contingent on a given history. As discussed by Koop, Pesaran, and Potter (1996), calculation of the GIRF is accomplished by averaging out future shocks through numerical integration.

I closely follow Taylor, Peel, and Sarno (2001), and van Dijk, Franses, and Paap (2002) in the calculation of the GIRF for the non-linear ESTAR and FI-ESTAR models. For the FI-ESTAR model, it is necessary to forecast the future values of a process that has both long memory and non-linearity. This can be accomplished through recursion noting that for $h=1$, we have the following expression for the forecast of y_{t+1} conditional on information known at time t , where again $x_t \equiv (1-L)^d y_t$:

$$\hat{y}_{t+1|t} = -\sum_{k=1}^{\infty} c_k y_{t-k+1} + \phi_{1,0} + \sum_{j=1}^p \phi_{1,j} x_{t-j+1} + (\phi_{2,0} + \sum_{j=1}^p \phi_{2,j} x_{t-j+1})(1 - \exp[-\gamma(y_{t-d^*+1} - c)^2]). \quad (14)$$

The above expression must be truncated, and similar to van Dijk, Franses, and Paap, I truncate the first 100 values. To calculate the GIRF, I start with the observation in April 1981 and condition on the information up to and including that point. Following the forecasting procedure outlined in Granger and Teräsvirta (1993), I then forecast the process h periods into the future, both with the initial shock equal to δ and with the initial shock equal to 0. For a given history, this procedure is repeated 200 times, where I randomly sample with replacement from the estimated residuals of the FI-ESTAR model. I record the disparity between the two forecasted series at each point in time for $i=1, \dots, h$ as in equation (13). I then update my sample one period,

and condition on the history up until $t = \text{May } 1983$. Once more, I calculate the conditional expectations of the processes h steps ahead with $\varepsilon_t = \delta$ and $\varepsilon_t = 0$. I again record the disparity for $i=1, \dots, h$. The sample is updated, and this procedure continues until I reach the end of the sample. For the euro-zone countries, this results in the calculation of 42,400 simulations of the quantity in equation (13). The estimated GIRF is the mean of these values for each $i=1, \dots, h$.⁸

Here, I consider perturbations to the initial conditions of 1%, 10%, 25%, 40%, and 50%, which results in setting $\delta = \log(1+k/100)$, for $k=1, 10, 25, 40$, and 50, where k is the percentage size of the shock. To make the quantity comparable to linear models, the half lives are calculated as the number of months for the GIRF to fall below $\delta/2$. To get a sense of the impulse response functions, refer to Figure 2, which depicts the estimated GIRFs for the CPI based real exchange rates for every country in my sample for each of the models I employ. The figure depicts the estimated GIRFs with the size of the shock set equal to 25% and $h=60$. Each model tends to generate a hump shaped impulse response function that is common to a large set of macroeconomic variables. Figure 2 suggests that the ARFIMA model tends to have difficulty producing an impulse response function for the real exchange rate that decays rapidly enough to generate half lives of a reasonable magnitude. Given the hyperbolic nature of the ARFIMA model, the impulse response functions tend to flatten out after about 10 periods, suggesting little support for PPP. The introduction of non-linearity, even when long memory is present, produces a dramatically different picture. With the exception of Canada, the non-linear impulse response functions decay rapidly, and it is also interesting to note that the long memory non-linear model

⁸ To be consistent, I chose to truncate the first 100 observations even when I calculate the impulse response functions for the ESTAR models. The estimated GIRF is calculated in precisely the same way for the ESTAR model as the FI-ESTAR model with the first quantity on the right hand side of equation (14) set equal to 0.

can produce impulse response functions that decay faster at some horizons than a model in which d is restricted to be zero (e.g. the UK, Germany, and eventually Canada).

To formally assess if the estimated models can mitigate the half lives of shocks to the real exchange rate, I report the computed half lives for each estimated model for every country in Table 7.⁹ Since the half lives calculated for the linear ARFIMA models are independent of the size of the shock, I simply report their half lives on the right hand side of the table. For the impulse response functions for the ARFIMA model, I calculate up to 1,000,000 coefficients in the MA representation of the model. For the estimated ARFIMA models for France and the UK, the initial shock had not dissipated by half even after 1,000,000 values were calculated. For the remaining countries, the half lives range from over 11 years (for Canada) to over 11,363 years (for Italy). Thus, the estimates recorded here strongly suggest that the linear ARFIMA model is incapable of reducing the estimated half lives of shocks to the real exchange rate. Either PPP is highly questionable, or linear long memory ARFIMA models are too restrictive.¹⁰

For the non-linear models, we see that the estimated half lives have been dramatically reduced relative to the results employing the linear models when non-linearity is introduced for every real exchange rate except Canada. Given the computational complexity affiliated with

⁹ Because I wish to compare the estimated half lives across different models, I do not restrict the parameter space for any of the models. Previous authors (e.g. Taylor, Peel, and Sarno, 2001, and Paya, Venetis, and Peel, 2003) using STAR models impose the reasonable restriction that the inner regime is characterized by a unit root, and is more persistent. Without this restriction, it is possible to get half lives that increase with the size of the shock.

¹⁰ I also estimated d using an alternative estimator, namely the bias reduced log periodogram regression based estimator of Andrews and Guggenberger (2003). The results, which are available upon request, show that the estimate of d ranges from 0.7877 to 1.2617 with strong concentration near the unit root boundary. The results show the estimated half lives are likely robust to the selected estimator.

calculating the GIRF, I set the maximum value of $h=1,000$. In the case of Canada with the ESTAR model, the estimated impulse response function has not been reduced by half even after 1,000 values have been calculated. With the implementation of the ESTAR model, we are able to generate half lives less than 3 years for each EU country for at least one shock size. In fact, the use of the non-linear model goes a long way to resolving the PPP paradox, as the half lives to shocks can be as low as 1.7 years (Germany with a 1% shock size). When the FI-ESTAR model is employed, we are able to generate half lives of less than 3 years for the real exchange rates of France and the UK. For example, the non-linear model with fractional integration produces a half life of 1.8 years for the UK with a shock size equal to 25%. Further, the restriction $d=0$ does not necessarily yield smaller half lives. For at least one shock size, the estimated half lives of the FI-ESTAR model are smaller than the half lives with the restriction $d=0$ for every real exchange rate except Italy's. For example, for the French real exchange rate, the maximum half life for the FI-ESTAR model is 50 months, as compared to 79 months for the estimated ESTAR model. As a final point, I also estimated the non-linear models and associated half lives with the restriction that the constants in both regimes are zero. Although the results are similar to those reported in Table 7, it should be pointed out that the estimated half lives were generally smaller with this restriction. In fact, for each EU country and Japan, either the ESTAR or FI-ESTAR model was able to generate half lives of 2 or fewer years for at least one shock size. For example, with a shock size of 1%, the estimated half life for France and Germany was 24 and 22 months respectively when the FI-ESTAR model was employed.

VI. Conclusion

Long memory and non-linear models have proven to be important tools in analyzing the properties of a wide range of macroeconomic variables. It has become clear with the work of

Diebold and Inoue (2001) that the two modeling procedures may capture the same phenomena. While it is clear that long memory and non-linearity can be viewed as substitutes for each other, an important question that remains largely unanswered relates to the consequences of the modeling choice in addressing important economic questions. Since the two models can produce similar dynamics, it might seem as though the modeling choice is unimportant. This paper refutes this assertion and clearly demonstrates that, depending on the application, it might be quite important to carefully distinguish between long memory and non-linearity.

In this paper, I consider a model that allows the user to jointly test for long memory and non-linearity in the real exchange rates of the G-7 countries vis-à-vis the US. Interestingly, there is evidence of both long memory and non-linearity in the dynamics related to PPP for each of the EU countries in my sample, with little evidence of non-linearity for Canada and Japan. The use of long memory in isolation produces a bleak picture for the theory PPP, since the parameterizations that result from estimation of an ARFIMA model tend to be non-stationary with an estimated integration order, d , near unity.

In terms of impulse response analysis, I estimated median unbiased estimates of half lives based on linear AR models. The evidence is consistent with previous analysis with regard to the PPP paradox, as half lives range from 49 months to an infinite number of months. While the linear AR models are incapable of generating half lives of a reasonable magnitude, the linear long memory models do far worse. In fact, the impulse response functions for the estimated ARFIMA models decay slowly and produce half lives that can exceed 1,000,000 months. In contrast, the introduction of non-linearity through an ESTAR model dramatically reduces the half lives. The non-linear models are able to produce half lives of less than 3 years for each EU country in my sample when the order of integration, d , is constrained to zero. Constraining d to

be zero, which is generally rejected in the data, does not necessarily produce impulse response functions that decay faster than a model in which d is allowed to be positive. The results suggest that the use of a model with both long memory and non-linearity can be important for analyzing the dynamic behavior of macroeconomic data.

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FIGURES

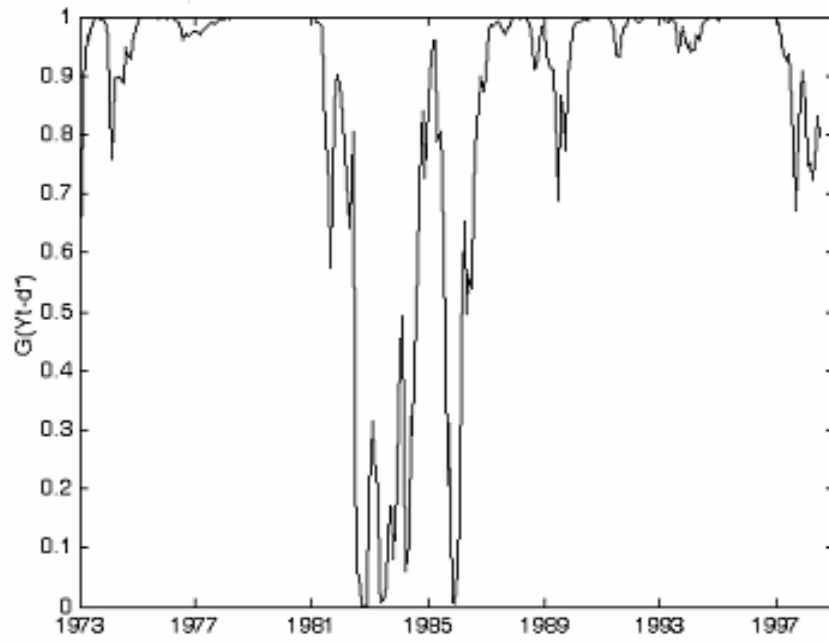


Figure 1
Estimated Transition Function from the FI-ESTAR Model
CPI Based French Real Exchange Rate

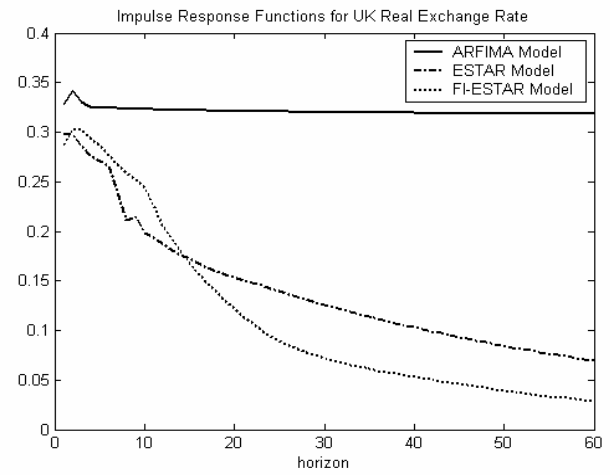
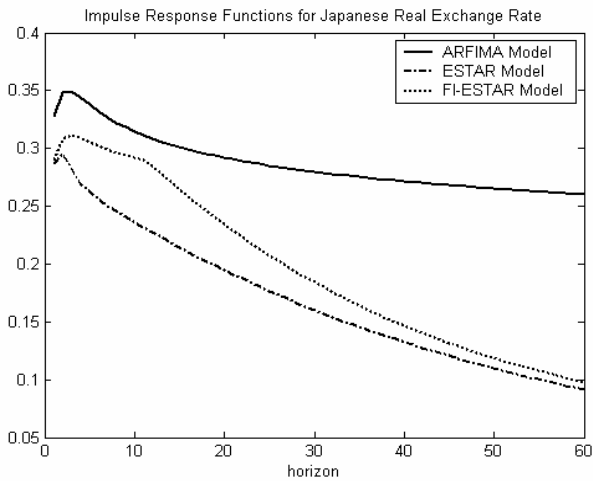
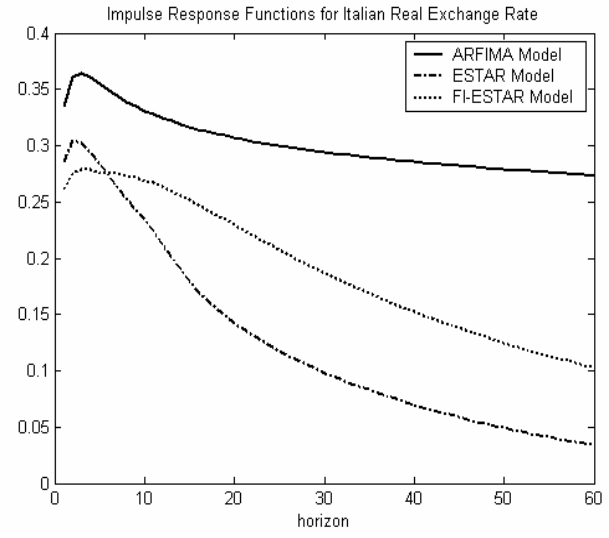
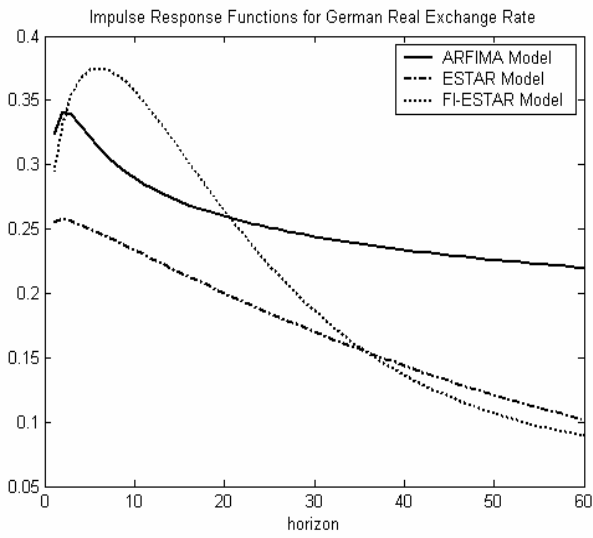
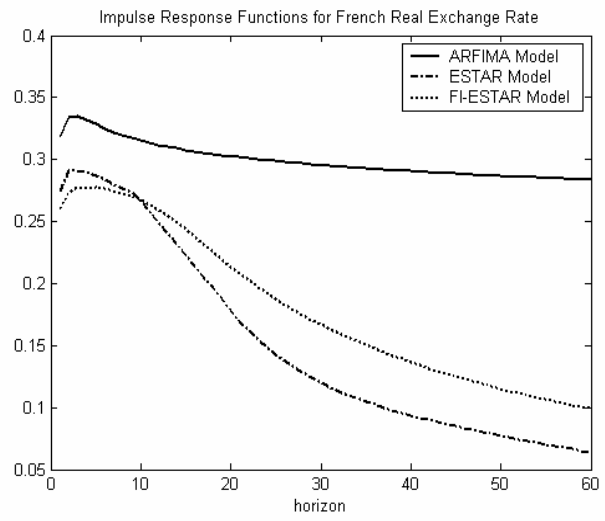
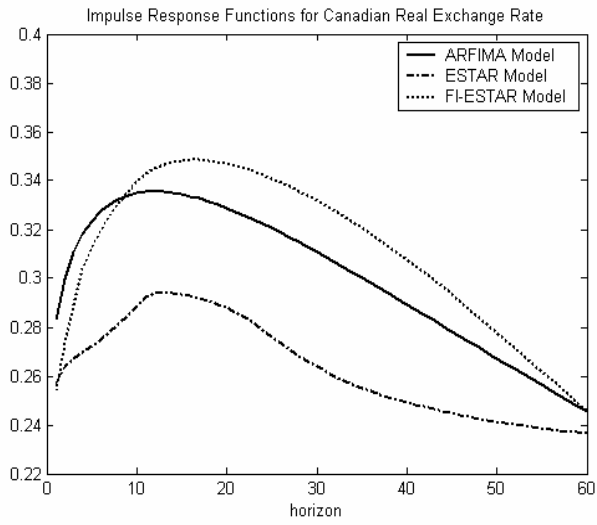


Figure 2
 Estimated Impulse Response Functions for ARFIMA, ESTAR, and FI-ESTAR Models
 Size of the Shock is Equal to 25% (with No Restrictions on the Estimated Models)

Table 1
Results for Estimation of ARFIMA (p, d, q) Model
Real Exchange Rates

Country	Canada	France	Germany	Italy	Japan	UK
ϕ_1	0.9868 [0.0130]	0.3326 [0.1686]	0.4466 [0.1811]	0.4426 [0.2025]	0.4141 [0.1564]	0.3721 [0.1414]
ϕ_2	N/A	N/A	N/A	N/A	N/A	-0.1254 [0.0776]
d	0.1445 [0.0667]	0.9411 [0.1424]	0.8499 [0.1599]	0.8977 [0.1798]	0.8961 [0.1361]	0.9910 [0.1278]
μ	0.2029 [0.1474]	-0.0929 [0.0590]	-0.1208 [0.0756]	-0.0314 [0.0615]	-0.1433 [0.0752]	-0.0375 [0.0464]
Ex. Kurt.	0.5579	1.0156	0.4533	0.9481	0.7445	1.5365
Skewness	0.0105	0.2772	0.0086	0.2444	-0.4851	-0.1009
Pval- Q (5)	0.6310	0.1961	0.8356	0.8779	0.6014	0.7176
Pval- Q (10)	0.0734	0.5147	0.6341	0.8607	0.4613	0.8470
Pval- Q (20)	0.0179	0.7511	0.9055	0.8199	0.1389	0.2112
Pval-Q ² (1)	0.5900	0.6225	0.0995	0.6700	0.4017	0.0083
Pval-Q ² (5)	0.0040	0.8309	0.0613	0.8232	0.0225	0.0002
Pval-Q ² (10)	0.0006	0.4966	0.1424	0.4939	0.0618	0.0010
AIC	-8.9011	-7.2939	-7.2046	-7.4322	-7.1482	-7.4113
SSE	0.0506	0.2073	0.2267	0.1806	0.2917	0.2231

Notes : The table yields the parameter estimates for an ARFIMA($p, d, 0$) model using the estimator of Beran (1995). Numerical standard errors using the outer product of the gradient are reported in brackets. P-valQ and Pval-Q² are the p-values for the Ljung-Box Q-statistics for the hypothesis of serial correlation and heteroskedasticity in the residuals.

Table 2
Testing Cycle for ESTAR(p) Model
Real Exchange Rates

	LM χ^2	LM F	p value χ^2	p value F	Selected delay	Number of AR Params
Canada	5.1553	1.2822	0.2717	0.2765	12	2
France	7.5837	1.8997	0.1081	0.1105	10	2
Germany	9.4164	2.373	0.0515	0.0523	1	2
Italy	10.1098	2.5544	0.0386	0.0391	10	2
Japan	3.4645	0.8579	0.4833	0.4894	6	2
UK	14.5968	2.4642	0.0236	0.0239	7	3

Notes : The table reports the χ^2 and F versions of the LM test statistics and p-values for the linearity tests employing the testing cycle of Teräsvirta (1994). The results reported here are for the level of the log of the real exchange rate.

Table 3
 ESTAR (p) Model Selected via Teräsvirta Rule
 Real Exchange Rates

Coefficient	Canada	France	Germany	Italy	Japan	UK
$\phi_0^{(1)}$	-0.0725 [0.1584]	-0.0380 [0.0344]	-0.0172 [0.0166]	-0.012 [0.0234]	-1.8269 [1.0043]	-0.0128 [0.0674]
$\phi_1^{(1)}$	1.4182 [0.6741]	0.2925 [0.5181]	1.4001 [0.1533]	0.5493 [0.6013]	1.7965 [0.6351]	-1.0347 [1.2091]
$\phi_2^{(1)}$	-0.2631 [0.5931]	0.6196 [0.5225]	-0.497 [0.1411]	0.6462 [0.7059]	-2.8921 [1.5855]	0.4447 [0.7806]
$\phi_3^{(1)}$	N/A	N/A	N/A	N/A	N/A	0.6004 [0.5874]
$\phi_0^{(2)}$	0.0738 [0.1585]	0.0384 [0.0344]	0.0175 [0.0164]	0.0099 [0.0236]	1.8193 [1.0043]	0.0107 [0.0681]
$\phi_1^{(2)}$	-0.2763 [0.6896]	0.9840 [0.5245]	-0.2836 [0.2180]	0.7913 [0.6077]	-0.5196 [0.6437]	2.4077 [1.2114]
$\phi_2^{(2)}$	0.1205 [0.6121]	-0.9032 [0.5307]	0.3627 [0.1959]	-1.0239 [0.7097]	2.6002 [1.5893]	-0.9507 [0.8055]
$\phi_3^{(2)}$	N/A	N/A	N/A	N/A	N/A	-0.4788 [0.6027]
γ	13.1639	4.8877	2.4893	31.2004	573.0928	5.6711
$1/\sigma_s^2$	60.4457 [32.0006]	43.1312 [5.3350]	38.3587 [2.8419]	45.5357 [26.2536]	21.0108 [380.0337]	60.6561 [3.1588]
c	0.4374 [0.0689]	0.3340 [0.0425]	-0.1569 [0.0389]	0.1990 [0.0079]	-0.8557 [0.0021]	0.1996 [0.0126]
Ex. Kurt.	0.3768	1.1082	0.1775	0.5706	0.8326	0.8509
Skewness	0.0255	0.3283	-0.0381	0.1331	-0.5165	0.1282
Pval- Q (5)	0.9326	0.3017	0.8416	0.9357	0.6030	0.8972
Pval- Q (10)	0.1121	0.5885	0.7811	0.7642	0.4861	0.7276
Pval- Q (20)	0.0755	0.7936	0.9114	0.8495	0.0577	0.0517
Pval-Q ² (1)	0.3001	0.4630	0.3638	0.2726	0.5184	0.0020
Pval-Q ² (5)	0.1550	0.9782	0.0770	0.3738	0.0677	0.0016
Pval-Q ² (10)	0.0364	0.9287	0.2239	0.4593	0.0741	0.0065
Pval No Rem. Nonlinear	0.7835	0.1608	0.2719	0.1167	0.9995	0.0870
AIC	-8.8870	-7.3489	-7.2128	-7.4699	-7.1817	-7.4669
SSE	0.0484	0.1842	0.2170	0.1633	0.2710	0.2010

Notes: The results here yield estimates of the ESTAR(p) model with no fractional differencing:

$$y_t = \phi_{1,0} + \phi_{1,1}y_{t-1} + \dots + \phi_{1,p}y_{t-p} + (\phi_{2,0} + \phi_{2,1}y_{t-1} + \dots + \phi_{2,p}y_{t-p})(1 - \gamma[y_{t-d} - c]^2) + \varepsilon_t,$$

where y_t denotes the log of the real exchange rate. The estimated model is based on the results in Table 2.

P-valQ and Pval-Q² are the p-values for the Ljung-Box Q-statistics for the hypothesis of serial correlation and heteroskedasticity in the residuals. P-val No Rem. Nonlinear records the probability value for the hypothesis of no remaining non-linearity as discussed by Eitrheim and Teräsvirta (1996) against an ESTAR alternative.

Table 4
 Tests Results for Linearity for the Fractionally Differenced Series
 Real Exchange Rates

	Differencing Parameter Estimated Under the Null		Differencing Parameter Estimated Under the Alternative		Selected delay
	LM χ^2	p value χ^2	LM χ^2	p value χ^2	
Canada	2.5401	0.2808	2.8278	0.2432	5
France	1.7172	0.4237	6.4786	0.0392	6
Germany	8.4053	0.0150	11.9704	0.0025	1
Italy	4.4460	0.1083	6.8969	0.0318	6
Japan	1.8446	0.3976	1.7617	0.4144	12
UK	3.5212	0.4747	11.4613	0.0218	11

Notes : This table reports the results of the χ^2 version of the LM tests for linearity against the fractionally differenced ESTAR model. The differencing parameter has been estimated in two ways. First, it is estimated under the null using an ARFIMA model as discussed by van Dijk, Franses and Paap (2002) and extended by Smallwood (2005). Alternatively, it has been estimated using the full FI-ESTAR model under the alternative. The selected delay has been obtained under the null.

Table 5
FI-ESTAR Estimation Results
Real Exchange Rate

Coefficient	Canada	France	Germany	Italy	Japan	UK
$\phi_0^{(1)}$	-0.1641 [0.0034]	0.0014 [0.0055]	-0.0292 [0.0035]	-0.2905 [0.0171]	0.0065 [0.0062]	-0.0416 [0.0173]
$\phi_1^{(1)}$	1.7801 [0.1367]	1.0356 [0.2381]	0.3494 [0.1828]	-2.5870 [0.2805]	0.4653 [0.2160]	0.2918 [0.3051]
$\phi_2^{(1)}$	N/A	N/A	N/A	N/A	N/A	0.5340 [0.2804]
$\phi_0^{(2)}$	0.1657 [0.0038]	-0.0059 [0.0063]	0.0270 [1.0711]	0.2898 [0.0170]	-0.0113 [0.0067]	0.0385 [0.0192]
$\phi_1^{(2)}$	-0.7929 [0.1349]	-0.1538 [0.2386]	0.5731 [59.0080]	3.5453 [0.2625]	-0.1523 [0.0067]	1.0755 [0.1927]
$\phi_2^{(2)}$	N/A	N/A	N/A	N/A	N/A	-0.9143 [0.2052]
γ	269.5053	1.2744	3.9135	326.5277	0.8623	5.499
$1/\sigma_s^2$	60.4457 [47.4677]	43.5493 [5.2323]	38.4246 [0.6716]	46.0501 [38.6125]	20.5398 [1.7786]	60.3479 [5.6230]
c	0.3993 [0.0133]	0.1400 [0.0324]	-0.3513 [0.0548]	-0.2576 [0.0155]	-0.8203 [0.1558]	0.1072 [0.0244]
d	0.1277 [0.0821]	0.2559 [0.1208]	0.3965 [0.1385]	0.2559 [0.1395]	0.9317 [0.1394]	-0.0475 [0.2487]
Ex. Kurt.	-0.1849	0.8952	0.2341	0.3132	0.7455	1.0342
Skewness	0.2124	0.3348	-0.0705	-0.0580	-0.4986	-0.0019
Pval- Q (5)	0.5115	0.1618	0.5470	0.0476	0.4442	0.2623
Pval- Q (10)	0.0321	0.4082	0.4640	0.1350	0.3375	0.4101
Pval- Q (20)	0.0049	0.5114	0.7546	0.2213	0.1606	0.0451
Pval-Q ² (1)	0.1973	0.8302	0.9127	0.3014	0.4628	0.0054
Pval-Q ² (5)	0.3544	0.9681	0.6207	0.4184	0.0705	0.0001
Pval-Q ² (10)	0.3361	0.5762	0.6322	0.3273	0.0968	0.0004
Pval No Rem. Nonlinear	0.0726	0.3940	0.5695	0.0684	0.8698	0.1333
AIC	-8.9173	-7.3070	-7.2134	-7.4480	-7.1383	-7.4017
SSE	0.0480	0.1954	0.2183	0.1697	0.2790	0.2126

Notes: The results here yield estimates of the FI-ESTAR model:

$$(1-L)^d y_t = \phi_{1,0} + \phi_{1,1}(1-L)^d y_{t-1} + \dots + \phi_{1,p}(1-L)^d y_{t-p} + (\phi_{2,0} + \phi_{2,1}(1-L)^d y_{t-1} + \dots + \phi_{2,p}(1-L)^d y_{t-p})(1 - \gamma[y_{t-d} - c]^2) + \varepsilon_t,$$

where y_t denotes the log of the real exchange rate. The estimated model is based on the test results in Table 4. P-valQ and Pval-Q² are the p-values for the Ljung-Box Q-statistics for the hypothesis of serial correlation and heteroskedasticity in the residuals. P-val No Rem. Nonlinear records the probability value for the hypothesis of no remaining non-linearity as discussed by van Dijk, Franses, and Paap (2002).

Table 6
Results for Estimation of the ADF Model with Implied Median Unbiased Parameter Estimates
Real Exchange Rates

Country	Canada	France	Germany	Italy	Japan	UK
α_{LS}	0.9938 [0.0047]	0.9779 [0.0098]	0.9799 [0.0100]	0.9792 [0.0094]	0.9855 [0.0066]	0.9775 [0.0099]
c	0.0016 [0.0010]	-0.0022 [0.0018]	-0.0023 [0.0019]	-0.0003 [0.0014]	-0.0076 [0.0034]	-0.0045 [0.0021]
ζ_{1LS}	0.1810 [0.0510]	0.3009 [0.0565]	0.2996 [0.0540]	0.3502 [0.0534]	0.3099 [0.0485]	0.3678 [0.0513]
ζ_{2LS}	N/A	-0.0931 [0.0587]	N/A	N/A	N/A	-0.1114 [0.0517]
ζ_{3LS}	N/A	0.1531 [0.0564]	N/A	N/A	N/A	N/A
Half Lives _{LS} (in months)	118	34	37	36	51	33
α_{MU}	1.0000	0.9860	0.9890	0.9870	0.9930	0.9850
ζ_{1MU}	0.1787	0.2947	0.2952	0.3463	0.3084	0.3662
ζ_{2MU}	N/A	-0.1017	N/A	N/A	N/A	-0.1171
ζ_{3MU}	N/A	0.1494	N/A	N/A	N/A	N/A
Half Lives _{MU} (in months)	∞	53	67	57	105	49
Ex. Kurt.	0.6595	1.0426	0.4078	0.6988	3.7333	0.1848
Skewness	-0.0401	0.3496	0.0316	0.2317	-0.5020	-0.0908
Pval- Q (5)	0.9310	0.8686	0.8398	0.8947	0.5766	0.6573
Pval- Q (10)	0.0934	0.8580	0.6760	0.8440	0.4794	0.8208
Pval- Q (20)	0.0193	0.9506	0.9235	0.8396	0.1408	0.1958
Pval-Q ² (1)	0.6586	0.6633	0.0607	0.7860	0.3463	0.0061
Pval-Q ² (5)	0.0094	0.8702	0.0129	0.8199	0.0379	0.0000
Pval-Q ² (10)	0.0865	0.4104	0.0412	0.5163	0.0805	0.0000
SSE	0.0504	0.1976	0.2243	0.1781	0.2887	0.2198
Implied AR(p) Model in Levels						
	Canada	France	Germany	Italy	Japan	UK
ϕ_1	1.1787	1.2807	1.2842	1.3333	1.3014	1.3512
ϕ_2	-0.1787	-0.3964	-0.2952	-0.3463	-0.3084	-0.4833
ϕ_3	N/A	0.2511	N/A	N/A	N/A	0.1171
ϕ_4	N/A	-0.1494	N/A	N/A	N/A	N/A

The specification used here is given by:

$$r_t = c + \alpha_{LS} r_{t-1} + \zeta_1 \Delta r_{t-1} + \dots + \zeta_{p-1} \Delta r_{t-p+1} + u_t$$

Notes: The table yields the parameter estimates for a ADF regression with a constant. P-valQ and P-valQ² are the p-values for the Ljung-Box Q-statistics for the hypothesis of serial correlation and heteroskedasticity in the residuals. Half Lives_{LS} denotes the half-lives based on the least squares estimate of α (α_{LS}), which is given by the number of months for the initial shock to decrease by 1/2. Half Lives_{MU} denotes the same quantity for the median unbiased estimate for α (α_{MU}). The median unbiased parameters have been calculated computationally where α ranges along a grid from -0.999 to 1 with a step size of 0.001. See Andrews and Chen (1994) for details.

Table 7
Estimated Half-Lives for Unrestricted Models
Real Exchange Rates

<u>Country:</u>	<u>Shock Size</u>	ESTAR Model					<u>ARFIMA Model</u>
		1%	10%	25%	40%	50%	
Canada	1,000+	1,000+	1,000+	1,000+	1,000+	1,000+	139
France	79	45	33	27	31	1,000,000+	
Germany	20	48	53	58	57	2,565	
Italy	25	24	26	23	22	136,366	
Japan	57	53	50	48	48	69,521	
UK	55	50	36	24	23	1,000,000+	

<u>Country:</u>	<u>Shock Size</u>	FI-ESTAR Model					<u>ARFIMA Model</u>
		1%	10%	25%	40%	50%	
Canada	110	110	106	104	108	139	
France	33	49	50	38	29	1,000,000+	
Germany	40	45	47	51	53	2,565	
Italy	68	58	56	56	56	136,366	
Japan	37	41	54	67	74	69,521	
UK	33	27	22	26	30	1,000,000+	

Notes : The table reports the estimated half lives of shocks to the CPI based real exchange rates for the estimated estimated ARFIMA, ESTAR, and FI-ESTAR models reported in Tables 1, 3, and 5. The half lives are calculated using the estimated impulse response functions (see equations (12)-(14) in the text).